The relativistic quantum force equation.

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Abstract.

The force equation of relativistic quantum mechanics is derived from ECE generally covariant unified field theory and the various terms defined in the relativistic hamiltonian. The correct origin of relativistic quantum mechanics is now known to be the fermion equation, the first single particle equation of the fermion because it can be derived without negative energy. Having derived and defined the hamiltonian in a Schroedinger like equation, the relativistic force equation follows. The first eigenvalues of the relativistic quantum force equation are computed.

Keywords: ECE generally covariant unified field theory, relativistic quantum force equation.

1. Introduction.

In recent papers of this series [1-10] two kinds of novel force equation of quantum mechanics have been derived from the structure of generally covariant unified field theory. The first one is the quantum Hamilton equation, and the second a more useful force equation which was derived in the preceding paper UFT 176 from the Schroedinger equation, itself a limit of the ECE fermion equation derived in UFT 172 ff. from Cartan geometry [11]. Hamilton´s dynamics [12] give the force equation of the second kind from the Schroedinger equation using the independence of Hamilton´s canonical variables. The non relativistic quantum force equation of the second kind uses the wavefunctions and eigenvalues of the Schroedinger equation to derive force eigenvalues straightforwardly for the first time in quantum mechanics. It is well known that computational quantum chemistry provides the necessary wavefunctions and eigenvalues for a range of systems: atoms, molecules, semiconductors, nanostructures, bionanostructures, DNA self structuring and so forth. In UFT 176 simple examples of force eigenvalues were given from exact solutions such as the harmonic oscillator and H atom. For the harmonic oscillator every force eigenvalue was found to be the same as the classical value, including the zero point force eigenvalue which indicates the presence in the vacuum of a hitherto unknown force of utility for new energy research. For the H atom it was found that the eigenvalue of force for the 1s orbital is zero, giving the first correct explanation for the stability of the 1s orbital. Characteristic patterns of force eigenvalues were found for other H atom orbitals, patterns that can be extended to all aspects of quantum mechanics to give an entirely new science of materials, a new type of quantum field theory, and new type of quantum optics.

In Section 2 this work is extended to the relativistic quantum force equation, which is derived from the fermion equation of ECE theory without the use of negative energy. The fermion equation is written as two simultaneous equations which can also be derived [13] from the Lorentz transform of the Pauli spinor. These equations are subject to a well defined mathematical transformation which gives the various terms in the relativistic hamiltonian in a Schroedinger like approximation [14, 15]. These include the mass term, spin orbit term, and Darwin term, and a relativistic correction of the kinetic energy term of the Schroedinger equation´s hamiltonian operator.

In Section 3 some of the force eigenvalues of relativistic quantum mechanics are computed for the first time from the force equation of the second kind extended to special relativity. These include force eigenvalues for spin orbit interaction, which is responsible for the important fine structure in atomic and molecular spectroscopy.

2. Derivation from the fermion equation.

Consider the fermion equation of UFT 172 ff. in the format:

$$(\hat{E} - \mathrm{c}\,\boldsymbol{\sigma}.\hat{\boldsymbol{p}})\,\varphi^R = m\,c^2\varphi^L \tag{1}$$

$$(\hat{E} + \mathrm{c}\,\boldsymbol{\sigma}.\hat{\boldsymbol{p}})\,\varphi^L = m\,c^2\varphi^R \tag{2}$$

where E is the total energy, regarded as an eigenvalue, and where $\hat{\boldsymbol{p}}$ is the operator defined by Schroedinger´s axiom, the basis of quantum mechanics:

$$\hat{\boldsymbol{p}}\,\psi = -\,i\,\hbar\,\nabla\psi \tag{3}$$

where $\hbar$ is the reduced Planck constant and ψ a wavefunction. Here $\boldsymbol{\sigma}$ is the Pauli vector made up of Pauli matrices, m is the mass, c the speed of light in vacuo, and φ^L and φ^R the left and right Pauli spinors, which in ECE theory are derived from elements of the tetrad in Cartan geometry. In the presence of a potential energy, denoted:

$$V = \mathrm{e}\,\varphi \tag{4}$$

where φ is the scalar potential and - e the charge on the electron, Eqs. (1) and (2) become:

$$(\hat{E} - V - c\,\boldsymbol{\sigma}\,.\,\hat{\boldsymbol{p}}\,)\,\varphi^R = m\,c^2\,\varphi^L \tag{5}$$

$$(\hat{E} - V + c\,\boldsymbol{\sigma}\,.\,\hat{\boldsymbol{p}}\,)\,\varphi^L = m\,c^2\,\varphi^R \tag{6}$$

As in UFT 173 make the mathematical transformations:

$$\varphi^R = \varphi_S^R + \varphi_S^L \tag{7}$$

$$\Phi^L = \varphi_S^R - \varphi_S^L \tag{8}$$

which transform Eqs. (5) and (6) into:

$$(\,E - V - m\,c^2\,)\,\varphi_S^R = c\,\boldsymbol{\sigma}\,.\,\hat{\boldsymbol{p}}\,\varphi_S^L \tag{9}$$

$$(E - V + m\,c^2\,)\,\varphi_S^L = c\,\boldsymbol{\sigma}\,.\,\hat{\boldsymbol{p}}\,\varphi_S^R \tag{10}$$

The energy relative to the rest energy, the relativistic kinetic energy, is defined as:

$$\square = T = E - m\,c^2 \tag{11}$$

so Eqs. (9) and (10) become:

$$c\,\boldsymbol{\sigma}\,.\,\hat{\boldsymbol{p}}\,\varphi_S^L = (\square - V\,)\,\varphi_S^R \tag{12}$$

$$c\,\boldsymbol{\sigma}\,.\,\hat{\boldsymbol{p}}\,\varphi_S^R = (\square - V + 2\,m\,c^2)\,\varphi_S^L \quad . \tag{13}$$

From Eq. (13),

$$\varphi_S^L = \frac{c\,\boldsymbol{\sigma}\,.\,\hat{\boldsymbol{p}}}{\square + V + 2\,m\,c^2}\,\varphi_S^R \tag{14}$$

so in Eq. (12):

$$\in \varphi_S^R = V\,\varphi_S^R + c^2 \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}} \left(\frac{c\,\boldsymbol{\sigma}\,\hat{\boldsymbol{p}}}{\square\square V + 2m c^2}\right) \tag{15}$$

in which the operator $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}$ operates on the complete term inside the brackets using the Leibniz Theorem.

