EXACT SOLUTIONS OF THE TETRAD POSTULATE AND ECE WAVE EQUATION IN TERMS OF THE COVARIANT MASS.

by

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ABSTRACT

Exact and general solutions are given of the fundamental tetrad postulate of differential geometry for the tetrad and spin connections. These solutions are propagating waves of geometry, and indicate the origin of wave mechanics and classical physics in wave geometry when the philosophy of relativity is rigorously applied. The solutions must be expressed, furthermore, in terms of the covariant mass, whose existence is indicated by the recent collapse of any particle collision theory based on the concept of elementary particle of constant mass. A comparison is given of the covariant mass and older metrical theories of general relativity.

Keywords: ECE theory, solutions of the tetrad postulate, wave geometry, covariant mass, R theory.

UFT 180

1. INTRODUCTION.

In order to find the origin of wave (or quantum) mechanics in the philosophy of relativity it is necessary to define the concept of wave geometry, missing completely from the mathematically incorrect standard model of gravitational physics. The most fundamental property of differential geometry is the tetrad postulate, which implies that the complete vector field in any space of any dimension is independent of the way in which it is expressed in terms of components and basis vectors. The tetrad postulate is the link between Riemann and Cartan geometry and between the Riemann and spin connections. During the development of ECE theory {1 - 10} the tetrad postulate was developed into the second order ECE wave equation whose exact and general solutions are given in Section 2 of this paper for the first time. These solutions represent a propagating wave of geometry and there is a direct link between this wave and a wave in boson and fermion equations in quantum mechanics and a potential wave in gravitation or electromagnetism. The solution is developed by writing the ECE wave equation in the classical limit and using the Einstein and de Broglie quantum postulates that make total energy (E) proportional to angular frequency (ω) and relativistic total momentum ($\underline{p}$) proportional to wavenumber ($\underline{\kappa}$) through the reduced Planck constant $\hbar$. This procedure produces an equation linking the covariant mass m to ω and κ . In general, m is not a constant as in the received opinion of particle collision theory. Recently, in UFT 158 ff and 171 it has been shown conclusively that constant mass is not compatible with energy and momentum conservation in scattering and absorption processes and that this result necessitates the development of the concept of covariant mass m, defined directly in terms of a parameter R that can be obtained from the tetrad postulate. Exact and general solutions of the tetrad postulate and ECE wave equation are given for both the tetrad and spin connections, and expressed in terms of the covariant mass m.

In Section 3 a comparison is made of the covariant mass method as applied to the theory of gravitation, and the metric based theory of gravitation of the received twentieth century opinion. Both theories are founded in the philosophy of relativity, but are profoundly different. The covariant mass method has the advantages of elegance and simplicity, and rigorously correct geometry. Surprisingly, the older metric based method was allowed to develop for nearly a century while continuing to use incorrect mathematics caused by the arbitrary neglect of spacetime torsion.

2. WAVE GEOMETRICAL SOLUTIONS OF THE TETRAD POSTULATE

During the development of ECE theory {1 - 10} it has been shown that the tetrad postulate of differential geometry can be developed as the second order differential equation:

$$(\square + R)\, q^a_\mu = 0 \qquad -(1)$$

where q^a_μ is the Cartan tetrad and in which R is defined to be:

$$R = q^\nu_a\, \partial^\mu\, \Omega^a_{\mu\nu}. \qquad -(2)$$

Here $\Omega^a_{\mu\nu}$ is defined by the spin connection $\omega^a_{\mu b}$ and the Riemann connection $\Gamma^\kappa_{\mu\nu}$ as follows:

$$\Omega^a_{\mu\nu} = \omega^a_{\mu b}\, q^b_\nu - \Gamma^\kappa_{\mu\nu}\, q^a_\kappa = \omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}. \qquad -(3)$$

If this equation is applied for example to the gravitation, the free particle limit is:

$$\left(\square + \left(\frac{m_0 c}{\hbar}\right)^2\right) q^a_\mu = 0 \qquad -(4)$$

in which m_0 is the constant, observed, mass of the free particle. Here c is the vacuum speed of light and $\hbar$ the reduced Planck constant. Eq. (4) means that gravitational theory is built in to quantum mechanics, in other words a satisfactory unification of quantum mechanics and general relativity can be achieved straightforwardly.

Using the Schroedinger axiom:

$$\hat{p}^{\mu} = i\hbar \partial^{\mu} \quad -(5)$$

Eq. (4) is transformed into its classical equivalent:

$$E^2 = c^2 (p^2 + \hbar^2 R). \quad -(6)$$

This equation has the structure of the Einstein energy equation of special relativity provided R is defined in terms of the concept known as covariant mass, denoted m:

$$R = \left(\frac{mc}{\hbar} \right)^2. \quad -(7)$$

The effect of gravitation is therefore as follows:

$$\left(\frac{m_0 c}{\hbar} \right)^2 \rightarrow R \quad -(8)$$

which means that the constant or measured mass of the free particle, m_0 , is replaced by the covariant mass m. The latter is not constant. This is precisely what was indicated by the results of UFT 158 ff and UFT 171.

