Some applications of the ECE metric theory: electron Sagnac Effect, Tomita Chiao Effect and Faraday disk.

by

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Abstract

The ECE metric method has been developed in papers 145 onwards of this series and is based on the ECE orbital theorem of paper 111. All metrics of a spherically symmetric spacetime in four dimensions obey this theorem. The metric method is used in this paper to derive the electron Sagnac effect, first observed in the mid nineties, the Tomita Chiao effect, the first observation of the Berry phase, and the Faraday disk effect, first observed in 1837.

Keywords: ECE metric method, electron Sagnac effect, Tomita Chiao effect, Berry phase, Faraday disk.

1. Introduction

In paper 111 of this series [1-10] the orbital theorem of Einstein Cartan Evans (ECE) unified field theory was derived straightforwardly for a spherically symmetric spacetime in four dimensions. Familiar metrics such as the Minkowski metric and the mis-called "Schwarzschild" metric [11] are solutions of this theorem. In ECE theory this metric is known as the gravitational metric and is derived not from the incorrect [1-10] Einstein field equation, but from the orbital theorem. In papers 145 onwards of the series the Minkowski metric has been rotated to give the Thomas precession [12], and the gravitational metric has been rotated to give the de Sitter precession [13] or geodetic effect. The photon Sagnac effect [14] has been shown to be the Thomas precession for a null geodesic and the effect of gravitation on the Sagnac phase has been worked out using the rotating gravitational metric in the null geodesic condition for a plane. This theory has produced new and very accurate instrument designs such as gravimeters, a speedometer and an altimeter, and an interferometer system capable of measuring the effect of mechanical macroscopic rotation on spectra. These effects and instruments are consequences of the orbital theorem using only two of the simplest possible solutions thereof. There are many other solutions which are as yet undeveloped and / or unknown. For example there are in principle electromagnetic metrics as well as gravitational metrics, because the structure of dynamics and electrodynamics in ECE theory is the same. So the metrical structures of dynamics and electrodynamics must be the same also. Using the rotated gravitational metric, the effect of gravitation on several types of instrument can be worked out straightforwardly.

In Section 2 the Tomita Chiao effect [15] is derived straightforwardly from a static Minkowski metric in the null geodesic condition, and the effect of rotating the metric calculated. The Tomita Chiao effect is the rotation of the plane of polarization of light after propagating through a helical optical fibre and is generally regarded [1-10] as the first observation of the Berry phase. It is closely related to the photon Sagnac effect as becomes clear from a simple consideration of the static and rotating Minkowski metric. Using this theory, a design is given for a high accuracy and compact fibre optic gyro. The effect of gravitation on the Sagnac and Tomita Chiao effects is developed using an adaptation of the rotating gravitational metric. The calculations are straightforward but give incisive results and useful instrument designs. A comparison is given of the rotating frame ECE theory of the Sagnac effect, given in earlier papers, and the rotating metric method, showing that both methods are equivalent.

In Section 3 the electron Sagnac effect [16] is derived using a static and rotating Minkowski metric without use of the null geodesic condition, i.e. by using the metric for a particle with finite mass such as the electron. The null geodesic condition applies to the hypothetical "massless" particle, the photon travelling at the speed of light c in a vacuum in received opinion. The Sagnac effect for electrons was first observed experimentally in the mid nineties [16] and relies on the de Broglie wave particle dualism in that the electron is a wave as well as a particle. A classical method is sufficient to derive a phase shift for a rotating electron beam. There is an angular velocity intrinsic to the Minkowski metric, and given for a particle with mass by v / r, where v is the tangential linear velocity of the particle (electron for example) going around a circle of radius r in a plane. This angular velocity

exists in the absence of platform rotation, which increases it. The phase induced by this angular velocity is then vt/r, where t is the time taken, for example, to traverse the circumference of the circle. This time interval can be measured directly in a high accuracy instrument. This phase affects the wavefunction of the electron, and the effect of gravitation on the rotating electron can be worked out straightforwardly using the rotating gravitational metric and measured by direct timing, giving another high accuracy instrument design. For the photon, the angular velocity intrinsic to the static Minkowski metric is c/r, not v/r.

In Section 4 a theory of the Faraday disk [1-10] is developed using a combination of the metric method and the ECE field theory. In so doing the Einstein energy equation and relativistic Hamilton Jacobi equation are derived from the Minkowski metric in order to show that mechanical rotation has an effect on the vector potential of electrodynamics. This theory is applied to the Faraday disk whose experimental characteristics are first summarized. The metrical part of the theory allows the effect of extra mechanical rotation and of gravitation to be worked out for the Faraday disk. This theory results in the design of an instrument in which the effect of gravitation is measured through the voltage of the Faraday disk generator. The general theory can be applied to different types of instrument in which mechanical rotation is combined with electromagnetic components of any type.