In the approximation:

$$\left(1 + \frac{\square\square V}{2m c^2}\right)^{-1} \square\ 1 - \left(\frac{\square\square V}{2m c^2}\right) \tag{16}$$

the term inside the brackets becomes:

$$\in \varphi_S^R = V\,\varphi_S^R + \frac{1}{2m}\,\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}} \left(\left(1 - \left(\frac{\square\square V}{2m c^2}\right)\right) \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\,\varphi_S^R\right. \tag{17}$$

so Eq. (15) becomes the Schroedinger-like equation:

$$\in \psi = V\,\psi + \frac{\hat{p}^2}{2m}\,\psi - \frac{\boldsymbol{\sigma}\,\hat{p}}{4m^2c^2}\left((\in - V)\,\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\,\psi\right) \tag{18}$$

which can be expanded to:

$$\in \psi = \left(V + \frac{\hat{p}^2}{2m}\right)\psi - \frac{1}{4m^2c^2}\,\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\,(\in \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\,\psi) + \frac{1}{4m^2c^2}\,\sigma \cdot \hat{p}\,(V\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\,\psi)\ ;\ \psi = \varphi_S^R \tag{19}$$

In the approximation:

$$v << c \tag{20}$$

the energy $\in$, the relativistic kinetic energy T is approximated by:

$$T = E - m\,c^2 \longrightarrow \frac{p^2}{2m} \tag{21}$$

so using this approximation in the third term of the right hand side of Eq. (19) gives:

$$\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\,\frac{p^2}{2m}\,\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}} = \frac{\hat{p}^4}{2m} \tag{22}$$

and Eq. (19) becomes:

$$\in \psi = \left(V + \frac{\hat{p}^2}{2m}\right)\psi - \frac{\hat{p}^4}{8m^3c^2}\,\psi + \frac{1}{4m^2c^2}\,\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\,(V\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\,\psi) \tag{23}$$

where we have used:

$$\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\, \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}} = \hat{p}^2 \quad . \tag{24}$$

The Schroedinger equation is the non relativistic approximation:

$$\in \psi = (V + \frac{\hat{p}^2}{2m}) \psi \qquad . \tag{25}$$

The third term on the right hand side of Eq. (23) is the mass term, omitted in some approximations [13]. The fourth term is:

$$\frac{1}{4m^2c^2}\, \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\, (V \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\, \psi) = \frac{\hbar i \hbar}{4m^2c^2}\, \boldsymbol{\sigma} \cdot \boldsymbol{\nabla}\, (V \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\, \psi) \tag{26}$$

The right hand side is expanded with the Leibniz Theorem:

$$\boldsymbol{\sigma} \cdot \boldsymbol{\nabla}\, (V \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\, \psi) = \boldsymbol{\sigma} \cdot \boldsymbol{\nabla}\, V \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\, \psi + \mathrm{V}\, (\boldsymbol{\sigma} \cdot \boldsymbol{\nabla})\, (\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\, \psi) \quad . \tag{27}$$

The electric field strength $\boldsymbol{E}$ is defined as

$$\boldsymbol{E} = -\, \boldsymbol{\nabla}\, \Phi \tag{28}$$

so:

$$\boldsymbol{\sigma} \cdot \boldsymbol{\nabla}\, V \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\, \psi = -\, e\, \boldsymbol{\sigma} \cdot \boldsymbol{E}\, \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\, \psi \quad . \tag{29}$$

By Pauli algebra:

$$\boldsymbol{\sigma} \cdot \boldsymbol{E}\, \boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}} = \boldsymbol{E} \cdot \hat{\boldsymbol{p}} + i\, \boldsymbol{\sigma} \cdot \boldsymbol{E} \times \hat{\boldsymbol{p}} \quad . \tag{30}$$

Therefore:

$$\hat{H}\, \psi = \in \psi \quad , \tag{31}$$

$$\hat{H} = e\, \varphi + \frac{\hat{p}^2}{2m} - \frac{\hat{p}^4}{8m^3c^2} - \frac{e\hbar}{4m^2c^2}\, \boldsymbol{\sigma} \cdot \boldsymbol{E} \times \hat{\boldsymbol{p}} - \frac{\hbar^2}{4m^2c^2}\, (\boldsymbol{\nabla}\, V)\, \boldsymbol{\nabla} - \frac{\hbar^2}{4m^2c^2}\, V \nabla^2 \tag{32}$$

Eq. (31) therefore becomes the Schroedinger type equation:

$$\hat{H}\, \psi = \in \psi \quad , \tag{33}$$

where the hamiltonian is:

$$\hat{H} = \mathbf{V} - \frac{\hbar^2}{2m} (1 + \frac{V}{2m^2c^2\hbar}) \nabla^2 + \frac{\hbar^4 \hbar^4}{8m^3c^2\hbar} - \frac{e\hbar}{4m^2c^2}\, \boldsymbol{\sigma} \cdot \boldsymbol{E} \times \hat{\boldsymbol{p}} - \frac{\hbar^2}{4m^2c^2}\, (\boldsymbol{\nabla}\, V)\, \boldsymbol{\nabla} \quad . \tag{34}$$

The fourth term is the spin orbit term with the correct Thomas factor, the fifth term is the Darwin term, and the sixth term a relativistic correction of the kinetic energy operator of the Schroedinger equation. In a Coulomb potential in radial coordinates:

$$\varphi = -\frac{e}{4\pi\epsilon_0 r} \tag{35}$$

the electric field strength is:

$$\boldsymbol{E} = -\frac{e^2}{4\pi\epsilon_0}\frac{\boldsymbol{r}}{r^3} \tag{36}$$

where ϵ_0 is the S. I. vacuum permittivity. So the spin orbit hamiltonian is:

$$\hat{H}_{SO} = \frac{e^2}{16\pi\epsilon_0 m^2 c^2 r^3}\,\boldsymbol{\sigma}\,.\,\hat{\boldsymbol{L}} \tag{37}$$

where $\hat{\boldsymbol{L}}$ is the orbital angular momentum operator. The spin angular momentum operator is defined as:

$$\hat{\boldsymbol{S}} = \frac{1}{2}\,\hbar\,\hat{\boldsymbol{\sigma}} \tag{38}$$

where $\hat{\boldsymbol{\sigma}}$ is also a vector operator. Therefore:

$$\hat{H}_{SO} = \frac{e^2}{8\pi\epsilon_0 m^2 c^2 r^3}\,\hat{\boldsymbol{S}}\,.\,\hat{\boldsymbol{L}} = \xi\,(r)\,\hat{\boldsymbol{S}}\,.\,\hat{\boldsymbol{L}} \tag{39}$$

and the complete hamiltonian operator in this approximation to the fermion equation is:

$$\hat{H} = \mathbf{V} - \frac{\hbar^2}{2m}\left(1 + \frac{V}{2m^2c^2}\right)\nabla^2 + \frac{\hbar^4}{8m^3c^2} + \hat{\boldsymbol{H}}_{SO} - \frac{\hbar^2}{4m^2c^2}\,(\boldsymbol{\nabla}\,V)\,\boldsymbol{\nabla}\ . \tag{40}$$

Finally, the force equation of the second kind is obtained as:

$$(\hat{H} - \in)\,\boldsymbol{\nabla}\,\psi = \boldsymbol{F}\,\psi \tag{41}$$

in spherical polar coordinates. In contemporary code packages in computational quantum chemistry a hamiltonian of this type is used as a starting point, so the force equation (41) can be incorporated in code packages to give an unlimited amount of new information.

