# Failure of Einstein´s theory of light deflection

# and non-zero photon mass.

by

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## Abstract

The Einstein theory of light deflection due to gravitation is shown to be erroneous and self inconsistent. The integral used by Einstein himself is evaluated numerically and is severely inconsistent with Einstein's claim to have produced the observed deflection at closest approach. The reasons for this result are discussed in detail and a new theory suggested in terms of finite photon mass. The Einstein field equation uses an incorrect connection symmetry, so all metrics from this equation are erroneous. Valid metrics can be obtained from the Orbital Theorem of UFT 111 of this series, and in order to apply these metrics to the problem of light deflection by gravitation, the mass of the photon must be identically non-zero. Numerical studies determine this value to be 3.35 x $10^{-41}$ kg.

*Keywords:* ECE theory of light, light deflection, gravitation, errors in the Einstein theory of light deflection by gravitation.

# 1. Introduction

It is well known [1-10] that the Einstein field equation is fundamentally erroneous due to the use of a symmetric connection and subsequent neglect of spacetime torsion. The connection must take the symmetry of the commutator as shown in UFT 139 of this series and so it is clear that the connection must be antisymmetric in its lower two indices. In consequence any metric derived from the Einstein field equation is meaningless. In 1916, Schwarzschild derived the first solution of the erroneous Einstein field equation, but in his original two papers [2] he did not derive the metric that is commonly known as "the Schwarzschild metric". This misnamed metric was apparently derived from Schwarzschild's original work by Hilbert, and used by Einstein to predict dynamical properties from his then new field equation of 1915. It is now known that metrics can be derived from the ECE Orbital Theorem of UFT 111 of this series [1-10]. These are metrics of spherically symmetric spacetime. The simplest solution of the Orbital Theorem is the Minkowski metric, and the metric that is always mis-named "the Schwarzschild metric" is another possible solution of the Orbital Theorem. Metrics from the latter must be used to derive tetrads, torsion, and new field equations, and the Einstein field equation must be discarded. It is well known that there are no black holes in consequence of the fact that the theory behind them (the Einsteinian theory) is incorrect. Big Bang is now known observationally never to have happened.

In Section 2, several self inconsistencies are shown in the application of the misnamed "Schwarzschild metric". Notably, Einstein's method of calculation light deflection due to gravitation is shown to be erroneous by several orders of magnitude. The cause of this is that Einstein assumed a circular orbit, for which the denominator of the integral used in Einstein's calculation is zero. The method used by Einstein [11] to solve this integral is also erroneous. He also assumed that the photon mass m is identically zero, thus eliminating it from consideration, and introducing a null geodesic. This method means that the effective potential used in Einstein's calculation is mathematically indeterminate, it is essentially a balance of limits, the way in which these limits are considered cannot be objective. There is no reason why the orbit of the photon should be a circle.

In Section 3 it is shown by direct numerical integration that the integral used by Einstein does not give his claimed result for light deflection due to gravitation:

$$\Delta\,\varphi = \frac{4MG}{c^2 R_0} \tag{1}$$

where $G$ is Newton's constant, $M$ is the mass of the Sun, $c$ is the vacuum speed of light and $R_0$ the distance of closest approach, essentially the radius of the Sun. The result claimed by Einstein does not produce Eq. (1) to within machine precision, and in fact the integral uses a denominator which in Einstein's own theory vanishes if a rigorous method is used. New numerical methods are suggested which can be used with any valid metric, i.e. any metric which is a solution of the Orbital Theorem of UFT 111, but not a solution of the erroneous Einstein field equation of 1915.

## 2. Criticisms of Einstein´s method

The method is based on what is known in ECE theory as the gravitational metric:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 \left(1 - \frac{r_0}{r}\right) - dr^2 \left(1 - \frac{r_0}{r}\right)^{-1} - r^2 d\varphi^2 \tag{2}$$

in the XY plane and in cylindrical polar coordinates. In the obsolete physics this is always incorrectly referred to as "the Schwarzschild metric" and in the obsolete physics it is always claimed uncritically and dogmatically that the Einstein field equation is correct, despite the fact that it is easily shown to use the wrong connection symmetry as in UFT 139. In Eq. (2):

$$r_0 = \frac{2(M+m)G}{c^2} \tag{3}$$

The lagrangian from Eq. (2) is pure kinetic in nature, in relativity there is no potential energy or force:

$$\mathcal{L} = T = \frac{1}{2}\mu c^2 = \frac{\mu}{2}\left(\left(\frac{dt}{d\tau}\right)^2\left(1 - \frac{r_0}{r}\right) - \left(1 - \frac{r_0}{r}\right)^{-1}\left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\varphi}{d\tau}\right)\right) \tag{4}$$