2. Derivation of the Tomita Chiao Effect.

Consider the Minkowski metric in cylindrical polar coordinates (see papers 145 and 146 on www.aias.us and accompanying notes):

$$ds^2 = c^2\, dt^2 - d\,r^2 - r^2\, d\varphi^2 - dZ^2 \quad . \tag{1}$$

The usual definitions of the cylindrical polar system are used [17]:

$$X = r \cos \varphi \quad , \quad Y = r \sin \varphi \quad , \quad Z = Z \quad . \tag{2}$$

The circular helix is parameterized by [17]:

$$Z = Z_0\, \varphi \tag{3}$$

where the pitch of the helix is:

$$p = 2\pi Z_0 \quad . \tag{4}$$

The pitch is the distance along the helical axis (Z) that results in one full turn of the helix. Therefore the helical metric is:

$$ds^2 = c^2\, dt^2 - d\,r^2 - (r^2 + Z_0{}^2)d\varphi^2 \tag{5}$$

for one turn of the helix. For n turns:

$$R := n\, Z_0 \tag{6}$$

and:

$$ds^2 = c^2 dt^2 - d\,r^2 - (r^2 + R^2)\,d\varphi^2 \quad . \tag{7}$$

Consider electromagnetic radiation propagating through a helical optical fibre. In the particulate representation of the de Broglie wave particle dualism, there is a photon propagating at c, so the metric has a null geodesic:

$$ds^2 = 0 \quad . \tag{8}$$

The radius of the fibre is considered to be a constant, so:

$$d\,r^2 = 0 \quad . \tag{9}$$

Therefore:

$$c^2 dt^2 = (r^2 + R^2) d\varphi^2 \tag{10}$$

and the intrinsic angular frequency of the metric is:

$$\omega = \frac{d\varphi}{dt} = \frac{c}{(r^2 + R^2)^{\frac{1}{2}}} \tag{11}$$

thus defining a phase angle in radians:

$$\theta_R = \omega\, t \tag{12}$$

where t is time, for example the time taken to transcribe the circumference of a circle of radius r. This is the time taken to cover $2\pi r$, and can be measured directly with a contemporary digital timer. This time interval is given by:

$$t = \frac{2\pi}{\omega} = \frac{1}{c}\, 2\pi\, (r^2 + R^2)^{\frac{1}{2}} \quad . \tag{13}$$

From Eqs. (11) and (12) it is seen that as

$$R \longrightarrow \infty \tag{14}$$

then

$$\omega \longrightarrow 0 \quad . \tag{15}$$

This means that if the helical fibre is drawn out into a straight line the phase θ disappears.

On the other hand if

$$R \longrightarrow 0 \tag{16}$$

then

$$\omega \longrightarrow \frac{c}{r} \qquad (17)$$

which is the Sagnac effect for platform at rest (see paper 145).

Eq. (11) and the phase (12) produce a rotation of the plane of polarization of light propagating through the helical optical fibre - the Tomita Chiao effect [1-10] or Berry phase. This effect is shown as follows. Consider the linearly polarized unit vector:

$$e_l^{(1)} = e^{(1)} (\exp(i\,\theta) + \exp(-i\,\theta)) \qquad (18)$$

i.e.

$$\text{Real}\,(e_l^{(1)}) = 2\,i\,\cos\theta \qquad (19)$$

where the electromagnetic phase is:

$$\theta = \omega\,t - \kappa Z \qquad (20)$$

where ω is the electromagnetic angular frequency at instant t, and where κ is the wave-vector at point Z. The phase (12) results in:

$$e_l^{(1)\,\prime} = e^{i\theta_R}\,e_l^{(1)} \qquad (21)$$

where:

$$e^{(1)} = \frac{1}{\sqrt{2}}\,(i - ij) \qquad (22)$$

is the unit vector of the complex circular system [1-10]. Therefore:

$$e_l^{(1)\,\prime} = \frac{1}{\sqrt{2}}\,(i - ij)(\exp(i\,(\theta + \theta_R)) + \exp(-i\,(\theta - \theta_R)) \qquad (23)$$

where:

$$\exp(i\,(\theta + \theta_R)) = \cos(\theta + \theta_R) + i\,\sin(\theta + \theta_R) \qquad (24)$$

$$\exp(-i\,(\theta - \theta_R)) = \cos(\theta - \theta_R) - i\,\sin(\theta - \theta_R)\quad. \qquad (25)$$

Using the formulae [17]:

$$\cos(\theta \pm \theta_R) = \cos\theta\,\cos\theta_R \mp \sin\theta\,\sin\theta_R\quad, \qquad (26)$$

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and

$$\sin(\theta \pm \theta_R) = \sin\theta \cos\theta_R \pm \cos\theta \sin\theta_R \quad , \tag{27}$$

then

$$e^{i(\theta+\theta_R)} + e^{-i(\theta-\theta_R)} = 2(\cos\theta_R - i\sin\theta_R)\cos\theta \tag{28}$$

and

$$e_l^{(1)'} = \frac{2}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})(\cos\theta_R - i\sin\theta_R)\cos\theta \tag{29}$$

so

$$\text{Real}\,(e_l^{(1)'}) = \frac{2}{\sqrt{2}}(\boldsymbol{i}\cos\theta_R - \boldsymbol{j}\sin\theta_R) \quad . \tag{30}$$

Comparing Eqs. (19) and (30) it is seen that the phase θ_R has rotated the plane of light after it has propagated through the helical optical fibre, and this is precisely what is meant by the Tomita Chiao effect. The latter has been derived by considering simply the static Minkowski metric, and has been related to the Sagnac effect for a static platform. These results illustrate the elegance of the metric method.