Now apply the force equation (41) to atomic H with the simplified hamiltonian [14]:

$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V + \hat{H}_{SO}\quad . \tag{42}$$

The wavefunction of the state $|nlm_l>$ of H is:

$$|nlm_l > R_{nl}(r)\, Y_{lm_l}(\theta, \varphi) \tag{43}$$

which is a product of well known radial and spherical harmonic components. The radial average of the spin orbit hamiltonian [14] is:

$$\hbar\, c\, \xi_{nl} = \hbar^2 \int_0^\infty \xi(r)\, R_{nl}{}^2(r) r^2 dr \tag{44}$$

and gives the average energy of interaction of an electron in a given orbital with its own spin.

In a hydrogen-like atom with atomic number *Z*, the Coulomb potential energy is:

$$V = \frac{\square Z e^2}{4\pi\epsilon_0 r} \quad . \tag{45}$$

For H:

$$Z = 1\ , \tag{46}$$

and for H:

$$\xi(r) = \frac{e^2}{8\pi\epsilon_0 m^2 c^2 r^3} \quad . \tag{47}$$

Therefore:

$$< \frac{1}{r^3} >_{nl} = \frac{1}{a_0^3 n^3 l(l+\square)\square l+1\square} \quad , \tag{48}$$

where the Bohr radius is

$$a_0 = \frac{4\pi\square^2\epsilon_0}{me^4} \quad . \tag{49}$$

Therefore for an electron in H with quantum number *n* and *l*:

$$\xi_{nl} = \frac{R_\infty \alpha^2}{n^3 l(l+\square)\square l+1\square} \tag{50}$$

where the fine structure constant is:

$$\alpha = \frac{e^2}{4\pi\square c\epsilon_0} = 0.007297351 \tag{51}$$

and

$$\hbar c R_\infty = \frac{me^4}{8\epsilon_0{}^2 \square^2} = 2.179908 \times 10^{-18} \text{ J.} \quad (52)$$

In general:

$$\xi_{nl} = \frac{R_\infty \alpha^2 Z^4}{n^3 l(l+\square)\square l+1\square} \quad (53)$$

so spin orbit interaction is proportional to Z^4 and in large atoms is very important.

The first order correction of a state $|nls\ ;jm_j>$ from perturbation theory is:

$$E_{SO} = < nls\ ;jm_l\ |\hat{H}_{SO}\ |\ nls\ ;jm_l > \quad (54)$$

so:

$$(\hat{H} - E + E_{SO}\ )\ \nabla\psi = \boldsymbol{F}\ \psi \quad (55)$$

We have:

$$\hat{\boldsymbol{L}}\ .\ \hat{\boldsymbol{S}}\ |\ nls\ ;jm_l > = \frac{1}{2}\ (\ j^2 - l^2 - s^2\ )\ |\ nls\ ;jm_l > \quad (56)$$

because:

$$\hat{\boldsymbol{J}}^{\,2} = |\hat{\boldsymbol{L}} + \hat{\boldsymbol{S}}|^2 \ = \hat{\boldsymbol{L}}^2 + \hat{\boldsymbol{S}}^2 + 2\hat{\boldsymbol{L}}.\hat{\boldsymbol{S}} \quad . \quad (57)$$

So:

$$E_{SO} = \frac{1}{2}\ \hbar^2\ (\ \mathrm{j(j+1)} - l(l+1) - s(\mathrm{s}+1)) < nls\ ;jm_l\ |\ \xi\ (r)\ |\ nls\ ;jm_l >$$

$$= \frac{1}{2}\ \hbar c\ \xi_{nl}\ (\ \mathrm{j(j+1)} - l(l+1) - s(\mathrm{s}+1))$$

$$= \alpha^2 \hbar c R_\infty \left[\ \frac{\mathrm{j}\square\mathrm{j}\square+\square 1\square\square\square l\square l\square+1\square\square\square s\square\mathrm{s}\square+1\square}{n^3 l(l+\square)\square l+1\square}\ \right] \quad (58)$$

Therefore the force eigenvalues are evaluated from Eqs. (55) and (58) using the H wavefunctions ψ in the first approximation. This is because spin orbit splitting in H is very small. For example for the 2 p electron:

$$n = 2\ ,\ l = 1 \quad (59)$$

energy level separations and transition wavenumbers are of the order

$$R_\infty \approx 10^5 \text{ cm}^{-1} \tag{60}$$

The magnitude of the spin orbit interaction constant is $\alpha^2 R_\infty/24$, i..e $R_\infty/ 4.51 \text{ x } 10^5$ fractions of a wavenumber. So using the H wavefunctions ψ is a very good approximation. The permitted states of angular momentum from a system made up of two sources of angular momentum are given by the well known [14, 15] Clebsch Gordan series:

$$\left.\begin{aligned} &j = j_1 + j_2 , j_1 + j_2 - 1, \ldots , | j_1 - j_2 | , \\ &m_j = m_{j1} + m_{j2} \end{aligned}\right\} \tag{61}$$

so for a 2p electron:

$$j = \frac{3}{2} \quad \text{and} \quad \frac{1}{2} \tag{62}$$

and in Eq. (58):

$$n = 2 \quad , \quad l = 1 \quad , \quad j = \frac{3}{2} \text{ and } \frac{1}{2} \quad , \quad s = \frac{1}{2} \tag{63}$$

so $\boldsymbol{F}$ can be worked out from Eq. (55).

3. Computer evaluation of force eigenvalues.

In the previous section the spin-orbit splitting of force eigenvalues has been evaluated. Here we give the corresponding analytical values for Hydrogen obtained by computer algebra. After that we present the splitting of force eigenvalues by a magnetic field.