The Lagrange equation gives the total energy $E$ and angular momentum $L$ as the following constants of motion:

$$E = \mu c^2\left(1 - \frac{r_0}{r}\right)\frac{dt}{d\tau} \quad , \quad L = \mu r^2 \frac{d\varphi}{d\tau} \tag{5}$$

In Eq. (4) the reduced mass is used:

$$\mu = \frac{mM}{m+M} \tag{6}$$

in order to reduce correctly to Newtonian dynamics (see notes accompanying this paper). The photon mass is $3.35 \times 10^{-41}$ kilograms, so for all practical purposes:

$$\mu = m \tag{7}$$

and the inverse square law may be used for $r_0$ :

$$r_0 = \frac{2MG}{c^2} \tag{8}$$

In order that this law be valid, the photon mass $m$ must be identically non-zero. Einstein assumed it to be zero and in consequence his calculations fail drastically as shown straightforwardly in Section 3 by evaluating his OWN integral [11] numerically. Any desk top can be used to do this.

The equation of motion is obtained from Eq. (2) by multiplying both sides by $(1 - r_0/r)$ to give:

$$m\left(\frac{dr}{d\tau}\right)^2 = \frac{E^2}{mc^2} - \left(1 - \frac{r_0}{r}\right)\left(mc^2 + \frac{L^2}{mr^2}\right) \tag{9}$$

The infinitesimal of proper time $d\tau$ is eliminated as follows:

$$\frac{dr}{d\tau} = \frac{d\varphi}{d\tau}\frac{dr}{d\varphi} = \left(\frac{L^2}{mr^2}\right)\frac{dr}{d\varphi} \tag{10}$$

to give the orbital equation:

$$\left(\frac{dr}{d\varphi}\right)^2 = r^4\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right) \tag{11}$$

where the two constant lengths $a$ and $b$ are defined by:

$$a = \frac{L}{mc} \quad , \quad b = \frac{cL}{E} \quad . \tag{12}$$

The solution of Eq. (11) is:

$$\varphi = \int \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-½} dr \tag{13}$$

and the deflection of light due to gravitation is [11]:

$$\Delta\varphi = 2\int_{R_0}^{\infty} \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-½} dr - \pi \tag{14}$$

where $R_0$ is the distance of closest approach, essentially the radius of the Sun. Using:

$$u = \frac{1}{r} \quad , \quad du = -\frac{1}{r^2}\, dr \tag{15}$$

the integral (14) may be rewritten as:

$$\Delta\varphi = 2\int_0^{1/R_0}\left(\frac{1}{b^2} - (1 - r_0 u)\left(\frac{1}{a^2} + u^2\right)\right)^{-½} du - \pi \quad . \tag{16}$$

If we are to accept the gravitational metric for the sake of argument, its correct use must be to assume an identically non zero photon mass m and to integrate Eq. (16), producing an equation for the experimentally observed deflection $\Delta\varphi$ in terms of $m$, $a$ and $b$ .

Einstein used the null geodesic condition:

$$ds^2 = 0 \tag{17}$$

which means that the photon mass $m$ vanishes identically and that the concept of massless photon propagates at $c$ in vacuo. This is the dogma of the obsolete physics, a dogma which leads to disaster as shown in Section 3. For the gravitational metric, the correct equation of motion is:

$$\frac{1}{2}\left(\frac{E^2}{mc^2} - mc^2\right) = \frac{1}{2}m\left(\frac{dr}{d\tau}\right)^2 - \frac{mMG}{r} + \frac{L^2}{2mr^2} - \frac{MGL^2}{mc^2r^3} \quad . \tag{18}$$