If the helical metric is rotated such that:

$$\mathrm{d}\varphi \longrightarrow \mathrm{d}\varphi \mp \Omega\,\mathrm{d}t \tag{31}$$

where the angular velocity Ω is defined by

$$\Omega = \frac{v}{r} \tag{32}$$

then:

$$\frac{d\varphi}{dt} = \frac{c}{r_1} \pm \frac{v}{r} \tag{33}$$

where

$$r_1 = (r^2 + R^2)^{\frac{1}{2}} \quad . \tag{34}$$

Thus:

$$\frac{dt}{d\varphi} = \frac{rr_1}{rc \pm r_1 v} \tag{35}$$

and for a 2π rotation of φ :

$$t = \frac{2\pi r r_1}{rc \pm r_1 v} \quad .$$
(36)

The difference in this time interval for clockwise and counter clockwise rotation is:

$$\Delta t = 2\pi r r_1 \left(\frac{1}{rc - r_1 v} - \frac{1}{rc + r_1 v} \right) = \frac{\Omega Ar}{\left(c - \frac{r_1}{r} v \right)\left(c + \frac{r_1}{r} v \right)}$$
(37)

where the area Ar is defined by:

$$Ar := \pi r_1{}^2 = \pi (r^2 + R^2) \quad .$$
(38)

This result can be expressed as:

$$\Delta t = \frac{\Omega Ar \cos\lambda}{(c\cos\lambda - v)(c\cos\lambda + v)}$$
(39)

where the cosine is defined by:

$$\cos\lambda = \frac{r}{(r^2 + R^2)^{1/2}} \quad .$$
(40)

If:

$$c \gg v$$
(41)

then

$$\Delta t \longrightarrow \frac{\Omega Ar}{c^2} \frac{1}{\cos\lambda} = \frac{\pi\Omega}{c^2} \left(\frac{(r^2 + R^2)^{3/2}}{r} \right)$$
(42)

and if

$$r \gg R$$
(43)

the result reduces to the Sagnac effect:

$$\Delta t = \frac{\Omega\pi r^2}{c^2} \quad .$$
(44)

These equations are the basis for the design of a high accuracy and compact fibre optic gyro constructed by winding many turns of a fibre optic on a drum. Its effective area is

$$Ar = \pi(r^2 + n^2 Z_0{}^2) \quad .$$
(45)

As n and Z_0 become very large, the instrument has a very large effective area, yet is compact and practical, capable of measuring a very small Ω, or angular resolution, and being very sensitive to rotation. It is a rotating Tomita Chiao effect.

This ECE metric method allows the effect of gravitation on light to be worked out theoretically and observed in the laboratory in a much simpler way than the usual relativistic Kepler problem [1-10] leading to light bending (see earlier paper of this series [1-10]), orbital ecliptic precession, gravitational red-shift, geodetic precession, binary pulsar phenomena and so on. The Minkowski metric is simply replaced by the gravitational metric in cylindrical polar coordinates:

$$ds^2 = x^2 c^2 dt^2 - \frac{dr^2}{X^2} - r^2 d\varphi^2 - dZ^2 \tag{46}$$

where

$$x = (1 - \frac{2MG}{c^2 R})^{\frac{1}{2}} \quad . \tag{47}$$

Here G is Newton's constant, M is a gravitating mass, R is the distance between that mass and the photon or electron under consideration in the Sagnac and Tomita Chiao effects. When considering light, the null geodesic method is used:

$$ds^2 = 0 \quad . \tag{48}$$

This is the same procedure as used in the well known theory of light bending, as has been shown in comprehensive detail in earlier ECE papers (see www.aias.us). Intrinsic in this theory is the use of a photon mass m, attracted gravitationally by M. Rigorously therefore, the null geodesic condition is an approximation, because it is true exactly only of the mass m is zero, in which case no gravitational attraction between m and M can occur. The photon mass m is finite, but very small, and has not yet been measured experimentally.

One of the important by products of ECE theory [1-10] is the realization that the Einstein field equation is irretrievably incorrect because it uses an incorrect symmetric connection (ECE papers 122 ff.). It is quite simple to show that the connection in Riemann and Cartan geometry takes the antisymmetry of the commutator of covariant derivatives. The connection is always antisymmetric, a major discovery of ECE theory. So the metric (46) cannot be derived from the incorrect Einstein field equation and all metrics of the Einstein field equation are incorrect [1-10]. In ECE theory the metric (46) is derived simply as a possible solution of the orbital theorem of paper 111. Furthermore, it has been known experimentally for over half a century, since the discovery of the velocity curve of a whirlpool galaxy, that the metric (46) can describe only a very limited sample of astronomical data, mainly confined to the solar system. It fails completely for whirlpool galaxies, described straightforwardly in ECE theory by spacetime torsion [1-10]. It is futile to claim that the Einstein field equation is precisely correct, as used to be done in the twentieth century. The Einstein field equation is quite easily refuted [1-10], both theoretically and experimentally. Finally the metric (46) is mis-named in twentieth century literature as the Schwarzschild metric. Contemporary scholarship [1-10] emphasises what should have been glaringly obvious, this metric was not inferred by Schwarzschild in his two original 1916 papers, the first to solve the Einstein field equation. Received opinion in this area of natural

philosophy brushed aside logic and scholarship and continues to do so among the unenlightened. The Einstein field equation is the archetypical idol of the cave, the result being a lazy minded, lapse into dark matter throughout the late twentieth century. The ECE unified field theory has removed this unscientific obscurity and replaced it with simple logic and the original Baconian principles of the sixteenth and seventeenth century enlightenment.