From Eqs. (6) and (8):

$$R = \frac{1}{c^2 \hbar^2} \left(E^2 - c^2 p^2 \right) \quad -(9)$$

in which the total relativistic energy E of special relativity is used:

$$E = \gamma m_0 c^2. \quad -(10)$$

The total relativistic momentum of special relativity is defined as the sum:

$$p^2 = \gamma^2 m_0^2 \left(\frac{dr}{dt}\right)^2 + \frac{L^2}{m_0 r^2} \quad -(11)$$

where L is the angular momentum. Here E, $\underline{p}$ and $\underline{L}$ are constants of motion. The Planck postulate is:

$$E = \hbar \omega \quad -(12)$$

where ω is the angular frequency, and the de Broglie postulate is:

$$\underline{p} = \hbar \underline{\kappa} \quad -(13)$$

where $\underline{\kappa}$ is the wavenumber. Using Eqs. (9) to (13) gives:

$$R = \kappa^\mu \kappa_\mu = \frac{\omega^2}{c^2} - \kappa^2 \quad -(14)$$

where the four wave vector is:

$$\kappa^\mu = \left(\frac{\omega}{c}, \underline{\kappa}\right). \quad -(15)$$

The covariant mass is therefore:

$$m = \frac{\hbar}{c}\left(\kappa^\mu \kappa_\mu\right)^{1/2} \quad -(16)$$

and the effect of gravitation on the Einstein energy equation is found very simply by the replacement:

$$p^2 \longrightarrow p^2 + \hbar^2 R, \quad -(17)$$

a procedure that develops the concept of minimal prescription to second order in $\underline{p}$.

The ECE wave equation and tetrad postulate become:

$$\left(\Box + \frac{\omega^2}{c^2} - \kappa^2\right) q^a_\mu = 0 \quad -(18)$$

which is the fundamental and generally covariant equation of wave geometry sought for in this section. The generally covariant wave equation of the unified field in physics is obtained directly from Eq. (18). For example the wave equation of gravitation is found from the postulate:

$$\Phi^a_\mu = \Phi^{(0)} q^a_\mu \quad -(19)$$

where Φ^a_μ is the gravitational potential, and the wave equation of electromagnetism from:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(20)$$

where A^a_μ is the electromagnetic potential.

In UFT 179 it was shown that the received opinion of gravitational theory, based on the use of the metric, can be expressed as:

$$E_1^2 - c^2 p_1^2 = m_0^2 c^4 \quad -(21)$$

an equation which has the same format as the Einstein energy equation of special relativity for the free particle in the absence of gravitation:

$$E^2 - c^2 p^2 = m_0^2 c^4 \quad -(22)$$

In Eq. (21) and (22) E_1 and p_1 are constants of motion defined by the Euler Lagrange equation and lagrangian and hamiltonian constructed form the metric. This point is further developed in Section 3. It follows that R can be defined in a number of equivalent ways as follows:

$$R = \left(\frac{m_0 c}{\hbar}\right)^2 \left(\frac{E^2 - c^2 p^2}{E_1^2 - c^2 p_1^2}\right) = \frac{\omega^2}{c^2} - \kappa^2 = q^\nu_a \partial^\mu \Omega^a_{\mu\nu} \quad -(23)$$

A solution of Eq. (18) is the propagating wave of geometry, chosen for convenience in the Z axis:

$$q_\mu^a = q_\mu^a(0) \exp\left(-i(\omega t - \kappa Z)\right) \quad -(24)$$

whose phase is:

$$\phi = \omega t - \kappa Z \quad -(25)$$

where ω is its angular frequency at instant t, and whose wave vector is κ at point Z. This is also a solution of the tetrad postulate of Cartan geometry:

$$D_\mu q_\nu^a = 0. \quad -(26)$$

Differentiating both sides of Eq. (26):

$$\Box q_\nu^a + \partial^\mu \Omega_{\mu\nu}^a = 0 \quad -(27)$$

where:

$$D_\mu q_\nu^a = \partial_\mu q_\nu^a + \omega_{\mu b}^a q_\nu^b - \Gamma_{\mu\nu}^\kappa q_\kappa^a \quad -(28)$$

so:

$$\partial^\mu\left(\omega_{\mu\nu}^a - \Gamma_{\mu\nu}^a\right) = \left(\frac{\omega^2}{c^2} - \kappa^2\right) q_\nu^a. \quad -(29)$$