The correct way of reaching the Newtonian limit is:

$$\frac{1}{2}\left(\frac{E^2}{mc^2} - mc^2\right) \longrightarrow \frac{1}{2}mv^2 - \frac{mMG}{r} \tag{19}$$

$$\left(\frac{dr}{d\tau}\right)^2 \longrightarrow \left(\frac{dr}{dt}\right)^2 \tag{20}$$

$$\frac{L^2}{2mr^2} \longrightarrow \frac{m}{2}r^2\left(\frac{d\varphi}{dt}\right)^2 \tag{21}$$

$$\frac{MGL^2}{mc^2r^3} \longrightarrow 0 \tag{22}$$

where we have used:

$$\frac{d\varphi}{d\tau} = \left(1 - \frac{v^2}{c^2}\right)^{-½}\left(1 - \frac{r_0}{r}\right)^{-½}\frac{d\varphi}{dt} \tag{23}$$

where $v$ is the total velocity of the photon. Clearly, if the photon mass $m$ is zero identically, the Newtonian limit is never reached because $v$ is always $c$ for the massless photon.

Einstein assumed:

$$a = \infty \tag{24}$$

The angular momentum $L$ is a constant of motion, so the assumption (24) means:

$$m = 0 \ , \quad \frac{d\varphi}{d\tau} = \infty \tag{25}$$

which in the obsolete dogma is known as "the ultrarelativistic limit". Despite the fact that relativity is defined in Eq. (4) to be purely kinetic in nature, the obsolete dogma uses the "effective potential":

$$V(r) = \frac{1}{2} mc^2 \left(-\frac{r_0}{r} + \frac{a^2}{r^2} - \frac{r_0 a^2}{r^3}\right) \tag{26}$$

Einstein assumed that:

$$-\frac{r_0}{r} + \frac{a^2}{r^2} - \frac{r_0 a^2}{r^3} = \frac{a^2}{r^2} - \frac{r_0 a^2}{r^3} \tag{27}$$

and also assumed circular orbits, so:

$$\frac{dr}{d\tau} = 0 \tag{28}$$

This assumption means, however, that:

$$\frac{1}{b^2} = \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right) \tag{29}$$

and the denominator of Eq. (13) becomes zero, the integral becomes infinite, and the method used by Einstein is incorrect.

His method is described in detail in the notes accompanying this paper on www.aias.us. It was to assume:

$$\frac{r_0}{r} \longrightarrow 0 \tag{30}$$

which must mean:

$$r \longrightarrow \infty \tag{31}$$

and:

$$m \longrightarrow 0 \;, \quad a \longrightarrow \infty \tag{32}$$

The effective potential was therefore defined as

$$V(r) \xrightarrow[m \to 0,\, a \to \infty,\, r \to \infty]{} mc^2 \left(\frac{a}{r}\right)^2 \left(1 - \frac{r_0}{r}\right) \tag{33}$$

which is mathematically indeterminate. Einstein also assumed:

$$mc^2 \longrightarrow 0 \tag{34}$$

so the equation of motion (18) becomes:

$$\frac{E^2}{2mc^2} = \frac{L^2}{mr^2} \left( \frac{1}{2} - \frac{MG}{c^2 r} \right) \tag{35}$$

He used:

$$r = R_0 \tag{36}$$

in this equation, thus finding an expression for $b_0$:

$$\frac{1}{{b_0}^2} = \frac{1}{{R_0}^2} - \frac{r_0}{{R_0}^3} \quad . \tag{37}$$

Finally he used Eq. (37) in Eq. (16) with:

$$a^2 \longrightarrow \infty \tag{38}$$

to obtain the integral:

$$\Delta\,\varphi \;=\; 2 \int_0^{1/R_0} \left(\frac{R_0 - r_0}{{R_0}^3} \; - u^2 + r_0 u^3\right)^{-½} du \; - \; \pi \tag{39}$$

It was claimed by Einstein [11] that this integral is:

$$\Delta\,\varphi \;=\; \frac{4MG}{c^2 R_0} \tag{40}$$

however, in Section 3, it is shown by direct numerical integration of Eq. (39) that it gives a result different from Eq. (40) and this is a disaster for the obsolete physics. It becomes clear that Einstein could and did make errors. It is also known that Eddington's claim to have "verified" the result (40) was unfounded. The correct result is now available with precision from NASA Cassini and is, experimentally:

$$\begin{aligned} \Delta\,\varphi \;=\; & 1.75 \text{ arc seconds} = 8.484 \times 10^{-6} \text{ radians} \\ & = 4MG / c^2 R_0 \end{aligned} \tag{41}$$

with the following parameters:

$$R_0 = 6.955 \times 10^8 \text{ m} \quad , \quad M = 1.9891 \times 10^{30} \text{ kg} \quad , \quad G = 6.67428 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \quad , \quad c = 2.9979 \times 10^8 \text{ m s}^{-1} \quad . \tag{42}$$