3.1 Spin-orbit splitting

For Hydrogen, the Hamilton operator with neglect of all relativistic effects except spin-orbit coupling has been given in Eq. (42) and the following equations. The relativistic quantum numbers j and m_j have to be used. For s states there is no spin-orbit splitting because the angular momentum is zero. Using the analytical wave functions given in [14], the force eigenvalues F listed in Table 1 are obtained from Eq. (55) where a is the Bohr radius and m the electron mass. The energy E_{so} has to be replaced by Eq. (58) for completeness. The force values in Table 1 are classified according to the non-relativistic quantum number l; the relativistic spin splitting is introduced by using the terms $j = l \pm s$ in the formula for E_{so}. The results have been graphed in Figs. 1-3. It can be seen that the splitting is symmetric outside the core region. There are poles where the wave function has a zero crossing, leading to finite values of $F \cdot \psi$ as already discussed in UFT paper 177.

n	*l*	E	F
1	0	$-\frac{\hbar^2}{ma^2}$	0
2	0	$-\frac{\hbar^2}{8ma^2}$	0
2	1	$-\frac{\hbar^2}{8ma^2}$	$-\frac{▫mr^3-2amr^2▫E_{so}+r\hbar^2-2a\hbar^2}{2amr^3}$
3	0	$-\frac{\hbar^2}{18ma^2}$	0
3	1	$-\frac{\hbar^2}{18ma^2}$	$-\frac{▫mr^4-12amr^3+18a^2mr^2▫E_{so}+4r^2\hbar^2-36ar\hbar^2+54a^2\hbar^2}{3amr^4-18a^2mr^3}$
3	2	$-\frac{\hbar^2}{18ma^2}$	$-\frac{▫mr^3-6amr^2▫E_{so}+2r\hbar^2-12a\hbar^2}{3amr^3}$

Table 1. Energy and force eigenvalues for the Hydrogen with spin-orbit splitting energy E_{so}.

3.2 Magnetic splitting

Degenerate spectral lines are split by an external magnetic field as known by the Zeeman and Paschen-Back effect. The Hamilton operator in non-relativistic approximation [13] is given in this case by

$$\hat{H} = \frac{\hat{p}^2}{2m} + V - \frac{e\hbar}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}. \tag{64}$$

Assuming only a Z component of **B** then leads to

$$\boldsymbol{\sigma}\cdot\mathbf{B} = \begin{bmatrix} B_Z & 0 \\ 0 & -B_Z \end{bmatrix}, \tag{65}$$

so we obtain two values for $\hat{H}$:

$$\hat{H}_+ = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V - \frac{e\hbar}{2m}B_Z, \tag{66}$$

$$\hat{\mathrm{H}}_{-} = -\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2}{\partial \mathrm{x}^2} + \mathrm{V} + \frac{\mathrm{e}\hbar}{2\mathrm{m}}\mathrm{B_Z}. \tag{67}$$

Assuming a weak magnetic field, the wave function of the Hydrogen atom in the undistorted state can be taken to evaluate the force eigenvalues F_+ and F_- from

$$\left(\hat{\mathrm{H}}_{+} - \mathrm{E}\right)\frac{d\psi}{dx} = \mathrm{F}_{+}\psi, \tag{68}$$

$$\left(\hat{\mathrm{H}}_{-} - \mathrm{E}\right)\frac{d\psi}{dx} = \mathrm{F}_{-}\psi, \tag{69}$$

The results are shown in Table 2, where only the positive value for B_Z has been taken. F_- is obtained by changing sign of B_Z. Setting $B_Z = 0$ gives the force values for the undistorted atom as already derived in UFT paper 177.

In Figs. 4-9 the force eigenvalues for H are shown in an enlarged scale in order to make the effects visible. The radially weighted probability density $\psi \cdot r^2$ is plotted for comparison. F_+ and F_- show a symmetric splitting of the original F (with $B_Z = 0$). Beyond a certain nodal point the splitting is nearly quadratically increasing with the radius coordinate. Also the $s=0$ state is split, where no F is present without a magnetic field. Near to the pole radii, the behaviour is similar to spin-orbit splitting.

n	l	E	$\mathrm{F_+}$
1	0	$-\frac{\hbar^2}{\mathrm{ma}^2}$	$\frac{\hbar \mathrm{eB_Z}}{2\mathrm{am}}$
2	0	$-\frac{\hbar^2}{8\mathrm{ma}^2}$	$\frac{\hbar\,\mathrm{er}^3\mathrm{B_Z} - 4\mathrm{aer}^2\mathrm{B_Z} - 4\mathrm{r}\hbar + 8\mathrm{a}\hbar}{4\mathrm{amr}^2}$
2	1	$-\frac{\hbar^2}{8\mathrm{ma}^2}$	$\frac{\hbar\,\mathrm{r} - 2\mathrm{a}\,\mathrm{er}^2\mathrm{B_Z} - 2\hbar}{4\mathrm{amr}^3}$
3	0	$-\frac{\hbar^2}{18\mathrm{ma}^2}$	$\frac{\hbar\,2\mathrm{er}^4\mathrm{B_Z} - 30\mathrm{aer}^3\mathrm{B_Z} + 81\mathrm{a}^2\mathrm{er}^2\mathrm{B_Z} - 20\mathrm{r}^2\hbar + 228\mathrm{ar}\hbar - 486\mathrm{a}^2\hbar}{6\mathrm{amr}^2\,2\mathrm{r}^2 - 18\mathrm{ar} + 27\mathrm{a}^2}$
3	1	$-\frac{\hbar^2}{18\mathrm{ma}^2}$	$\frac{\hbar\,\mathrm{er}^4\mathrm{B_Z} - 12\mathrm{aer}^3\mathrm{B_Z} + 18\mathrm{a}^2\mathrm{er}^2\mathrm{B_Z} - 8\mathrm{r}^2\hbar + 72\mathrm{ar}\hbar - 108\mathrm{a}^2\hbar}{6\mathrm{amr}^2\,\mathrm{r} - 6\mathrm{a}}$
3	2	$-\frac{\hbar^2}{18\mathrm{ma}^2}$	$\frac{\hbar\,\mathrm{r} - 6\mathrm{a}\,\mathrm{er}^2\mathrm{B_Z} - 4\hbar}{6\mathrm{amr}^3}$

Table 2. Energy and force eigenvalues for the Hydrogen with magnetic field B_Z.

