Computing the spectrum of elementary particles

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Abstract

A numerical method for computing wave functions and masses of elementary particles is developed. It is based on the ECE wave equation, which is a quantized form of the Einstein energy equation, extended for general relativity. The method is developed in analogy to eigenvalue problems of solid-state physics.

Keywords: elementary particles, numerical methods.

1 Introduction

To explain elementary particles, the so-called standard model is used today. This is a classification scheme based on symmetry groups. Although gilded by some mathematics, there is no equation set from which all quantities like mass, charge or spin can be derived ab initio. In contrast, consider equations of motion, which deliver all types of observables, in other fields of physics. For elementary particles, such a method does not exist. In addition, many "particles" are visible only for tiny fractions of a second, so it is questionable if these transitional states can be given the status of a particle at all. Quantum mechanics is applicable only in a rudimentary way. One would need a unification of quantum mechanics with general relativity, which is not existent in standard physics to date.

Alternative approaches come from the side of general relativity. Bruchholz [5] has developed a method of investigating the stability of equations of general relativity combined with electromagnetism, which were derived by Rainich [6]. Bruchholz solved these equations numerically in a way that depends on parameters which determine particle characteristics like mass, spin, and magnetic moment, for example. He found "islands of stability" for these quantities exactly where they coincide with their physically known values. Even masses of neutrinos were predicted.

The method used in this paper is based on standard procedures of computational quantum chemistry or solid state physics. The quantized form of the ECE wave equation [1–3], which has been extended in range to general relativity, is taken as the basis. This extension, called m theory, is an application of

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the principles of general relativity to a centrally symmetric spacetime. m theory has already been applied to interactions of elementary particles [3]. In this paper, the structure of particles itself is investigated. A practical computation scheme is developed, although its numerical application has yet to be realized.

2   Elaboration of the method

The Einstein energy equation in Minkowski space reads:

$$E^2 = c^2 p^2 + m^2 c^4, \tag{1}$$

where c is the vacuum velocity of light, p the particle momentum and m the particle mass. The term "m" for mass should not be confused with "m theory", where m stands for a function $m(r)$, describing the space compression or curvature for a radial coordinate r in a spherically symmetric spacetime. The m space is a space of general relativity. The Einstein energy equation in m space is the generalization:

$$E^2 = c^2 p^2 + m(r) m^2 c^4. \tag{2}$$

Quantization is achieved by the replacement rules of ordinary quantum mechanics:

$$\mathbf{p} \to -i\hbar\nabla, \tag{3}$$

$$E \to i\hbar\frac{\partial}{\partial t}. \tag{4}$$

This transforms Eq. (2) into a quantized energy equation for time-independent quantum states ψ:

$$\nabla^2\psi + m(r)\left(\frac{mc}{\hbar}\right)^2 \psi = 0. \tag{5}$$

For $m(r)$=1 this is the standard wave equation:

$$\nabla^2\psi + \left(\frac{mc}{\hbar}\right)^2 \psi = 0 \tag{6}$$

with oscillatory solutions. In the case where $m(r) \neq 1$, this equation resembles the Bessel differential equation, as was shown in [4].

Eq. (5) is an equation of the generalized eigenvalue problem

$$A\psi + \lambda B\psi = 0 \tag{7}$$

where A and B are matrices, ψ is an eigenvector and λ the corresponding eigenvalue. This equation can be developed as follows. We expand the wave function ψ by a set of basis functions ϕ_i:

$$\psi(r) = \sum_i c_i \phi_i(r) \tag{8}$$

with development coefficients c_i. The aim is to find the coefficients c_i and eigenvalues λ from which the particle masses emerge via

$$\lambda = \left(\frac{mc}{\hbar}\right)^2. \tag{9}$$

Inserting Eq. (8) into (5) gives

$$\sum_i \nabla^2 c_i \phi_i + \mathrm{m}(r) \left(\frac{mc}{\hbar}\right)^2 \sum_i c_i \phi_i = 0. \tag{10}$$

Such an equation can be obtained in more suitable form from the variational principle, which is used, for example, to solve the Schrödinger equation. Applying the minimization of total energy,

$$\delta \left\langle E^2 \right\rangle \to \min., \tag{11}$$

means that we have to minimize the expectation value of E, or E^2 in our case. According to the quantized form (5) of the energy equation, we have to evaluate

$$\left\langle E^2 \right\rangle = \int \psi^* \nabla^2 \psi d\tau + \int \psi^* \mathrm{m}(r) \left(\frac{mc}{\hbar}\right)^2 \psi d\tau. \tag{12}$$