For our present purposes we accept the metric (46) simply as a possible solution of the orbital theorem, one which happens by accident to be accurate for a limited set of astronomical data in our own, entirely insignificant, solar system. This procedure is the only possible one at present, in the absence of a metric that can describe both solar system data and whirlpool galaxy data self consistently and, of course, without use of merely phenomenological dark matter, a fudge factor having no basis in the philosophy of relativity. At present different aspects of ECE theory describe solar system and whirlpool galaxy data, but a single self consistent metric is still needed, one which must also be a solution of the orbital theorem. Such a metric would be a significant advance in knowledge. Therefore accept the metric (46) for the time being and rotate it to give the de Sitter precession:

$$ds'^{2} = x^2 c^2 dt^2 - \frac{dr^2}{X^2} - r^2 (d\varphi^2 \mp \Omega dt)^2 - dZ^2 \quad . \tag{49}$$

In the condition:

$$ds' = dr = dZ \tag{50}$$

Eq. (49) gives the effect of gravitation on the Sagnac effect as described in papers 145 and 146 of this series. It has been shown in this section that the Tomita Chiao effect and Berry phase are described by Eq. (10), so it follows that the effect of gravitation on the Tomita Chiao effect is given by:

$$\omega = \frac{d\varphi}{dt} = \frac{x\, c}{r_1} \quad . \tag{51}$$

If the gravity affected coil is spun around Z at an angular velocity $\mp \Omega$, the resulting phase change is:

$$\Delta \theta = (\frac{x\, c}{r_1} \pm \Omega) \, t \quad . \tag{52}$$

This is the basis for a design of a high accuracy gravimeter.

The rotating frame method of the ECE theory [1-10] used in earlier papers to describe the Sagnac effect and the Faraday disk (see Section 4 of this paper) uses a rotating tetrad:

$$q^{(1)} = e^{(1)} \exp (\mp i\, \Omega t) \tag{53}$$

where Ω is the angular frequency of rotation of the Sagnac platform or Faraday disk. This method simply rotates spacetime in a plane to give a phase angle Ωt , and this is the same as

rotating the metric in a plane to give the same phase angle.

3. Electron Sagnac Effect

The ECE metric theory of this effect, first demonstrated experimentally [16] in the mid nineties, is similar to the metric theory of the photon Sagnac effect. The difference is that the null geodesic condition is not used, so:

$$ds^2 \neq 0 \tag{54}$$

in a plane defined by:

$$d r^2 = dZ^2 = 0 \quad . \tag{55}$$

Details of this procedure and the detailed derivation of the Lorentz transform are given in notes 147(5) accompanying this paper on www.aias.us. These notes show that the proper time infinitesimal is defined by:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - d r^2 - r^2 d\varphi^2 - dZ^2 \tag{56}$$

where:

$$v = \frac{dr}{dt} \tag{57}$$

is the velocity. Eq. (56) is:

$$ds^2 = c^2 dt^2 - | d r^2 | = c^2 dt^2 \left(1 - \frac{v^2}{c^2} \right) \tag{58}$$

so

$$d\tau^2 = \frac{1}{\gamma^2} dt^2 \tag{59}$$

or

$$d\tau = \left(1 - \frac{v^2}{c^2} \right)^{\frac{1}{2}} dt \quad . \tag{60}$$

The proper time τ is the time in a frame of reference in which the particle of mass m is not moving, in other words the proper time is the time as measured in a frame of reference in which the particle is at rest. The proper time is the least possible time. In any other frame, the time infinitesimal is longer than the infinitesimal of proper time. This is known as time dilatation [18]:

$$dt \geq d\tau \quad . \tag{61}$$

The infinitesimal dt is that in a frame of reference with respect to which the particle is moving. So the infinitesimal of time dt is the one measured by an observer in the laboratory frame of reference. The particle moves in the laboratory frame and is observed to move.

Therefore:

$$c^2 \, (dt^2 - d\tau^2) = r^2 \, d\varphi^2 \tag{62}$$

in the plane defined by:

$$d\,r = dZ = 0 \quad . \tag{63}$$

In Eq. (62):

$$d\tau^2 = \left(1 - \frac{v^2}{c^2}\right) dt^2 \tag{64}$$

so:

$$\omega = \frac{d\varphi}{dt} = \frac{v}{r} \quad . \tag{65}$$

This is the familiar looking expression for angular frequency, but it has been derived in a rigorously relativistic way. Eq. (65) gives the phase angle generated by an electron moving around a closed loop, for example a circle of radius r:

$$\theta = \omega \, t = \frac{v}{r} \, t \quad . \tag{66}$$

The time taken to traverse the circumference (2π radians) is

$$t = \frac{2\pi}{\omega} \tag{67}$$

and this can be measured experimentally by direct timing. The phase angle (θ) is the Sagnac effect for an electron on a static platform.