From Eqs. (24) and (29):

$$\partial^\mu \Omega_{\mu\nu}^a = \left(\frac{\omega^2}{c^2} - \kappa^2\right) q_\nu^a(0) \exp\left(-i(\omega t - \kappa Z)\right). \quad -(30)$$

Summing over repeated indices in the Z axis (indices 0 and 3) gives:

$$\partial^0 \Omega_{0\nu}^a + \partial^3 \Omega_{3\nu}^a = \left(\frac{\omega^2}{c^2} - \kappa^2\right) q_\nu^a(0) \exp\left(-i(\omega t - \kappa Z)\right) \quad -(31)$$

i.e. :

$$\frac{1}{c}\frac{\partial \Omega^a_{0\nu}}{\partial t} - \frac{\partial \Omega^a_{3\nu}}{\partial Z} = \left(\frac{\omega^2}{c^2} - \kappa^2\right) q^a_\nu(0) \exp\left(-i\left(\omega t - \kappa Z\right)\right). \quad -(32)$$

Therefore the following are wave geometrical solutions of the tetrad postulate itself:

$$\Omega^a_{0\nu} = \frac{i\omega}{c} q^a_0(0) \exp\left(-i\left(\omega t - \kappa Z\right)\right), \quad -(33)$$

$$\Omega^a_{3\nu} = i\kappa q^a_3(0) \exp\left(-i\left(\omega t - \kappa Z\right)\right), \quad -(34)$$

$$q^a_\nu = q^a_\nu(0) \exp\left(-i\left(\omega t - \kappa Z\right)\right), \quad -(35)$$

meaning that Cartan's differential geometry has a very fundamental wave structure. In the philosophy of relativity, this geometrical wave structure is the origin of wave or quantum mechanics in physics.

3. COMPARISON WITH THE METRICAL THEORY OF GRAVITATION.

In the received opinion of the twentieth century in physics, gravitation was thought to be a change in the metric. In the absence of gravitation the metric was thought to be the Minkowski metric. It is convenient to define the latter in cylindrical polar coordinates for X Y planar motion:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2. \quad -(36)$$

It can be generalized if needed to any motion. In the philosophy of relativity, c is thought to be a constant, although this definition is not accepted by all. Thus the following is a constant:

$$c^2 = \left(\frac{ds}{d\tau}\right)^2 = c^2\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\phi}{d\tau}\right)^2. \quad -(37)$$

In these definitions $d\tau$ is the infinitesimal of proper time, the time seen in the frame moving with the particle. The infinitesimal of time dt is that in the frame of the static observer, the laboratory or observing frame in which the particle moves. In this theory the hamiltonian was defined as the constant:

$$H = \frac{1}{2} m_0 c^2 \quad -(38)$$

in which m_0 is the constant observed mass, for example the mass of the electron as measured in the standards laboratories. In this metrical theory m_0 does not change. In the absence of gravitation the equation of motion of the free particle is therefore:

$$H = \frac{1}{2} m_0 c^2 = \frac{1}{2} m_0 \left(c^2 \left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dr}{d\tau}\right)^2 - r^2 \left(\frac{d\phi}{d\tau}\right)^2 \right). \quad -(39)$$

As shown in previous work {1 - 10} this equation is the Einstein energy equation:

$$E^2 = p^2 c^2 + m_0^2 c^4 \quad -(40)$$

in which the total relativistic energy and momentum are defined as in Eqs. (10) and (11), i.e. as:

$$E = \gamma m_0 c^2 = m_0 c^2 \frac{dt}{d\tau}, \quad -(41)$$

$$\underline{p}_l = \gamma m_0 \frac{d\underline{r}}{dt}, \quad -(42)$$

$$p^2 = p_l^2 + \frac{L^2}{r^2}, \quad -(43)$$

where the Lorentz factor γ is defined as:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \quad -(44)$$

Here $\underline{v}$ is the velocity of the particle as measured in the laboratory frame by the static observer.

The total linear momentum is:

$$p^2 = p_\ell^2 + \frac{L^2}{r^2} \quad -(45)$$

where the angular momentum L is a constant of motion:

$$L = m_0 r^2 \frac{d\phi}{d\tau}. \quad -(46)$$

The momentum p_ℓ is the constant of motion:

$$p_\ell = m_0 \frac{dr}{d\tau} = \gamma m_0 \frac{dr}{dt}. \quad -(47)$$

From Eq. (41) it follows that:

$$\frac{dt}{d\tau} = \gamma \quad -(48)$$

and

$$d\tau = \left(1 - \frac{v^2}{c^2}\right)^{1/2} dt. \quad -(49)$$

The infinitesimal of proper time $d\tau$ is shorter than the infinitesimal dt of time as measured by the observer.