Inserting the expansion (8), the right-hand side can be written

$$\int \sum_j (c_j \phi_j)^* \nabla^2 \sum_j c_i \phi_i d\tau + \int \sum_j (c_j \phi_j)^* \mathrm{m}(r) \left(\frac{mc}{\hbar}\right)^2 \sum_j c_i \phi_i d\tau. \tag{13}$$

Allowing for complex coefficients and basis functions, this can be rewritten as

$$\int \sum_j c_j^* \phi_j^* \nabla^2 \sum_j c_i \phi_i d\tau + \int \sum_j c_j^* \phi_j^* \mathrm{m}(r) \left(\frac{mc}{\hbar}\right)^2 \sum_j c_i \phi_i d\tau, \tag{14}$$

and finally, by pulling the constant terms out of the integrals,

$$\sum_{i,j} c_j^* c_i \int \phi_j^* \nabla^2 \phi_i d\tau + \left(\frac{mc}{\hbar}\right)^2 \sum_{i,j} c_j^* c_i \int \phi_j^* \mathrm{m}(r) \phi_i d\tau. \tag{15}$$

The coefficients have to be determined in a way that the total energy is minimal. This is achieved by requiring that

$$\frac{\partial}{\partial c_j^*} \left\langle E^2 \right\rangle = 0 \tag{16}$$

for all coefficients c_j^*. (Alternatively, one could take the derivative for c_i.) Then the equation

$$\sum_{i,j} c_i \int \phi_j^* \nabla^2 \phi_i d\tau + \lambda \sum_{i,j} c_i \int \phi_j^* \mathrm{m}(r) \phi_i d\tau = 0 \tag{17}$$

follows with eigenvalue λ defined by Eq. (9). For N basis functions we obtain N eigenvectors c_{ik} and N eigenvalues λ_k where k numbers the eigen states. Eq. (17) has the form of a generalized eigenvalue problem:

$$A\psi + \lambda B\psi = 0 \tag{18}$$

with matrix elements

$$A_{ij} = \int \phi_j^* \nabla^2 \phi_i d\tau \tag{19}$$

and

$$B_{ij} = \frac{c^2}{\hbar^2} \int \phi_j^* \mathrm{m}(r) \phi_i d\tau \qquad (20)$$

and eigenvalues m^2 which must be positive. This implies certain mathematical properties of A and B. For example, if the matrices are Hermitian, then the eigenvalues must be real. If a matrix is positive definite, all eigenvalues are positive.

It is not necessary for the basis set $\{\phi_i\}$ to be orthogonal. This would be required if Eq. (17) had to be an ordinary eigenvalue equation with B being the unit matrix. However, the presence of the function $\mathrm{m}(r)$ inhibits this. In the case where $\mathrm{m}(r)=1$, matrix B is called the overlap matrix of basis functions and defines a scalar product:

$$\langle \phi_i, \phi_j \rangle = \int \phi_j^* \phi_i d\tau. \qquad (21)$$

The eigenvalue equation (17) can be solved numerically, if the integrals can be computed analytically. With additional effort, such a solution is also possible in the case where the integrals have to be solved numerically. The matrix elements $\langle \phi_i, \phi_j \rangle$ always represent a kind of structure constants.

3   Summary

As developed in this paper, the internal structure of elementary particles may be calculated by an eigenvalue computation scheme. The results will be the matter density $|\psi|^2$ and the corresponding mass. This method can be extended to electric and spin structures by using other fundamental equations. Instead of the Einstein energy equation, the ECE fermion equation may be used. Also, an alternative method such as that of Bruchholz could be implemented, which is based on stability studies of solutions. As a refinement of the method developed in this paper, the original ECE wave equation

$$\Box\, A_\nu^a + R\, A_\nu^a = 0 \qquad (22)$$

could be used [2]. Here $\Box$ is the d'Alembert operator, A_ν^a is the electromagnetic 4-potential and R is a curvature scalar. Even time-dependent processes could be investigated. However, this equation requires knowledge of the full spacetime geometry via R.

The next step of development will be the definition of suitable basis functions ϕ_i, and the preparation of a numerical package like LAPACK for solving the eigenvalue problem.

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