This is a classical theory of relativity. Whether it is described as special or general relativity is not relevant, because these labels are accidents of history. In the quantum theory the wavefunction of the electron [18] is:

$$\psi = \psi_0 \, \exp\left(\frac{i}{\hbar} p^\mu x_\mu\right) \tag{68}$$

where

$$p^\mu = \left(\frac{E}{c}, \boldsymbol{p}\right) \tag{69}$$

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$$x_\mu = (ct, -\boldsymbol{r}) \quad . \tag{70}$$

So:

$$\psi = \psi_0 \exp\left(i(Et - \boldsymbol{p}.\boldsymbol{r})/\hbar\right) \tag{71}$$

which has the same format as the wavefunction of a photon. However for the electron (see following Section) the Einstein energy equation [1-10, 19] applies

$$E^2 = c^2 p^2 + m^2 c^4 \tag{72}$$

with non zero mass m. Wave particle dualism implies:

$$E = \hbar\omega \quad , \quad \boldsymbol{p} = \hbar\boldsymbol{\kappa} \quad , \tag{73}$$

where $\hbar$ is the reduced Planck constant, E the energy and $\boldsymbol{p}$ the momentum, $\boldsymbol{\kappa}$ being the wave-vector of the electron. Note carefully that in the familiar looking Eq. (73) E is the relativistic energy:

$$E = \gamma\, mc^2 \tag{74}$$

and $\boldsymbol{p}$ is the relativistic momentum:

$$\boldsymbol{p} = \gamma\, m\boldsymbol{v} \quad . \tag{75}$$

This is because the Einstein energy equation (66) is simply a different way of writing the relativistic momentum [19]:

$$p^2 = \gamma^2 m^2 v^2 \quad . \tag{76}$$

For the photon

$$m = 0 \tag{77}$$

so:

$$p = E/c \tag{78}$$

$$\kappa = \omega/c \tag{79}$$

but for the electron there exists the rest energy:

$$E_0 = mc^2 \tag{80}$$

$$= (\omega^2 - c^2\kappa^2)^{\frac{1}{2}}/\hbar$$

and so:

$$\kappa \neq \omega / c \quad . \tag{81}$$

The Sagnac effect for counter rotating electron beams produces the phase shift:

$$\psi \longrightarrow e^{\pm i\omega_0 t} \psi \tag{82}$$

where

$$\omega_0 = \frac{v}{r} \quad . \tag{83}$$

Consider an electron beam rotating around a loop such as a circle in one direction, and spin the platform by $\pm\Omega$. The relevant metric becomes:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - r^2 (d\varphi \mp \Omega \, dt)^2 \tag{84}$$

i.e.:

$$r^2 (d\varphi \mp \Omega \, dt)^2 = c^2(dt^2 - d\tau^2) = v^2 \, dt^2 \tag{85}$$

so the intrinsic angular frequency of the metric is:

$$\omega = \omega_0 \pm \Omega \quad . \tag{86}$$

The difference in time taken for an electron beam to traverse 2π radians in a clockwise and anticlockwise direction is:

$$\Delta t = 2\pi \left(\frac{1}{\omega_0 - \Omega} - \frac{1}{\omega_0 + \Omega} \right) \quad . \tag{87}$$

For an electron:

$$\omega_0 = \frac{v}{r} \tag{88}$$

and for a photon:

$$\omega_0 = \frac{c}{r} \quad . \tag{89}$$

These times can be measured directly using contemporary digital timers, or by a shift in the

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fringe of a Sagnac interferogram, the usual method [1-10].

The extra effect of gravitation on these fringe shifts and time intervals can be calculated straightforwardly using the gravitational metric (46) without a null geodesic and the plane, so:

$$d\,r^2 = d\,Z^2 = 0 \tag{90}$$

in Eq. (46) and in consequence Eq. (46) becomes:

$$r^2 d\varphi^2 = c^2(x^2 dt^2 - d\tau^2) \tag{91}$$

in which:

$$d\tau^2 = \left(1 - \frac{v^2}{c^2}\right) dt^2 \quad . \tag{92}$$

So for a static platform, gravitation changes the intrinsic angular frequency (88) of the Minkowski metric to:

$$\omega_0 = \frac{d\varphi}{dt} = \frac{1}{r}\left(v^2 - \frac{2MG}{R}\right)^{\frac{1}{2}} \tag{93}$$

of the gravitational metric. As shown in detail in note 147(7) accompanying paper 147 on www.aias.us the result (93) is entirely consistent with classical dynamics, although obtained relativistically. The classical gravitational potential [19] is:

$$\Phi = - \frac{MG}{R} \tag{94}$$

from which is obtained the classical potential energy:

$$U = m\,\Phi \quad . \tag{95}$$

From Eq. (93):

$$v_1^2 = v^2 - \frac{2MG}{R} \tag{96}$$

so:

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 - \frac{MG}{R} \tag{97}$$

is the hamiltonian:

$$H = T + U \tag{98}$$

where the kinetic energy is:

$$T = \frac{1}{2} m v^2 \tag{99}$$

and the gravitational potential energy is:

$$U = - \frac{mMG}{R} \,. \tag{100}$$

The classical gravitational force between the electron mass m and the gravitating mass M is:

$$\mathbf{F} = - \nabla U = - \frac{mMG}{R^2} \mathbf{k} \tag{101}$$

in the Z axis of the Cartesian unit vector $\mathbf{k}$. This is the Newton inverse square law. The magnitude of the acceleration due to gravity is:

$$g = - \frac{MG}{R^2} \tag{102}$$

so the equivalence principle is:

$$\mathbf{F} = m\,\mathbf{g} = - \frac{GmM}{R^2} \mathbf{k} \tag{103}$$

as derived in an earlier paper from the ECE antisymmetry law.

