Derivation of O(3) Electrodynamics from Generally Covariant Unified Field Theory

Summary. The equations of O(3) electrodynamics are derived as an example of Evans' generally covariant unified theory of radiated and matter fields, a theory which is based on the equations of differential geometry and which extends Einstein's theory of general relativity to electrodynamics. The latter is therefore developed into a correctly and generally covariant sector of unified field theory, one of the basic outcomes of which is the Evans spin field $\boldsymbol{B}^{(3)}$ observed in the inverse Faraday effect (magnetization by circularly or elliptically polarized electromagnetic radiation) and in many other ways now known. Another direct result of making electrodynamics a generally covariant field theory is that it is described in terms of the covariant derivatives appropriate to spacetime with torsion. The electromagnetic field is thereby recognized as the spinning of spacetime itself rather than an entity superimposed on a frame in flat spacetime. Consequently Maxwell Heaviside field theory is developed into a generally covariant form, one example of which is O(3) electrodynamics. The latter can be thought of as a gauge field theory with O(3) orthonormal space symmetry. The equations of O(3) electrodynamics are derived in detail from the generally covariant unified field theory.

Key words: Generally covariant unified field theory, O(3) electrodynamics, Evans spin field $\boldsymbol{B}^{(3)}$ inverse Faraday effect.

18.1 Introduction

General relativity is in essence the geometrization of physics. The laws of natural philosophy are derived from the theorems of geometry, a giant leap forward in thought brought about by Hilbert and independently by Einstein in 1915, to whom the theory of general relativity is attributed [1]–[2]. Einstein's original theory was developed for the gravitational field and resulted in the Einstein field equation. In its most condensed form the latter is:

$$R = -kT \tag{18.1}$$

where R is scalar curvature, k the Einstein constant and T the index contracted canonical energy-momentum tensor. The left hand side is geometry,

the right hand side is physics. In logic therefore, such an equation applies for all radiated and matter fields, not only gravitation. This logical outcome of the 1915 Eq. (18.1) was finally achieved in 2003 by Evans [3]–[16] and is a generally covariant unified field theory based on differential geometry. In Section 18.2 of this paper the relevant equations of differential geometry are summarized, equations from which the whole of theoretical physics can be derived logically, given Eq. (18.1). In Section 18.3 the theory is used to derive in detail the equations of O(3) electrodynamics, and to infer the existence of the Evans spin field [17] $\boldsymbol{B}^{(3)}$, which has turned out to be the key to field unification sought after by Einstein for forty years (1915 to the mid fifties). The Evans spin field is observed experimentally in numerous ways now known [16], for example in the inverse Faraday effect [18], the magnetization of matter by circularly polarized electromagnetic radiation, and in the whole of physical optics via the Evans phase law. The latter encompasses both the dynamical and the geometrical phase and produces the correctly and generally covariant Berry phase [3]–[16]in unified field matter theory

18.2 The Fundamental Geometrical Equations of the Unified Field Matter Theory

The theory has been developed in comprehensive detail elsewhere [3]–[18]. It is the purpose of this Section to conveniently summarize the equations of differential geometry upon which the field matter theory is built. These equations therefore define the fundamental structure of the theory and provide the guidelines for the development of any generally covariant theory of physics, i.e. any field matter theory, classical or quantum, provided one accepts the axioms of general relativity encapsulated in Eq. (18.1).

Adopting the condensed but well known notation [2] of contemporary differential geometry the four equations of differential geometry from which the unified field theory is developed are as follows:

$$D \wedge V^a := d \wedge V^a + \omega^a{}_b \wedge V^b \qquad (18.2)$$

$$Dq^a = 0 \qquad (18.3)$$

$$\tau^c = D \wedge q^c \qquad (18.4)$$

$$R^a{}_b = D \wedge q^a{}_b. \qquad (18.5)$$

These equations define the fundamental properties of any non-Euclidean spacetime with both curvature and torsion in terms of differential forms. Eq. (18.2) defines the covariant exterior derivative of any vector V^a, where $\omega^a{}_b$ is the spin connection and where $d\wedge$ denotes the ordinary exterior derivative [2] of differential geometry. Eq. (18.3) is the tetrad postulate, which asserts that the ordinary (as distinct for the exterior) covariant derivative of the vector valued tetrad one form vanishes for any spacetime. The covariant ordinary

derivative is therefore denoted by D and the covariant exterior derivative by $D\wedge$ where $\wedge$ is the wedge operator [2]. Eqs. (18.4) and (18.5) are the first and second Maurer Cartan structure relations [2], defining respectively the torsion (τ^c) and Riemann or curvature $(R^a{}_b)$ forms in terms of the tetrad and spin connection respectively.

