Reduction of the ECE Theory of Electromagnetism to the Maxwell-Heaviside Theory, Part III

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Abstract

In this third paper of the series on how the ECE theory of electromagnetism reduces to the Maxwell-Heaviside theory, it is shown that given the assumption that nature abhors a singularity, if a Maxwellian state is ever achieved, even if only briefly, it remains Maxwellian, unless very strong measures are brought to create an instability.

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1. Introduction

The ECE theory of electromagnetism is rich in non-linear behaviour [1-6]. In the first paper in this series [2], it was shown that only one combination of spin connection variables and vector and scalar potentials allowed the ECE theory of electromagnetism to reduce to the standard Maxwell-Heaviside theory [7]. In the second paper[3] it was shown that if the scalar potential is a separable and continuous function of space and time, which is what is typically used in analysis, then the ECE equations reduce to those of Maxwell-Heaviside, with the non-linear richness vanishing. The reasoning can be extended to include any component of the magnetic vector potential also. In this paper it is shown, with the assumption that nature abhors a singularity, that the Maxwell-Heaviside state is very robust. If somehow one creates a non-Maxwellian state, then if even for an instant a Maxwellian state occurred, then the Maxwellian state would remain intact until some very significant event occurred to shift the state away from this stable point.

Given this, but beyond the scope of this paper, it is conjectured that the Maxwellian state represents some very robust minimum of some functional in ECE theory, or perhaps a strong attractor in a chaotic system of equations. Such a minimum could be envisioned to be surrounded by a very high "potential wall" of the form $\frac{1}{\phi}$ as ϕ approaches zero, for example.

2. Further Reduction of ECE EM Theory Reduces to Maxwell-Heaviside EM Theory

Let us start with the basic antisymmetry equations of the ECE electromagnetic theory for a single polarization, and assumed to govern the ECE electromagnetic field under all conditions. These equations are given by [2]

$$\frac{\partial \mathbf{A}}{\partial t} - \underline{\nabla}\phi + \omega_0 \mathbf{A} + \boldsymbol{\omega}\phi = 0, \tag{1}$$

$$\frac{\partial A_k}{\partial x_j} + \frac{\partial A_j}{\partial x_k} + \omega_j A_k + \omega_k A_j = 0 \tag{2}$$

where *i, j, k* are a cyclic permutation of the coordinates. Equation (2) has been solved to give the vector spin connection as a differentio-algebraic function of the magnetic vector potential [5]:

$$\omega = \begin{pmatrix} \dfrac{1}{2A_2 A_3}\left(A_1\dfrac{\partial A_3}{\partial y} + A_1\dfrac{\partial A_2}{\partial z} - A_2\dfrac{\partial A_3}{\partial x} - A_2\dfrac{\partial A_1}{\partial z} - A_3\dfrac{\partial A_2}{\partial x} - A_3\dfrac{\partial A_1}{\partial y} \right) \\ \dfrac{1}{2A_1 A_3}\left(-A_1\dfrac{\partial A_3}{\partial y} - A_1\dfrac{\partial A_2}{\partial z} + A_2\dfrac{\partial A_3}{\partial x} + A_2\dfrac{\partial A_1}{\partial z} - A_3\dfrac{\partial A_2}{\partial x} - A_3\dfrac{\partial A_1}{\partial y} \right) \\ \dfrac{1}{2A_1 A_2}\left(-A_1\dfrac{\partial A_3}{\partial y} - A_1\dfrac{\partial A_2}{\partial z} - A_2\dfrac{\partial A_3}{\partial x} - A_2\dfrac{\partial A_1}{\partial z} + A_3\dfrac{\partial A_2}{\partial x} + A_3\dfrac{\partial A_1}{\partial y} \right) \end{pmatrix} \tag{3}$$

For brevity where needed, we shall adopt the notation

$$\omega = \underline{\mathfrak{L}}(A) \tag{4}$$

where the operator $\underline{\mathfrak{L}}$ is known from equation (3).

Taking the cross product of equation (1) with A and using equation (2) we have

$$\omega \times A = \frac{1}{\phi}\left(\underline{\nabla}\phi - \frac{\partial A}{\partial t} \right) \times A \quad . \tag{5}$$

If ϕ becomes zero in equation (5) there exists the potential for a singularity. If however, ϕ is separable and continuous, it was shown in a previous paper [2] that $\underline{\nabla}\phi$, A, $\frac{\partial A}{\partial t}$, and ω are all parallel, and that $\omega \times A = 0$, even if $\phi = 0$. This system of equations is then Maxwellian.