From these considerations it is deduced that the time taken for a light beam to traverse 2π radians in a gravitational field is:

$$t = \frac{2\pi r}{(c^2 + 2\,R\,g)^{\frac{1}{2}}} \tag{104}$$

and that the time taken for an electron to do the same is:

$$t = \frac{2\pi r}{(v^2 + 2\,R\,g)^{\frac{1}{2}}} \,. \tag{105}$$

These times can be measured directly and form the basis for a simple high accuracy gravimeter. These times depend on g, so are different in various situations in which g is different, e.g. in various orbits and points on the Earth's surface. Therefore these times provide an accurate gravitational map.

4. Faraday disk

In order to apply these metrical methods to the Faraday disk, the idea of particles such as a photon or electron traversing an enclosed loop such as a circle must be developed to encompass a solid disk - the Faraday disk. As shown in detail in note 147(8) accompanying

this paper on www.aias.us the gravitational potential at a point P, a distance R from the centre of gravity of a solid disk of mass M is:

$$\Phi = - \frac{MG}{R} \qquad .$$

(106)

If the radius of the disk is a, and P′ is a point on the rim of the disk, the angle between a and R being θ, and if the distance between P and P′ is r, then

$$\Phi = - MG / (a \cos \theta + (a^2 \sin^2 \theta + r^2)^{\frac{1}{2}}) \qquad .$$

(107)

If:

$$r \gg a$$

(108)

then

$$\Phi \sim - \frac{MG}{r}$$

(109)

and the problem reduces to the same mathematical format as Eq. (94). So the effect of gravitation on the Faraday disk can be developed using the same type of metrical method.

To approach this problem it is shown firstly that the Einstein energy equation (72) is a direct consequence of the Minkowski metric. The latter is:

$$c^2 d\tau^2 = c^2 dt^2 - d\boldsymbol{r}.\, d\boldsymbol{r}$$

(110)

and in cylindrical polar coordinates:

$$d\boldsymbol{r}.\, d\boldsymbol{r} = dr^2 + r^2 d\varphi^2 + dZ^2 \qquad .$$

(111)

From Eq. (110):

$$\left(\frac{dt}{d\tau}\right)^2 = Y^2 = 1 + \frac{1}{c^2} \left(\frac{dr}{d\tau}\right)^2 \qquad .$$

(112)

Multiply both sides of Eq. (112) by m^2:

$$m^2 \left(1 - \frac{1}{Y^2}\right) = \frac{m^2}{Y^2 c^2} \left(\frac{dr}{d\tau}\right)^2 \qquad .$$

(113)

The relativistic momentum [19] is

$$\boldsymbol{p} = m\frac{d\boldsymbol{r}}{d\tau} = Y m \frac{d\boldsymbol{r}}{dt} = Y m \boldsymbol{v} \qquad .$$

(114)

From Eqs. (113) and (114):

$$\gamma^2 m^2 c^4 (1 - \frac{1}{\gamma^2}) = p^2 c^2 \qquad (115)$$

i.e.

$$E^2 = (\gamma m c^2)^2 = p^2 c^2 + E_0^2 \qquad (116)$$

where the rest energy is:

$$E_0 = m c^2 . \qquad (117)$$

This is the Einstein energy equation, Q.E.D.

Eq. (110) is:

$$x^\mu x_\mu = c^2 \tau^2 \qquad (118)$$

and Eq. (116) is:

$$p^\mu p_\mu = m^2 c^4 \qquad . \qquad (119)$$

Here:

$$x^\mu = (ct , r) \qquad (120)$$

$$x_\mu = (ct , - r) \qquad (121)$$

$$p^\mu = (\frac{E}{c} , p) \qquad (122)$$

$$p_\mu = (\frac{E}{c} , - p) \qquad . \qquad (123)$$

The relativistic Hamilton Jacobi equation is a simple consequence in Eq. (119) of the minimal prescription [1-10, 15]:

$$p^\mu \longrightarrow p^\mu - e A^\mu \qquad (124)$$

so:

$$(p^\mu - e A^\mu)(p_\mu - A_\mu) = m^2 c^4 \qquad . \qquad (125)$$

Here -e is the charge on the electron and A^μ is the electromagnetic four-potential

$$A^\mu = (\frac{\phi}{c} , A) \qquad (126)$$

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where ϕ is the scalar potential and $\boldsymbol{A}$ is the vector potential. For simplicity without loss of generality consider:

$$p^\mu = p_\mu = 0 \tag{127}$$

to obtain:

$$A^\mu \, A_\mu = \left(\frac{mc}{e}\right)^2 \ . \tag{128}$$

      With these preliminaries the effect of rotation on the electromagnetic potential may be developed in the plane of the Faraday disk, a plane defined by:

$$\mathrm{d}r = \mathrm{d}Z = 0 \tag{129}$$

so:

$$\mathrm{d}\boldsymbol{r} \cdot \mathrm{d}\boldsymbol{r} = r^2 \mathrm{d}\varphi^2 \ . \tag{130}$$

Therefore:

$$\left(\frac{dr}{d\tau}\right)^2 = r^2 \left(\frac{d\varphi}{d\tau}\right)^2 \tag{131}$$

and the relativistic momentum is:

$$\boldsymbol{p} = Y m \, \boldsymbol{r} \, \frac{d\varphi}{d\tau} = Y m \, \omega \, \boldsymbol{r} \ . \tag{132}$$

Therefore in Eq. (116):

$$E^2 - E_0^2 = (Y^2 - 1) \, m^2 \, c^4 = (Y m \, \omega \, c \, r)^2 \tag{133}$$

and the angular frequency is, self consistently:

$$\omega = \left(\frac{Y^2 - 1}{Y^2}\right)^{\frac{1}{2}} \frac{c}{r} = \frac{v}{r} \ . \tag{134}$$