These four equations are inter-connected in free space by the recently inferred and fundamental first and second Evans duality equations of differential geometry [3]–[17]:

$$\omega^a{}_b = -\kappa\epsilon^a{}_{bc}q^c \tag{18.6}$$

$$R^a{}_b = -\kappa\epsilon^a{}_{bc}\tau^c \tag{18.7}$$

where κ is wave-number. The novel Evans duality equations were inferred in free space from the fact that the torsion and Riemann forms are both anti-symmetric in their base manifold indices μ and ν, so that one must be the dual of the other in the orthonormal space of the tetrad [2]:

$$R^a{}_{b\mu\nu} = -\kappa\epsilon^a{}_{bc}\tau^c{}_{\mu\nu} \tag{18.8}$$

where:

$$\epsilon^{abc} = \eta_{da}\epsilon^d{}_{bc}. \tag{18.9}$$

Here ϵ_{abc} is the Levi Civita symbol (totally anti-symmetric third rank unit tensor) and where η_{da} is the metric in this orthonormal (orthogonal and normalized [2]) space. The torsion form, a vector valued two form with index c, is therefore dual to the curvature or Riemann form, a tensor valued two form anti-symmetric in its a, b indices [2]. Using the Maurer Cartan structure relations it is therefore inferred that the tetrad is dual to the spin connection, the second Evans duality equation (18.6).

The well known tetrad postulate has been developed [3]–[17] into the fundamentally important Evans Lemma (or subsidiary proposition) of differential geometry:

$$\Box q^a = Rq^a \tag{18.10}$$

which gives the Evans wave equation using Eq. (18.1):

$$(\Box + kT)\, q^a = 0 \tag{18.11}$$

Eq. (18.11) unifies general relativity and quantum mechanics, making the latter a causal theory of physics and rendering the Copenhagen interpretation of the wave function unnecessary. The Evans wave equation is the generally covariant development of all the well known wave equations of physics, for example the Dirac equation. In well defined limits the Evans wave equation reduces to the generally covariant form of the Proca equation, indicating conclusively that the photon must have a non-zero mass.

The scalar curvature appearing in Eq. (18.10) may always be defined from dimensionality as the square of a wave-number:

$$R := \kappa^2 \qquad (18.12)$$

and in the limit of special relativity (a free particle translating with constant velocity), this wave-number becomes the Compton wave-number of any particle (including the neutrino and photon, which must both have finite mass m):

$$\kappa_c = \frac{2\pi}{\lambda_c} = 2\pi \frac{mc}{\hbar}. \qquad (18.13)$$

Here $\hbar$ is the Planck constant, and c the speed of light in vacuo. The well known and observed Compton wave-number (or wavelength) of any particle is therefore recognized for the first time to be the Evans least curvature [3]–[17] that defines mass.

These are therefore the equations of geometry, specifically the equations of differential geometry, from which all the known equations of physics may be derived, and new equations and fundamental properties such as the Evans spin field $\boldsymbol{B}^{(3)}$, inferred.

18.3 The Equations of O(3) Electrodynamics

The field theory of O(3) electrodynamics [3]–[18] is a special case of Evans' generally covariant unified field theory outlined in Section 18.2 and has been extensively tested against experimental data [19]. It has numerous known advantages [3]–[18]over the older Maxwell Heaviside field theory, and produces novel properties of physics such as the Evans spin field $\boldsymbol{B}^{(3)}$, now known to be a fundamental spin invariant of the Einstein group missing from the original 1915 theory because the latter is confined to spacetimes with zero torsion form [2] (Christoffel symbols symmetric in the lower two indices and Riemann geometry). The electrodynamical sector of Evans' unified field theory recognizes the potential field of electrodynamics to be [3]–[18]:

$$A^a = A^{(0)} q^a \qquad (18.14)$$

where $A^{(0)}$ is a fundamental scalar, negative under charge conjugation symmetry ($\hat{C}$) and with the units of tesla m. The origin of the scalar $A^{(0)}$ is the universal constant $\hbar/e$, the magnetic fluxon, with units of magnetic flux (weber = volts). Here $\hbar$ is the reduced Planck constant ($h/(2\pi)$) and e the charge on the proton. (The charge on the electron is $-e$).