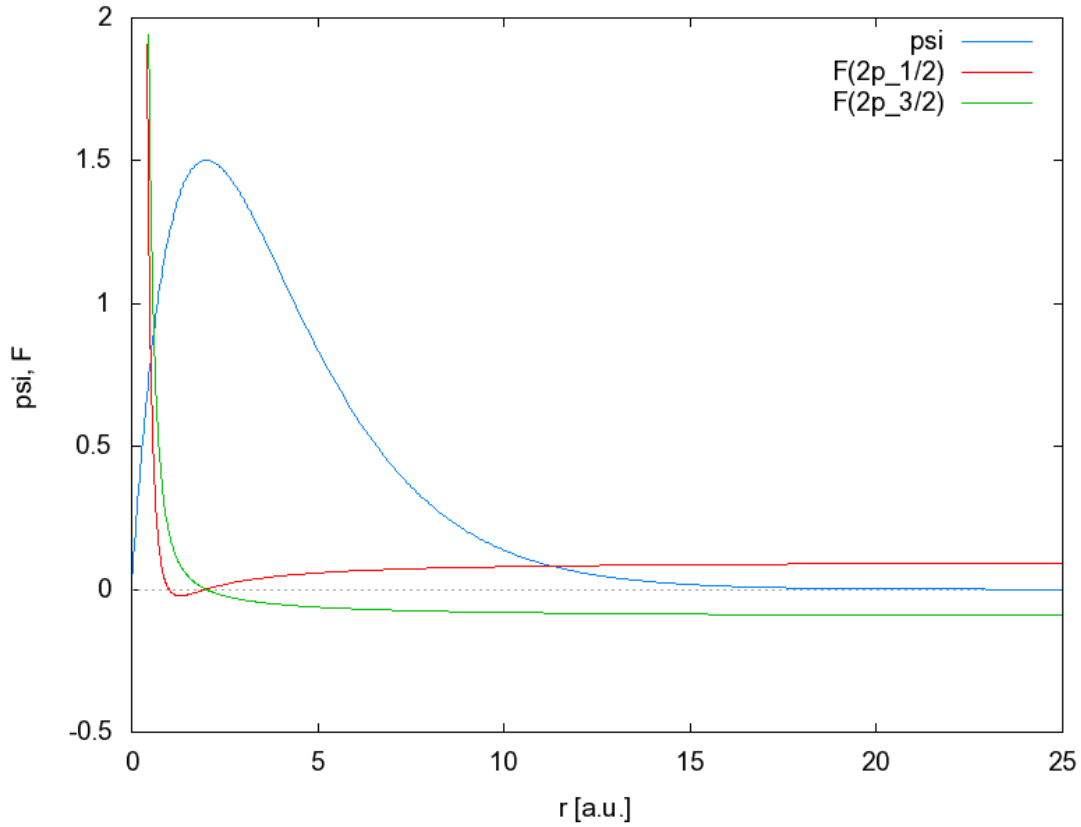

Fig. 1. Spin-orbit splitting of Force eigenvalue for H 2p.

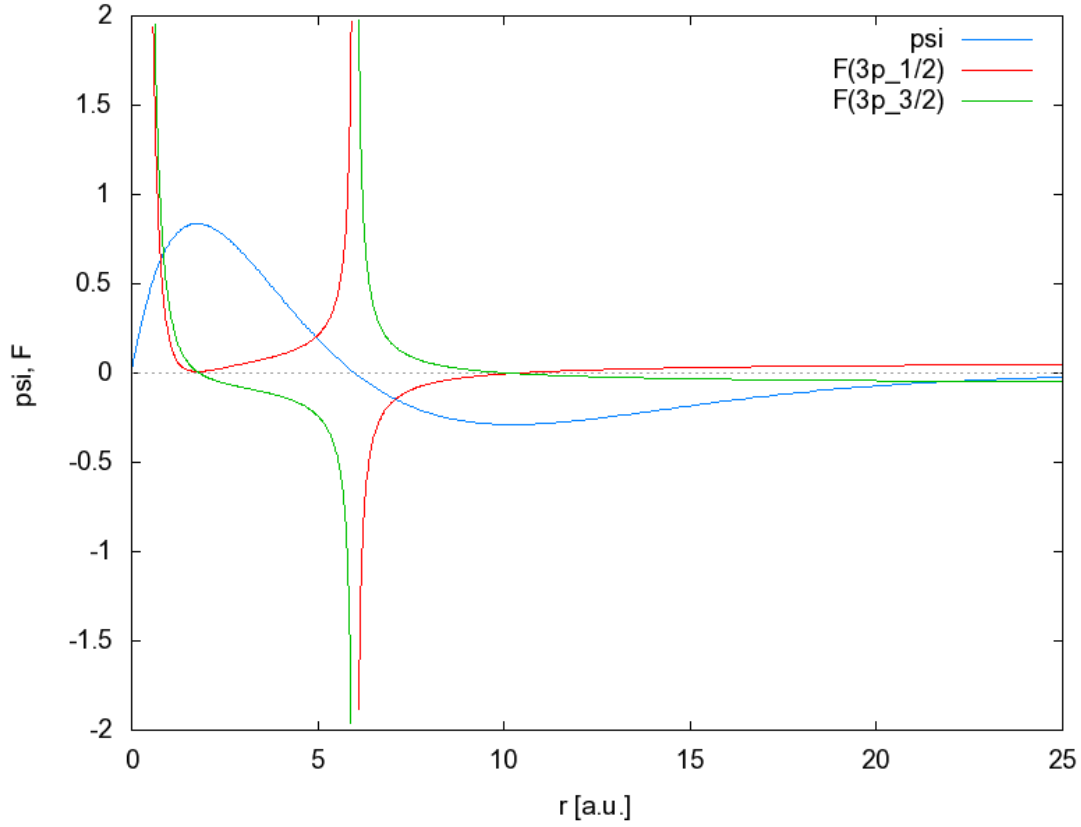

Fig. 2. Spin-orbit splitting of Force eigenvalue for H 3p.