However, the experimental result cannot possibly be due to Einsteinian general relativity at all. It is suggested that if we accept the gravitational metric, the correct experimental result must be evaluated from Eq. (16) with finite photon mass, and independent methods used to evaluate $a$ and $b$ . The correct method of evaluating Newtonian dynamics is to integrate:

$$\left(\frac{dr}{d\varphi}\right)^2 = r^4\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\frac{1}{a^2} - \frac{1}{r^2}\right) \tag{43}$$

$$a = \frac{L}{\mu c} \quad , \quad b = \frac{cL}{E} \quad , \quad r_0 = \frac{2(m+M)}{c^2}$$

to give the well known result:

$$\frac{\alpha}{r} = (1 + \epsilon \cos \varphi) \tag{44}$$

where the parameters $\alpha$ and $\epsilon$ are given [12] by:

$$\alpha = \frac{L^2}{\mu k} \quad , \quad \epsilon = \left(1 + \frac{2EL^2}{\mu k^2}\right)^{-1/2} \quad , \quad k = mMG \quad . \tag{45}$$

In the obsolete dogma it is claimed that the Newtonian result is:

$$\Delta \varphi = \frac{2MG}{c^2 R_0} \tag{46}$$

but this is obtained by guesswork - using an heuristic method. It is not a true Newtonian result because Newtonian dynamics do not contain the speed of light *c*.

So in the obsolete dogma, confusion is compounded, the subject has become an out of control fantasy based on clearly incorrect mathematics. In consequence cosmology has been badly damaged throughout the twentieth century.

## 3. Numerical integration of Einstein´s own integral (Eq.(39)).

Einstein's formula (39) for light deflection depends on the radius parameters $R_0$ and $r_0$. $R_0$ represents the radius of the Sun ($6.955 \times 10^8$ m) while $r_0$, sometimes called the "Schwarzschild radius", is only 2,954 m. Therefore we have

$$r_0 << R_0$$

which implies according to Eq. (37) that

$$b_0 \approx R_0.$$

The integral (39)

$$\Delta \varphi = 2\int_0^{1/R_0} \left(\frac{R_0 - r_0}{{R_0}^3} - u^2 + r_0\, u^3\right)^{-1/2} du - \pi \tag{47}$$

is not solvable analytically and needs to be evaluated numerically. First it is to be noted that the square root in the integrand has zero crossings, leading to infinite values of the integrand.

The argument of the square root

$$A(u) = \frac{R_0 - r_0}{{R_0}^3} - u^2 + r_0\, u^3 \tag{48}$$

is plotted in Fig. 1 where $u$ is the inverse radius parameter

$$u = \frac{1}{r} \tag{49}$$

and the relevant range for integral (47) is 0 to 1.4378 x $10^{-9}$ $m^{-1}$. Numerical analysis shows that there is a zero crossing exactly at this value so that the argument $A(u)$ is positive in the definition range of the integral. The integrand of (47) itself is graphed in Fig. 2. It has a sharp pole at $u=1/R_0$. The numerical result is precise to twelve numerical places and is:

$\Delta\varphi = 8.4934 \pm 10^{-6}$ microradians

and Einstein´s result is:

$\Delta\varphi = 8.4955$ microradians.