This is the familiar classical result but one which is, at the same time, fully relativistic. The vector potential from the minimal prescription is

$$\boldsymbol{A} = \frac{1}{e} \boldsymbol{p} = \frac{m}{e} Y \omega \, \boldsymbol{r} \tag{135}$$

and in the limit:

$$v \ll c \tag{136}$$

becomes:

$$A \sim \frac{m}{e} \omega \boldsymbol{r} \quad . \tag{137}$$

It is seen that mechanical rotation at an angular frequency Ω affects the vector potential as follows:

$$A \longrightarrow \frac{m}{e} (\omega + \Omega) \boldsymbol{r} \tag{138}$$

and this is the type of phenomenon observed in the Faraday disk.

The received opinion and details of this development of the Faraday disk theory are described in considerable detail in note 147(10) accompanying this paper on www.aias.us. . The received opinion is that the disk demonstrates the Lorentz force law:

$$\boldsymbol{E} = \boldsymbol{v} \times \boldsymbol{B} \tag{139}$$

i.e.:

$$E = v\, B = \omega\, r\, B \tag{140}$$

so that an electric field strength is generated by a magnetic flux density in a solid disk of radius r rotating at angular frequency ω. The Faraday disk uses a static magnet so:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0} \tag{141}$$

and in the received opinion the Faraday law of induction:

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0} \tag{142}$$

does not describe the Faraday disk because:

$$\nabla \times \boldsymbol{E} = \boldsymbol{0} \tag{143}$$

contradicting the experimental observation of an electric field strength circling around the rim of the disk. This is known as the Faraday paradox. The received opinion is that rotating the magnet does not change $\boldsymbol{B}$. However, recent and careful experiments by Kelly [20] show that the received opinion is incorrect, rotating the magnet is relevant and the magnetic field rotates with the magnet. In the received opinion the key relevant motion is that between the disk and observer (the return path or wire), meaning that the disk is a demonstration of relativity. There has been a protracted and irrelevant controversy about whether this is special or general relativity. As in the various Sagnac effects, this is irrelevant, merely a matter of semantics.

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Consider a point on the rim of a Faraday disk of radius r rotating at ω. From Eq. (132):

$$v = \gamma \omega r \tag{144}$$

is the tangential linear velocity. If

$$v \ll c \tag{145}$$

the familiar looking

$$v = \omega r \tag{146}$$

is regained. The vector potential from the minimal prescription in the limit (145) is:

$$\boldsymbol{A} = \frac{m}{e}\, \omega\, \boldsymbol{r} \quad . \tag{147}$$

From Eqs. (140) and (147):

$$A = \frac{m}{e}\frac{E}{B} = \frac{m}{e}\, v \quad . \tag{148}$$

In free space:

$$\frac{E}{B} = c \tag{149}$$

otherwise:

$$\frac{E}{B} = v \quad . \tag{150}$$

In ECE theory [1-10]:

$$\boldsymbol{E} = -\,\nabla\Phi - \frac{\partial \boldsymbol{A}}{\partial t} + \Phi\,\boldsymbol{\omega}_s - \omega_s\boldsymbol{A} \tag{151}$$

and

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} - \boldsymbol{\omega}_s \times \boldsymbol{A} \tag{152}$$

where the spin connection scalar is ω_s and the spin connection vector is $\boldsymbol{\omega}_s$.

If the vector potential is defined by Eq. (147):

$$A = \frac{m}{e}\ \boldsymbol{\omega}\ \mathsf{X}\ \boldsymbol{r} \tag{153}$$

and if the angular velocity $\boldsymbol{\omega}$ of the disk and the radius $\boldsymbol{r}$ of the disk are constant, then in the absence of a scalar potential gradient $\nabla\Phi$:

$$\boldsymbol{E} = \Phi\ \boldsymbol{\omega}_s - \omega_s\boldsymbol{A}\quad . \tag{154}$$

Using the ECE antisymmetry law [1-10]:

$$\boldsymbol{E} = -\ 2\ \omega_s\boldsymbol{A} \tag{155}$$

giving a direct relation between the electric field strength $\boldsymbol{E}$ and the vector potential $\boldsymbol{A}$. This relation is missing in the received opinion. Antisymmetry also implies, self consistently:

$$\nabla\Phi = \frac{\partial\boldsymbol{A}}{\partial t} = \boldsymbol{0}\quad . \tag{156}$$

If in Eq. (153):

$$\boldsymbol{\omega} = \omega_Z\,\mathbf{k} \tag{157}$$

$$\boldsymbol{r} = \mathsf{X}\,\mathbf{i} + \mathsf{Y}\,\mathbf{j} \tag{158}$$

then:

$$A = \frac{m\omega}{e}\ (-\mathsf{Y}\,\mathbf{i} + \mathsf{X}\,\mathbf{j}\,) \tag{159}$$

and

$$\nabla\,\mathsf{X}\,A = \frac{2m\omega}{e}\ \mathbf{k} \tag{160}$$

in which:

$$\frac{\partial A_{\mathsf{Y}}}{\partial\mathsf{X}} = -\frac{\partial A_{\mathsf{X}}}{\partial\mathsf{Y}} = \frac{m\omega}{e} \tag{161}$$

another example of the ECE antisymmetry law:

$$F_{ij} = \partial_i\,A_j - \partial_j\,A_i + \omega_{si}\,A_j - \omega_{sj}\,A_i \tag{162}$$

with:

$$\partial_i\,A_j + \omega_{si}\,A_j = -\,(\partial_j\,A_i + \omega_{sj}\,A_i)\quad . \tag{163}$$