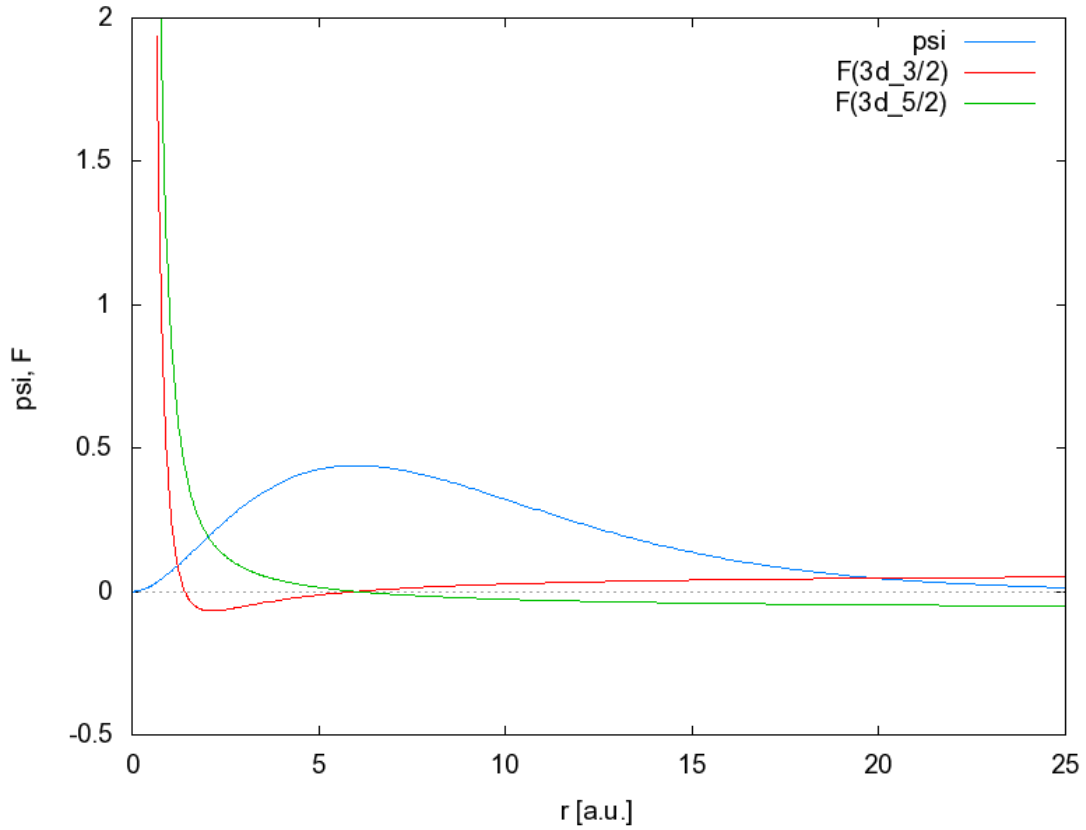

Fig. 3. Spin-orbit splitting of Force eigenvalue for H 3d.

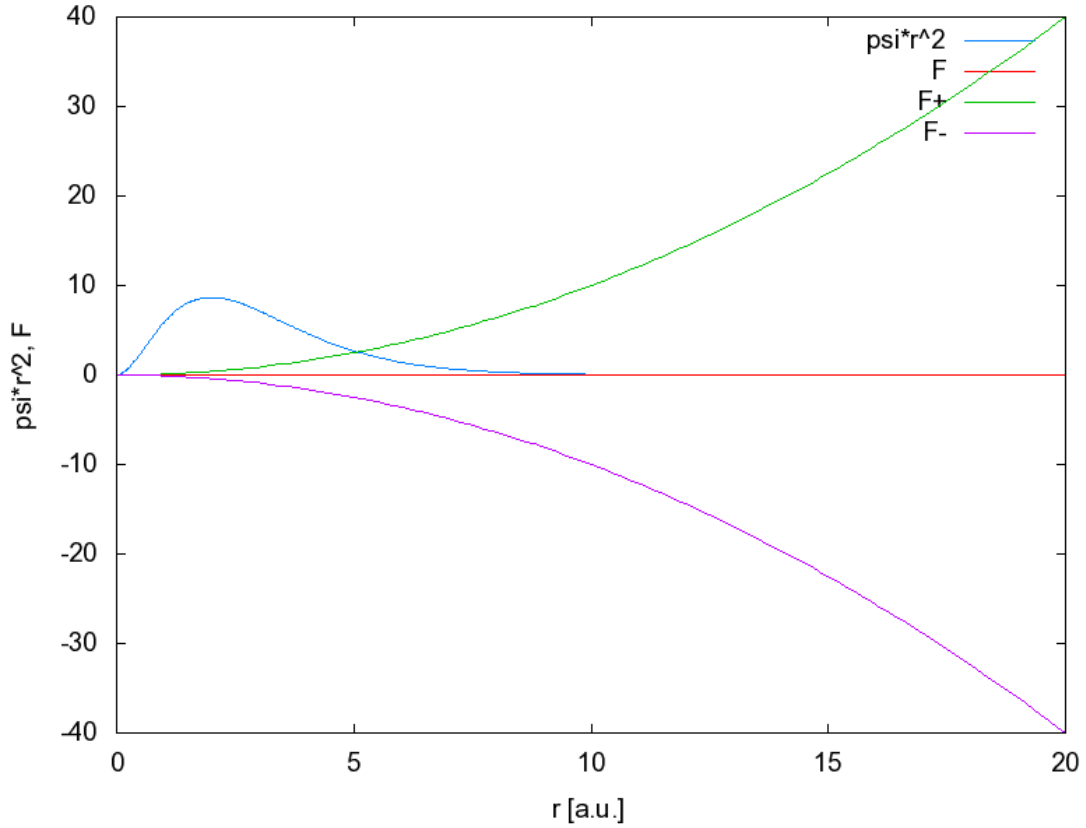

Fig. 4. Magnetic field splitting of Force eigenvalue for H 1s.