The experimental result from the latest satellite data is:

$$\Delta\varphi_{experimental} = 8.4848 \pm 0.003 \text{ microradians}$$
$$= 1.75 \pm 0.0005 \text{ arc seconds.}$$

This discrepancy deserves precise analysis. According to Fig. 2, the value of the integral is mainly determined by the region near to $1/R_0$. Increasing the boundary value $R_0$ by 10% leads to a decrease of $\Delta\varphi$ to 2.28. This may give a hint to the sensitivity of the result on the integration boundary. There is no change in orders of magnitude. The numerical accuracy of the integration was reported to be $10^{-12}$, much lower than the range of both results. The calculation was performed by the computer algebra system Maxima and was checked by evaluating the integral independently in Mathematica. Both programs yielded the identical result. So the discrepancy to Einstein's calculation cannot be explained by numerical instability of the integral value.

| $b_0$ | $R_0$ | $\Delta\varphi - \pi$ |
|---|---|---|
| 6.95501 x $10^8$ | 6.955 x $10^8$ | 3.1416 |
| 1 x $10^3$ | 6.955 x $10^8$ | 2.8756 x $10^{-6}$ |
| 6.95501 x $10^8$ | 6.955 x $10^{14}$ | 2.0000 x $10^{-6}$ |

**Table 1.** Variation of parameters in integral (47).

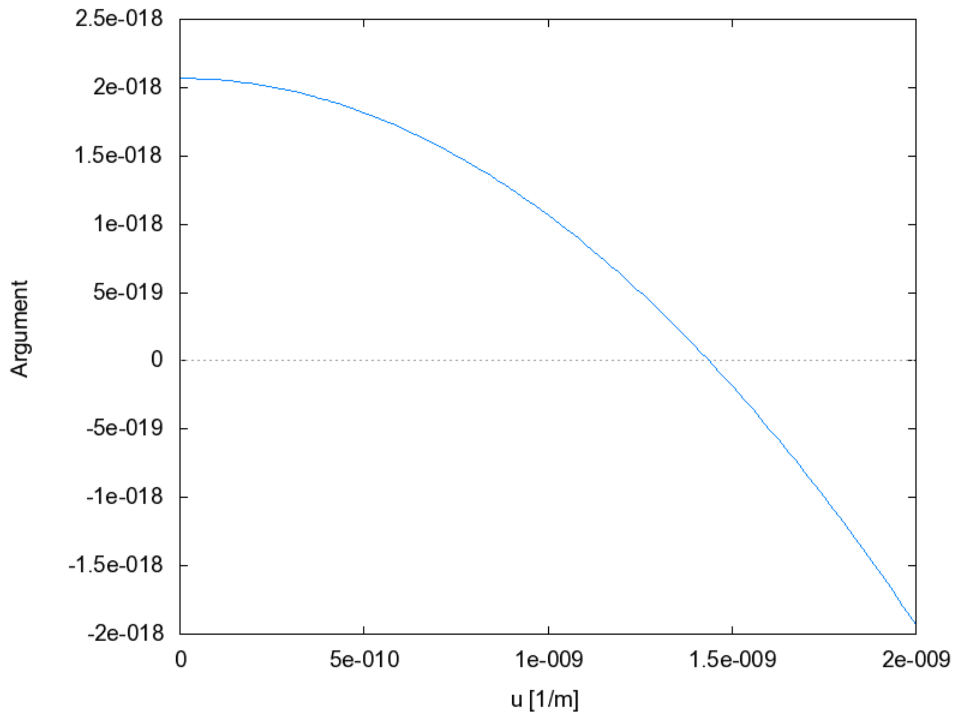


**Fig. 1.** Graph showing $\boldsymbol{u}$ dependence of square root argument in Eq. (47).

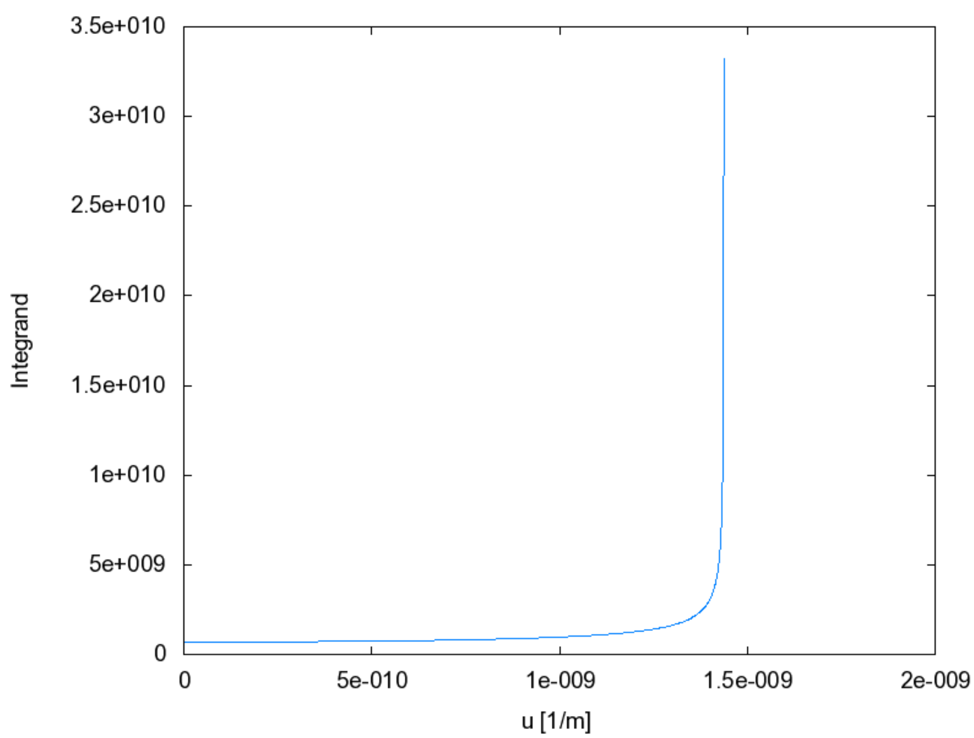


**Fig. 2.** Graph showing $\boldsymbol{u}$ dependence of integrand in Eq. (47).

In order to see the impact of the parameters $b_0$ and $R_0$ on the result we have changed both parameters separately as shown in Table 1. It is required to reduce $b_0$ by five orders of magnitude to have $\Delta\varphi$ covering the experimentally observed range. Alternatively, $R_0$ has to be increased by six orders of magnitudes. Thus Einstein's result is not consistent in any way.