Therefore the total magnetic flux density from Eq. (152) is:

$$B = \frac{2m\omega}{e} \, \mathbf{k} \; - \; \boldsymbol{\omega}_s \times \boldsymbol{A} \quad . \tag{164}$$

However, by antisymmetry:

$$E = -2 \, \omega_s A \; = \; 2 \, \Phi \, \boldsymbol{\omega}_s \tag{165}$$

so for the Faraday disk:

$$\boldsymbol{\omega}_s \times \boldsymbol{A} = \mathbf{0} \tag{166}$$

and:

$$E = 2 \, \Phi \, \boldsymbol{\omega}_s \tag{167}$$

$$B = \frac{2m\omega}{e} \, \mathbf{k} \tag{168}$$

is a complete description of the disk without need of any other consideration. The electric field strength is within $2 \, \Phi.$ The spin connection vector, and the magnetic flux density is the curl of the vector potential, proportional to the curl of the spin connection vector.

From Eqns. (167) and (168):

$$E = (\frac{\Phi}{A} \, |\boldsymbol{\omega}_s| \, r \,) \, B \quad . \tag{169}$$

Now use the following relation between the magnitudes of the scalar and vector potentials:

$$\Phi = c \, A \tag{170}$$

to find that

$$\begin{aligned} E &= (c \, r \, |\boldsymbol{\omega}_s| \,) \, B \\ &= v \, B \end{aligned} \tag{171}$$

if the magnitude of the spin connection vector is:

$$|\boldsymbol{\omega}_s| = (\frac{v}{c}) \frac{1}{r} \, . \tag{172}$$

The spin connection scalar is defined by:

$$\omega_s = c |\boldsymbol{\omega}_s| \tag{173}$$

so the spin connection four vector is defined by:

$$\omega_s^\mu = (\omega_s , c\boldsymbol{\omega}_s) \quad .$$ (174)

So the spin connection scalar is the angular velocity:

$$\omega_s = \omega = \frac{v}{r}$$ (175)

and is the intrinsic angular velocity of the Minkowski metric. Rotating the latter therefore produces a spacetime connection. Self consistently, it is the connection that is responsible for the Sagnac effect in a photon. The Maxwell Heaviside theory has no connection because it is based on a static Minkowki frame and for this reason cannot describe the Sagnac effect of the photon as is well known [15]. ECE theory therefore brings together several concepts in an entirely self consistent manner. For the Faraday disk, in summary:

$$\omega_s = \omega$$ (176)

$$\boldsymbol{\omega}_s = (\frac{v}{c})\frac{1}{r^2}\left(-r_Y \mathbf{i} + r_X \mathbf{j}\right)$$ (177)

and the metric method shows that $\boldsymbol{A}$ is affected by rotation, therefore so is $\boldsymbol{B}$, as observed by Kelly [20]. The metric method in the limit $v \ll c$ produces

$$v = \omega r$$ (178)

a result which is affected by both relativity and gravitation.

The familiar Lorentz induction on the other hand is:

$$\boldsymbol{E} = \boldsymbol{v} \times \boldsymbol{B}$$ (179)

$$\boldsymbol{B} = -\frac{1}{v^2} \boldsymbol{v} \times \boldsymbol{E} \quad ,$$ (180)

and comes from the Lorentz transform of the field tensor [15]. In ECE theory it comes from the coordinate transformation of the torsion [1-10]. It also has some hidden constraints which are revealed as follows. Multiply Eq. (179) on both sides as follows:

$$\boldsymbol{v} \times \boldsymbol{E} = \boldsymbol{v} \times (\boldsymbol{v} \times \boldsymbol{B}) = (\boldsymbol{v} . \boldsymbol{B}) \boldsymbol{v} - v^2 \boldsymbol{B}$$ (181)

so the usual result (180) is obtained only if:

$$\boldsymbol{v} . \boldsymbol{B} = 0 \quad .$$ (182)

Similarly from Eq. (180):

$$\boldsymbol{v} \times \boldsymbol{B} = - (\boldsymbol{v} \times (\boldsymbol{v} \times \boldsymbol{E})) / v^2 = \boldsymbol{E} - \frac{1}{v^2} \boldsymbol{v} (\boldsymbol{v} . \boldsymbol{E})$$ (183)

and we obtain the result:

$$v \cdot B = v \cdot E = 0 \ . \tag{184}$$

This is a constraint on the validity of the Lorentz induction law. It is not general, whereas the ECE theory of Eqs. (167) and (168) is general and simpler, so is preferred by Ockham's Razor. As is always the case, ECE is preferred to Maxwell Heaviside (MH) theory because the former is a generally covariant unified field theory and as we have seen MH is severely constrained in its inability to produce any effect involving rotation of the Minkowski metric. MH is, finally, unable to explain gravitational effects on light. Since Lorentz induction is based on MH theory, it is no surprise that Lorentz induction is also constrained as we have just demonstrated. So Lorentz induction is not satisfactory for the Faraday disk.

Acknowledgments

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