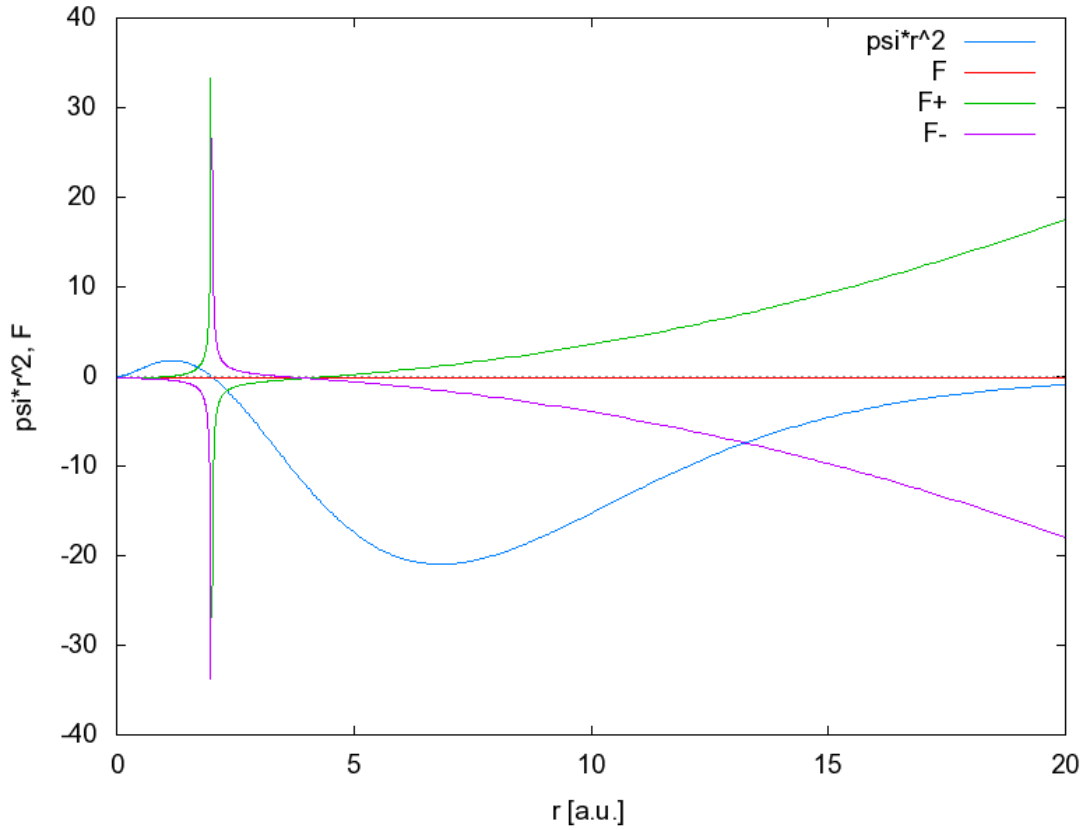

Fig. 5. Magnetic field splitting of Force eigenvalue for H 2s.

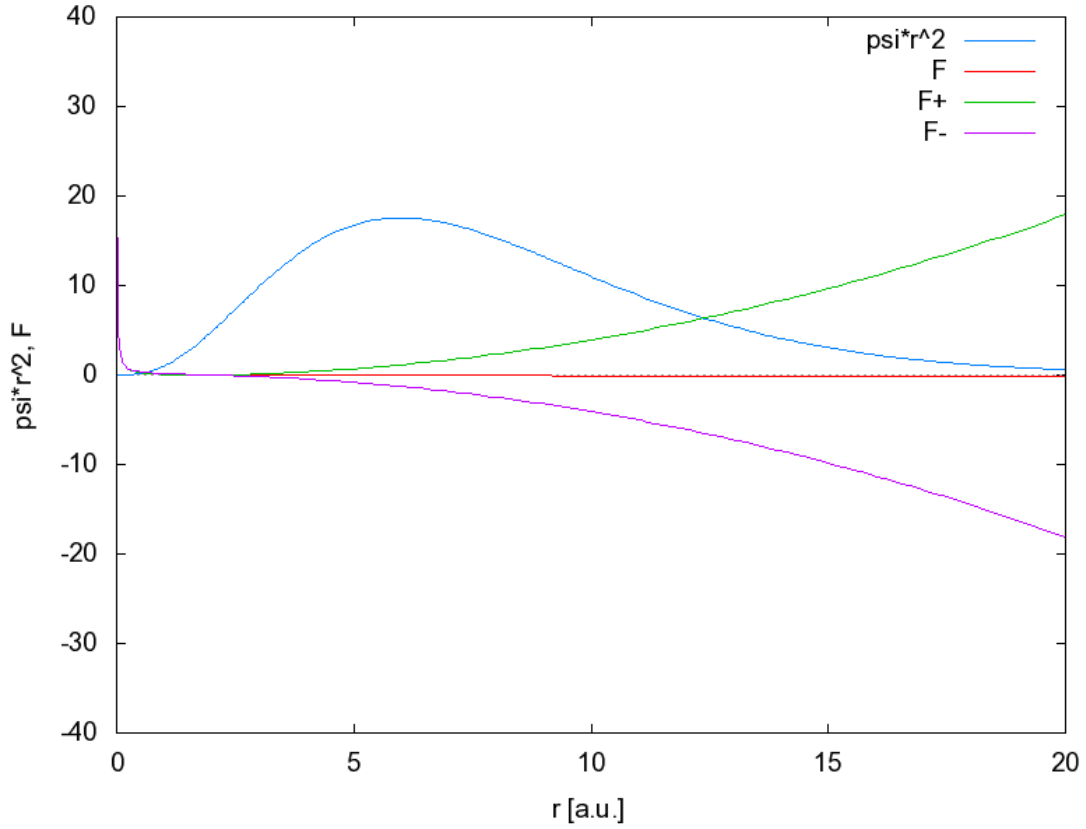

Fig. 6. Magnetic field splitting of Force eigenvalue for H 2p.

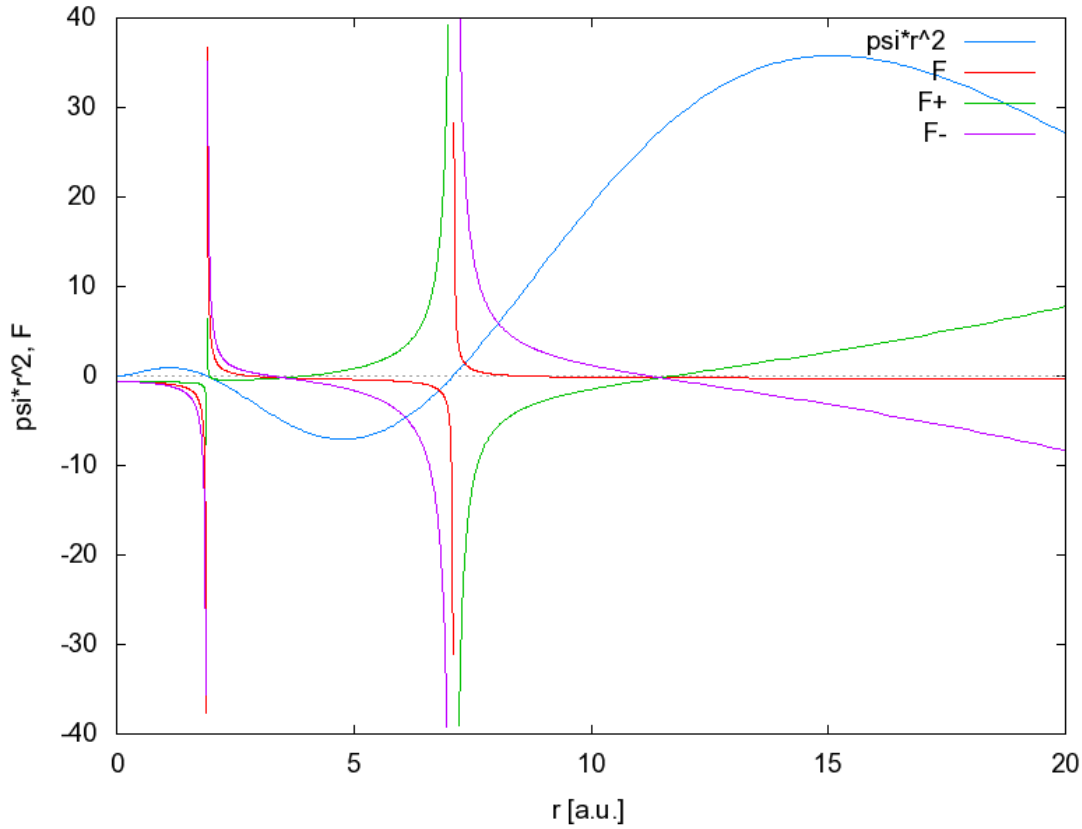

Fig. 7. Magnetic field splitting of Force eigenvalue for H 3s.