## 4. Analytical evaluation of the correct integral (Eq.(16)) and estimation of photon mass.

The correct formula for the light deflection is Eq. (16)

$$\Delta\varphi = 2\int_0^{1/R_0}\left(\frac{1}{b^2} - (1 - r_0 u)\left(\frac{1}{a^2} + u^2\right)\right)^{-1/2} du - \pi \tag{50}$$

with $a$ and $b$ being parameters having to be determined in such a way that the experimental result for $\Delta\varphi$ is obtained. From Eq. (12) we have

$$a = \frac{L}{mc} \quad , \quad b = \frac{cL}{E} \quad . \tag{51}$$

where $m$ is the photon mass and $E$ the photon energy

$$E = \hbar\omega. \tag{52}$$

From Eq. (51) follows

$$a = \frac{\hbar\omega}{mc^2}\, b\ . \tag{53}$$

The parameter $b$ is a constant of motion, and is determined by the need for zero deflection when the mass of the Sun, $M$ ,is absent, i.e. is determined by:

$$\Delta\varphi = 2\int_0^{1/R_0}\left(\frac{1}{b^2} - u^2\right)^{-1/2} du - \pi = 0 \tag{54}$$

The integral in Eq. (54) is:

$$2\int_0^{1/R_0}\left(\frac{1}{R_0{}^2} - u^2\right)^{-1/2} du = 2\sin^{-1}(R_0 u)\Big|_0^{1/R_0}$$

$$= \pi \tag{55}$$

so:

$b$ = $R_0$ = 6.955 x $10^8$ metres. (56)

The integral in Eq. (50) is approximated excellently by:

$$\Delta\varphi \sim 2\int_0^{1/R_0}\left(\frac{1}{b^2}-\frac{1}{a^2}-u^2\right)^{-1/2} du - \pi \quad . \tag{57}$$

For the lower bound:

$$ur_0 = 0 \quad ,$$

$$1 - ur_0 = 1 \quad , \tag{58}$$

and for the upper bound:

$$ur_0 = r_0/\,R_0 = 4.25 \text{ x } 10^{-6} \quad , \tag{59}$$

$$1 - ur_0 = 0.999996 \quad .$$

From Eq. (57):

$$\frac{1}{2}(\pi + \Delta\varphi) \quad = \int_0^{1/R_0}\left(\frac{1}{{R_0}^2}-\frac{1}{a^2}-u^2\right)^{-1/2} du = \sin^{-1}\left(\frac{1}{R_0}\left|\left(\frac{1}{{R_0}^2}-\frac{1}{a^2}\right)\right|^{-1/2}\right) \tag{60}$$