Case 1

For the case when ϕ is not separable but continuous, consider again equation (5) which is a form of the electric antisymmetry equation and is assumed to hold under all circumstances. If we make the assumption that "*nature abhors a singularity*", then when $\phi = 0$, one of two things must happen. To avoid a singularity, either

  i. $\underline{\nabla}\phi - \frac{\partial A}{\partial t} = 0$ or
  ii. $\underline{\nabla}\phi, , \frac{\partial A}{\partial t}$, and ω are all parallel.

Case (i) can be rejected because we are making the implicit assumption that ϕ and A are independent. This assumption means that the Maxwellian electric intensity is zero.

Case (ii) results when A becomes separable but continuous, making $\frac{\partial A}{\partial t}$ parallel to A, at which time ϕ becomes separable but continuous by examination of the Maxwell-Ampere equation. The system then in this instant of time becomes Maxwellian.

Consider A as separable but continuous, i.e.

$$A = A^{(r)}f(t).$$

Substitution of this into the Maxwell-Ampere equation gives,

$$f(t)\nabla \times \nabla \times A^{(r)} - f(t)\nabla \times \left(\omega \times A^{(r)}\right) + \frac{1}{c^2}\left(A^{(r)}\frac{\partial^2 f(t)}{\partial t^2}\right)$$
$$-\frac{1}{c^2}\frac{\partial}{\partial t}\left(-\underline{\nabla}\phi - \omega_0 A^{(r)}f(t) + \omega\phi\right) = \mu_0 J \quad .$$

Dividing by $f(t)$ we have

$$\nabla \times \nabla \times A^{(r)} =$$
$$\nabla \times \left(\omega \times A^{(r)}\right) - \frac{1}{c^2 f(t)}\left(A^{(r)}\frac{\partial^2 f(t)}{\partial t^2}\right) + \frac{1}{c^2 f(t)}\frac{\partial}{\partial t}\left(-\underline{\nabla}\phi - \omega_0 A^{(r)}f(t) + \omega\phi\right) + \mu_0 \frac{J}{f(t)}.$$

The left hand side of this equation is a function only of spatial coordinates. This then requires that

    i. ω is independent of time

    ii. ϕ is separable and of the form $\phi = \phi^{(r)}\frac{\partial f(t)}{\partial t}$

    iii. ω_0 is of the form $\omega_0 = \omega_0^{(r)}\frac{\int f(t)dt}{f(t)}$

i.e. the system is Maxwellian.

It was shown earlier [2], that when the system is Maxwellian, ω ceases to be a function of time.

We prove now that if ω is independent of time, that the ECE equations are Maxwellian, and that the system would tend to stay in that state.

By equation (1), if ω is independent of time,

$$\omega^{(r)} = -\frac{1}{\phi}\left(\frac{\partial A}{\partial t} - \underline{\nabla}\phi + \omega_0 A\right).$$

The left hand side of this equation does not depend on time. This requires that

$\frac{1}{\phi}\frac{\partial A}{\partial t}, \frac{\underline{\nabla}\phi}{\phi}$, and $\frac{\omega_0 A}{\phi}$ be all independent of time. For this to be so,

    i. $\phi = \phi^{(r)}f(t)$

    ii. $A = A^{(r)}\int f(t)dt$

    iii. $\omega_0 = \omega_0^{(r)}\frac{\int f(t)dt}{f(t)}$

This is the requirement for the Maxwellian field.

There is nothing in the Maxwell-Heaviside equations, if $\boldsymbol{J}$ and ρ are well defined (and continuous), to introduce a non-Maxwellian behaviour, once the Maxwellian behaviour exists. This could be regarded as an initial condition. Given this, then once a Maxwellian state is encountered, the system remains Maxwellian until some significant event breaks this robust state of stability.

Case 2

Consider the case when ϕ is not separable but continuous, but now let us consider the situation that any one of the components of the magnetic vector potential becomes zero. Let us examine the magnetic antisymmetry equation and make the assumption that at some point in space and at some time

$$A_3 = 0, \tag{6}$$

with the other two components of $\boldsymbol{A}$ being finite. If the assumption of nature's abhorrence of a singularity is valid, then we infer that components of $\boldsymbol{\omega}$ remain finite.