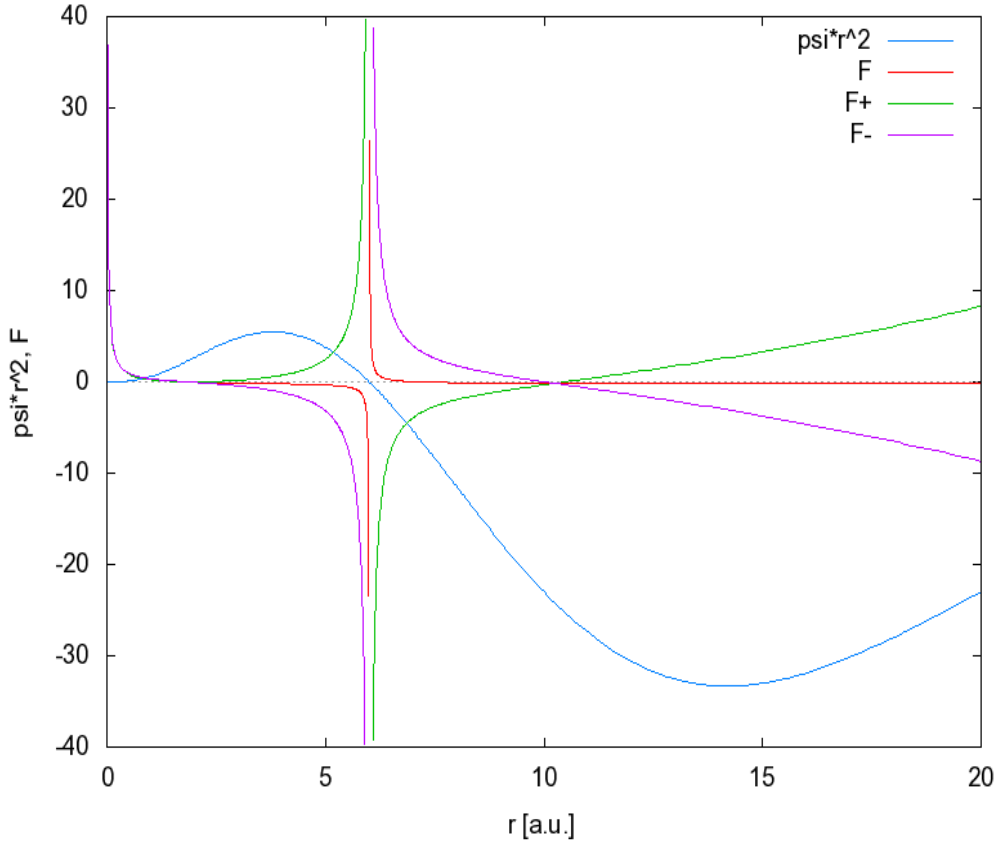

Fig. 8. Magnetic field splitting of Force eigenvalue for H 3p.

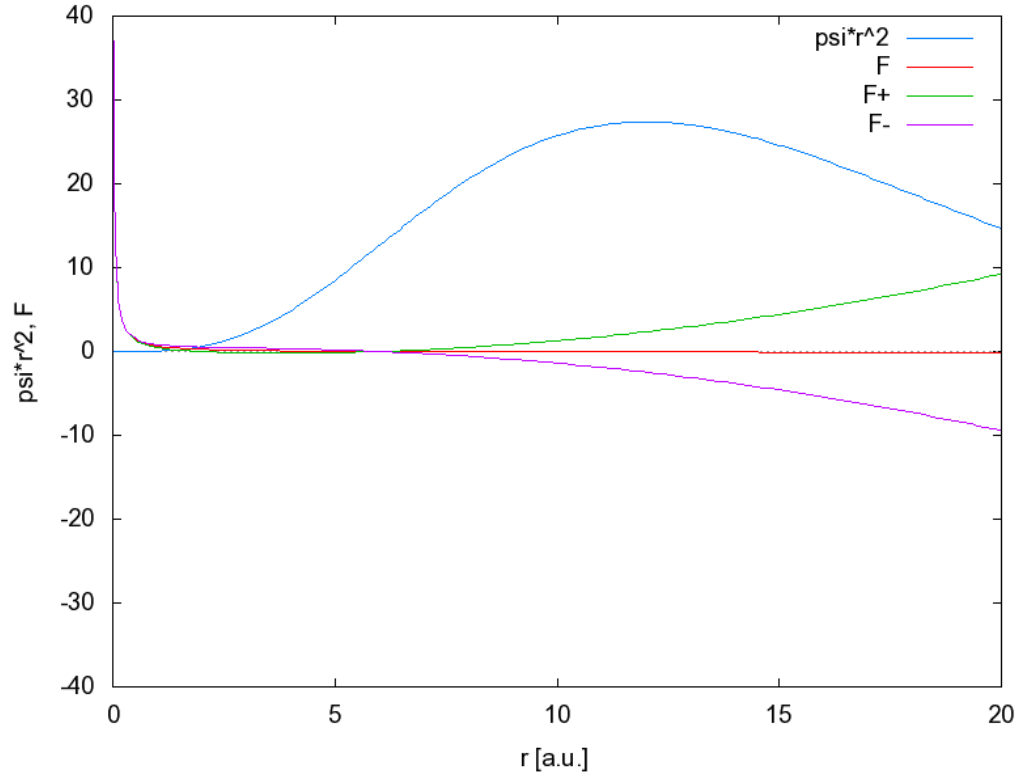

Fig. 9. Magnetic field splitting of Force eigenvalue for H 3d.

Acknowledgments.

The British Government is thanked for a Civil List Pension, and the staff of AIAS for many interesting discussions. Alex Hill is thanked for typesetting, David Burleigh for posting, and Simon Clifford for help with broadcasting.

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