An integral of this type can be evaluated [13] by:

$$\int\frac{dx}{(\alpha^2-x^2)^{1/2}} = \sin^{-1}\frac{x}{|\alpha|} \tag{61}$$

and is real valued for

$$x < |\alpha| \quad . \tag{62}$$

Therefore the valid upper bound for the integral:

$$\frac{\pi}{2}+\frac{\Delta\varphi}{2} = \int_0^{\alpha}\left(\frac{1}{{R_0}^2}-\frac{1}{a^2}-u^2\right)^{-1/2} du \tag{63}$$

is

$$\alpha = \left(\frac{1}{{R_0}^2}-\frac{1}{a^2}\right)^{1/2} \tag{64}$$

in which case:

$$\frac{\pi}{2} = \sin^{-1}\left(\frac{\alpha}{\alpha}\right) = \sin^{-1} 1 \quad . \tag{65}$$

Now approximate this result by:

$$\frac{\pi}{2} + \frac{\Delta\varphi}{2} \sim \sin^{-1}\left(\frac{1/\,\mathrm{R_0}^2 - 1/\,a^2}{1/\,\mathrm{R_0}^2}\right)^{½} \tag{66}$$

where it has been assumed that:

$$\frac{1}{\mathrm{R_0}^2} >> \frac{1}{a^2} \quad . \tag{67}$$

Then:

$$\sin^2\left(\frac{\pi}{2} + \frac{\Delta\varphi}{2}\right) = \cos^2\frac{\Delta\varphi}{2} \sim 1 - \left(\frac{\mathrm{R_0}}{a}\right)^2 \tag{68}$$

and in this approximation:

$$\left(\frac{\mathrm{R_0}}{a}\right)^2 \sim \frac{1}{2}\left(\frac{\Delta\varphi}{2}\right)^2 \tag{69}$$

This gives a photon mass of:

$$m = \left(\frac{\hbar\omega}{c^2}\right)\frac{\Delta\varphi}{2\sqrt{2}} = 3.35 \text{ x } 10^{-41} \text{ kg} \quad . \tag{70}$$

A recent review [14] of work on photon mass makes a best estimate of less than order $10^{-43}$ kilograms.

Therefore the photon mass has been determined for the first time. In a later paper the integral in Eq.(50) will be evaluated numerically, and a detailed analysis given of the result.

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## References.


[1] M. W. Evans et al., “Generally Covariant Unified Field Theory” (Abramis 2005 onwards) in seven volumes to date.

[2] M. W. Evans, S. Crothers, H. Eckardt and K. Pendergast, “Criticisms of the Einstein Field Equation” (Abramis in press, 2010).

[3] The main ECE website www.aias.us (in the British national website collection www.webarchive.org.uk), c/o National Library of Wales.

[4] Other relevant websites: www.atomicprecision.com, www.upitec.org, www.et3m.net.

[5] K. Pendergast, “The Life of Myron Evans” (Abramis, in press, 2010).

[6] M. W. Evans (ed.), “Modern Nonlinear Optics” (Wiley 2001, second edition), in three volumes.

[7] M. W. Evans and S. Kielich (eds.), ibid., first edition (Wiley, 1992, 1993, 1997), in three volumes.

[8] M. W. Evans and J.-P. Vigier, “The Enigmatic Photon” (Kluwer, 1994 to 2002) in five volumes.

[9] M. W. Evans and L. B. Crowell, “Classical and Quantum Electrodynamics and the B(3) Field” (World Scientific, 2001).

[10] M. W. Evans et al., papers on ECE in “Foundations of Physics Letters”, “Physica B” and “Acta Physica Polonica”, and two conference papers.

[11]} R. M. Wald, “General Relativity” (University of Chicago Press, 1984), Eq. (6.4.41), page 145.

[12] J. B. Marion and S. T. Thornton, “Classical Dynamics” (HB College Publishing, 1988, third edition).

[13] www.numberempire.com/intercalculator.php .

[14] L.- C. Tu, J. Luo and G.T. Gillies, Rep. Prog. Phys. , **68** , 77 (2005).