This being so, then substitution of equation (6) into equation (2) gives

$$\frac{1}{A_2}\left(\frac{\partial A_3}{\partial y} + \frac{\partial A_2}{\partial x}\right) = \frac{1}{A_1}\left(\frac{\partial A_3}{\partial x} + \frac{\partial A_1}{\partial z}\right). \tag{7}$$

A similar equation can be written with each of the other two A_i in turn being zero.

Two conditions exist, either

    i.   all the spatial derivatives of A_i must be zero, or
   ii.   all of the A_i become separable.

Clearly the first choice is not acceptable; it is too limiting. This requires that the second choice be valid. Again, nature has forced the system into a Maxwellian state if the mathematics are to remain valid.

3. Discussion and Conclusions

We have shown thus far in the series of papers that

    i.   If $\boldsymbol{\omega}\phi = \omega_0 \mathbf{A} = \frac{1}{2}\left(-\frac{\partial \mathbf{A}}{\partial t} + \underline{\nabla}\phi\right)$ then $\boldsymbol{\omega} \times \boldsymbol{A} = \boldsymbol{0}$ then the ECE theory reduces to Maxwell-Heaviside.

ii. If ϕ is separable and continuous, then so is $\boldsymbol{A}$, or if any component of $\boldsymbol{A}$ is separable and continuous then so is ϕ, then and the ECE system reduces to a Maxwell-Heaviside and $\boldsymbol{\omega}$ ceases to be a function of time.

iii. If ϕ or $\boldsymbol{A}$ is not separable, and if $\phi = 0$ or $A_i = 0$ become zero at any point in space and time, then if nature abhors a singularity, the system becomes Maxwell-Heaviside. $\boldsymbol{\omega}$ would then cease to be a function of time and the system would remain Maxwellian unless some events breaks down this stability.

If the traditional scalar potential is separable, then the vector potential is separable. This being the case, the ECE equations of electromagnetism reduce to those of the Maxwell-Heaviside theory. Non-Maxwellian effects can then only be expected when the scalar potential cannot be expressed as a separable function, not a common occurrence. This is certainly borne out by experiment, where very few non-Maxwellian effects have been observed.

We have identified many situations under which that ECE system becomes Maxwellian and is forced to stay there because of its stable nature. We shall now present the conditions for the opposite to be true, i.e. the ECE system to become non-Maxwellian state, and stay there.

Whether the state of an electromagnetic field is Maxwellian or not prior to "turning on the switch" is still open for discussion. Let us assume the worst, and assume that the state is Maxwellian, (or has become Maxwellian by one of the mechanisms discussed above). To achieve a non-Maxwellian ECE state, none of the potentials can ever be zero, nor can they ever become separable nor continuous. This means that the system has to placed in a state of potential that is either negative or positive, and remains that way, and that the potentials become discontinuous making their derivative multi-valued, or perhaps "near infinite". A pulsed potential, with extremely fast rise and collapse times would have this property, for example.

Another case is a ferromagnetic transition, i.e. the magnetization of a ferromagnetic material is changed in a state not being in saturation. This represents an effect of a multi-valued force field and potential. The potential is non-conservative because the energy required for the transition depends on the path in phase space. These examples have to be studied further.

References

1.   M. W. Evans, Generally Covariant Unified Field Theory (Abramis, 2005 onwards), in seven volumes to date

2.   D. W. Lindstrom and H. Eckardt, Reduction of the ECE Theory of Electromagnetism to the Maxwell-Heaviside Theory, Generally Covariant Unified Field Theory, Chapter 17, Volume 7, 2011

3.   D. W. Lindstrom and H. Eckardt, Reduction of the ECE Theory of Electromagnetism to the Maxwell-Heaviside Theory, Part II, www.aias.us

4.   M.W.Evans, H. Eckardt and D.W.Lindstrom, Antisymmetry constraints in the Engineering Model, Generally Covariant Unified Field Theory, Chapter 12, Volume 7, 2011

5.   H. Lichtenberg, Considerations on the Electro- and Magnetostatic ECE Equations, www.aias.us

6.   H. Eckardt and D. W. Lindstrom, Solution of the ECE Vacuum Equations, Generally Covariant Unified Field Theory, Chapter 16, Volume 7, 2011

7.   J. D. Jackson , Classical Electrodynamics, John Wiley & Sons, 1999