In the metrical theory, the effect of gravitation was to change the infinitesimal line element ds^2 but to leave H constant. A commonly used ds^2 was:

$$ds^2 = c^2 d\tau^2 = \left(1 - \frac{r_0}{r}\right) c^2 dt^2 - \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 - r^2 d\phi^2 \quad -(50)$$

in which

$$r_0 = \frac{2GM}{c^2} \quad -(51)$$

where G is Newton's constant, and M is the mass that attracts m_0 by gravitation. This procedure produced a mirage of agreement with data because it was used only in the solar system. In whirlpool galaxies for example it fails wildly. Despite this and many other mathematical flaws {2}, it was adhered to dogmatically throughout the twentieth century. The hamiltonian produced by Eq. (50) does not change, and is:

$$H = \frac{1}{2} m_0 c^2 = \frac{1}{2} m_0 \left(\left(1 - \frac{r_0}{r}\right) c^2 \left(\frac{dt}{d\tau}\right)^2 - \left(1 - \frac{r_0}{r}\right)^{-1} \left(\frac{dr}{d\tau}\right)^2 - r^2 \left(\frac{d\phi}{d\tau}\right)^2 \right), \quad -(52)$$

which is the equation of motion of the particle in the presence of gravitation, for example it produces a precessing Kepler ellipse. In previous work {1 - 10} it has been shown that Eq. (52) has the format of the Einstein energy equation of special relativity, but with:

$$E_1^2 = p_1^2 c^2 + m_0^2 c^4 \quad -(53)$$

i.e.:

$$E \rightarrow E_1 \, , \quad p^2 \rightarrow p_1^2 . \quad -(54)$$

Surprisingly this result does not appear to have been known until its discovery in UFT 179. In Eq. (53):

$$E_1 = \left(1 - \frac{r_0}{r}\right) m_0 c^2 \frac{dt}{d\tau} \quad -(55)$$

is a constant of motion and the total momentum is:

$$p_1^2 = p_{1\ell}^2 + \frac{L^2}{m_0 r^2} \quad -(56)$$

in which:

$$p_{1\ell} = \left(1 - \frac{r_0}{r}\right)^{-1} m_0 \frac{dr}{d\tau} , \quad L = m_0 r^2 \frac{d\phi}{d\tau} . \quad -(57)$$

So by comparing Eqs. (40) and (53) it becomes clear that the effect of gravitation in this obsolete metrical view was seen as keeping the following quantity constant:

$$E^2 - c^2 p^2 = E_1^2 - c^2 p_1^2 \quad -(58)$$

and used a constant mass m_0. This metrical view did not use the concept of classical potential energy at all, so did not use the classical concept of force. This is because the free particle hamiltonian did not change when a gravitational field was applied. For the free particle the potential energy is zero, so by keeping the hamiltonian constant in the presence of gravitation, no potential energy was introduced, and thus no classical force, defined by:

$$\underline{F} = -\underline{\nabla} V. \quad -(59)$$

In the metrical view there was no force therefore between m_0 and M. The two masses moved in an orbit because of the structure of the background geometry or spacetime. The metrical theory was forced to reduce to the Newtonian theory by choosing Eq. (51) empirically. So in this sense the old metrical theory was pure empiricism with many errors. In the metrical viewpoint the hamiltonian was always one half of the rest energy of the free particle, with fixed m_0. The old metrical view always erroneously neglected torsion, so was fundamentally untenable from its roots in the decade 1905 to 1915. This was in fact known as early as 1918, but the dogma prevailed for nearly a century until the emergence of ECE theory. The older view can now be replaced entirely by the simpler and more elegant covariant mass theory developed in this paper from previous work {1 - 10}. The ad hoc idea of dark matter in the old physics was used to try to construct a coherent cosmology, but dark matter has never been observed. Erroneous concepts such as Big Bang and black hole theory also emerged as the result of dogma, or Langmuir's pathological science - the uncritical repetition of incorrect opinion.

In great contrast the ECE and covariant mass methods are based directly on the correct geometry, one that from the outset includes torsion. The two fundamental properties of Cartan and Riemann geometry are torsion and curvature, defined by the two fundamental structure equations of differential geometry. Orbits in the covariant mass theory are defined by the correct identities of Cartan geometry, identities that link torsion and curvature. There is the fundamental and simple wave equation (18) and its equally simple classical equivalent (6); and also four field equations obtained from geometrical identities The fields and potentials are related by the structure equations themselves, equations which contain the connection of geometry. Sometimes during the course of the development of ECE theory, the metrical method was also corrected and developed and based on a new theorem of orbits (UFT 111) based on the properties of a spherical spacetime. The older Einstein field equation was not used and abandoned { 2 } because of its many flaws, notably the neglect of torsion. In ref. (2) every metric of the Einstein equation was shown by computer algebra to be erroneous or otherwise meaningless. The tremendous complexity of the old metric theory had hidden its flaws for nearly a century, and that sums up much of the standard model.

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