THE GENERALLY COVARIANT FERMION EQUATION

by

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ABSTRACT

The generally covariant fermion equation is developed from the tetrad postulate of Cartan geometry and the ECE wave equation. The concept of covariant mass is used to define the scalar curvature R of the wave equation and generally covariant fermion equation. It is shown that the format of the fundamental energy equation in general relativity is the same as that of the Einstein energy equation of special relativity. The effect of gravitation on atomic and molecular spectra is found by adjusting the mass, so the structure of the equations of atomic and molecular spectroscopy remains the same in the presence of gravitation.

Keywords: ECE theory, generally covariant fermion equation, covariant mass and R theory, the effect of gravitation on spectra.

UFT 179

1. INTRODUCTION

During the course of development of ECE theory {1 - 10} the well known {11} tetrad postulate of Cartan geometry has been developed straightforwardly into the ECE wave equation, whose eigenvalues are denoted R, a scalar quantity with the units of inverse square metres. The wave equation can be reduced to the second order differential format of the fermion equation developed in UFT 172 ff. (www.aias.us). The first order differential format of the fermion equation is the first true single particle equation of physics because it eliminates negative energy - a basically flawed concept that can never be observed (an "unobservable" of physics typical of the extinct standard model, one of many). In Section 2 the format of the ECE wave equation is used to deduce the first order differential format of the generally covariant fermion equation. This format is expressed as two simultaneous equations which can also be deduced from a general coordinate transformation of the Pauli spinors. The general coordinate transformation is the well known transformation of general relativity, it reduces to the Lorentz transform of the Pauli spinors in the limit of special relativity. This format is developed into a Schroedinger like equation by approximating its structure straightforwardly. It is shown that the structure of the fundamental energy equation in general relativity is the same as that of the well known Einstein energy equation of special relativity. Therefore the structures of the fermion equations of general and special relativity are the same. In Section 3 these novel concepts are combined to give an expression for the generally covariant fermion equation in terms of R and in terms of the metric.

2. ECE WAVE EQUATION AND REDUCTION TO FERMION EQUATION.

The ECE wave equation {1 - 10} is:

$$(\square + R) q^a_\mu = 0 \quad -(1)$$

where q^a_μ is the Cartan tetrad and where:

$$R = q^\nu_a \partial^\mu \left(\omega^a_{\mu b} q^b_\nu - \Gamma^\kappa_{\mu\nu} q^a_\kappa \right) \quad -(2)$$

where $\omega^a_{\mu b}$ is the spin connection of Cartan and $\Gamma^\kappa_{\mu\nu}$ the Riemann connection. Eq. (1) is a straightforward re-arrangement of the well known tetrad postulate {11} of differential geometry. The tetrad postulate is:

$$D_\mu q^a_\nu = \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\kappa_{\mu\nu} q^a_\kappa = 0 \quad -(3)$$

and asserts that the vector field in differential geometry is independent of which components and coordinates are used in the general space with torsion and curvature of any dimension. In three dimensional Euclidean space for example, the vector field is the same if expressed in different coordinates, for example Cartesian, circular polar, cylindrical polar, or any curvilinear system. Without the tetrad postulate physics would be meaningless, and only the most abstruse type of pure mathematics could be developed, a type of highly specialized mathematics that is of no use to physics and of little use to mathematics itself.

Eq. (2) can be simplified using the definitions {11}:

$$\omega^a_{\mu\nu} = \omega^a_{\mu b} q^b_\nu \, , \quad \Gamma^a_{\mu\nu} = \Gamma^\kappa_{\mu\nu} q^a_\kappa \quad -(4)$$

and can be reduced to the simple format:

$$R = q^\nu_a \partial^\mu \Omega^a_{\mu\nu} \, , \quad \Omega^a_{\mu\nu} = \omega^a_{\mu\nu} - \Gamma^a_{\mu\nu} . \quad -(5)$$

In order to investigate the effect of gravitation on atomic and molecular spectra, Eq. (1) must be reduced to a first order differential equation - the generally covariant fermion equation. In order to achieve this aim it is advantageous to define R in such a way that it can be related to any valid metric, because metrical theory is highly developed. It is well known

{1 - 11} that the metric tensor is defined in terms of Cartan tetrads:

$$g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab} \quad -(6)$$

where η_{ab} is the metric:

$$\eta_{ab} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \quad -(7)$$

The valid metric must be defined both in terms of curvature and torsion. The Einstein field equation considered curvature only, and for this reason has recently been refuted in comprehensive detail { 4 }. The rigorously correct way of finding a metric tensor is to solve the Cartan structure equations and Bianchi identities of differential geometry, a completely non trivial procedure. The ECE orbital theorem of UFT 111 has been suggested as an approximation to this procedure, an approximation valid in spherically symmetric spacetime.

In general relativity, the hamiltonian in the absence of gravitation is defined from the infinitesimal line element:

$$ds_0^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dZ^2 \quad -(8)$$

where τ is the proper time. In Eq. (8), the cylindrical polar coordinate system is used, in general any coordinate system can be used. Differentiating both sides of Eq. (8):

$$\left(\frac{ds}{d\tau}\right)^2 = c^2 = c^2\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\phi}{d\tau}\right)^2 - \left(\frac{dZ}{d\tau}\right)^2 \quad -(9)$$

a result of metrical geometry. Physics is introduced using the philosophy of relativity by multiplying by the mass m_0. This is defined in the received opinion as the constant quantity measured in standards laboratories, for example the ordinary mass of the electron. The hamiltonian is therefore:

$$H_0 = \frac{1}{2} m_0 c^2 \quad -(10)$$

and in the received opinion is a constant.

In UFT 158 ff however (www.aias.us) it was found that the received opinion is wholly untenable, because the concept of constant mass is not compatible with the Einstein de Broglie theory that is basic to twentieth century physics. For basic self consistency in physics it was found that the mass m_0 must be replaced by R. In UFT 158 ff the novel concept of covariant mass was introduced and defined by:

$$R = \left(\frac{mc}{\hbar}\right)^2. \quad -(11)$$

This definition is one possible definition of the covariant mass, and was developed in UFT 158 ff. In this Section it is shown that the format of the fundamental energy equation in metric based general relativity is the same as that of the well known Einstein energy equation in special relativity.

Consider the infinitesimal line element of the general metric in spherically symmetric spacetime:

$$ds^2 = c^2 d\tau^2 = n(r) c^2 dt^2 - m(r) dr^2 - r^2 d\phi^2 - dZ^2 \quad -(12)$$

where $n(r)$ and $m(r)$ are assumed to be functions of r. For ease of development and simplicity, consider motion in the XY plane

$$dZ = 0. \quad -(13)$$

The hamiltonian is defined as usual in metric based general relativity as:

$$H = \frac{1}{2} m \left(\frac{ds}{d\tau}\right)^2 = \frac{1}{2} mc^2 \quad -(14)$$

and is constant. The lagrangian is the same as the hamiltonian because in this theory potential

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