The generally covariant magnetic field in Evans' generally covariant unified field theory [3]–[18] must always be defined by:

$$F^a = D \wedge A^a \qquad (18.15)$$

and has the units of tesla = weber m^{-2}. The reason for this is that the covariant exterior derivative is always needed [2] in differential geometry for arbitrary spacetimes with in general non-zero torsion form and Riemann form.

So it is seen that differential geometry guides us towards the generally covariant definition of the magnetic field, (and also the electric field), and as we shall see, gives us the correct and generally covariant field equations of electrodynamics. The Maxwell Heaviside field theory, although well known, is not a correct theory of general relativity because it uses ordinary derivatives in a flat spacetime (zero Riemann and torsion forms).We have seen in Section 18.2 that the torsion form is sometimes the dual of the Riemann form for all spacetimes, (the second Evans duality equation (18.7) of differential geometry), so the existence of the Riemann form sometimes implies the existence of the torsion form. In other words curvature implies torsion. It is therefore seen that Einstein's omission of the torsion form is geometrically incorrect, and this explains why he was never able to develop a unified field theory. The contemporary "standard model" is not a theory of general relativity, and is therefore not correctly covariant, because within the standard model, a flat spacetime is used for three sectors out of four (electrodynamical, weak and strong fields). Unsurprisingly therefore , the standard model is unable to account for the fundamental and generally covariant Evans spin field $\boldsymbol{B}^{(3)}$, now known to be observable in many ways [3]–[18]. Contemporary string theory is a mathematical construct (i.e. string theory is not a theory of physics, it is a construct of mathematics that uses several unphysical "dimensions") and for this reason can make no predictions about nature. In other words string theory is an obscure and elaborate mathematical way of trying to describe things that are already known in physics, and already describable more simply with already known theories of physics. For this reason string theory is not capable of predicting anything new in physics, and is not a unified field theory. String theory may be interesting for pure mathematics, but the correct geometrical basis for physics is now well known to be the Evans field matter theory [3]–[18], whose origins in differential geometry are summarized briefly in Section 18.2. The Evans theory uses only the four physical dimensions: time and three space dimensions. These are used as in standard relativity to construct four dimensional spacetime. For this reason the Evans theory is a powerful and predictive theory of nature which also reduces to the known equations of both classical and quantum physics [3]–[18].

From Eqs. (18.4) and (18.6):

$$F^a = d \wedge A^a + \frac{\kappa}{A^{(0)}} A^b \wedge A^c. \tag{18.16}$$

Defining:

$$g := \frac{\kappa}{A^{(0)}} \tag{18.17}$$

we obtain the magnetic field in general relativity:

$$F^a = D \wedge A^a = d \wedge A^a + g A^b \wedge A^c. \tag{18.18}$$

The electric field in general relativity is similarly defined with appropriate indices. The precise way of doing this in O(3) electrodynamics is developed later

in this Section. The field theory of O(3) electrodynamics [3]–[18] is defined by:

$$a, b, c = (1), (2), (3) \tag{18.19}$$

so that:

$$\boldsymbol{B}^{(3)*} = \boldsymbol{\nabla} \times \boldsymbol{A^{(3)*}} - ig\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \tag{18.20}$$
$$et\ cyclicum.$$

Here $(1), (2)$ and (3) are the indices of the complex circular basis with O(3) group symmetry, whose three complex unit vectors, $\boldsymbol{e}^{(1)}$, $\boldsymbol{e}^{(2)}$ and $\boldsymbol{e}^{(3)}$, are cyclically inter-related:

$$\boldsymbol{e}^{(1)} \times \boldsymbol{e}^{(2)} = i\boldsymbol{e}^{(3)*} \tag{18.21}$$
$$et\ cyclicum$$

and related to the Cartesian $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ by:

$$\boldsymbol{e}^{(1)} = \boldsymbol{e}^{(2)*} = \frac{1}{\sqrt{2}} \left(\boldsymbol{i} - i\boldsymbol{j} \right) \tag{18.22}$$

$$\boldsymbol{e}^{(3)} = \boldsymbol{k}. \tag{18.23}$$

The inverse Faraday effect is then defined in general relativity by the magnetization:

$$\boldsymbol{M}^{(3)*} = \frac{1}{\mu_0} \frac{g^{'}}{g} \boldsymbol{B}^{(3)*} = \frac{-i}{\mu_0} g^{'} \boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \tag{18.24}$$

where μ_0 is the permeability in vacuo and $g^{'}$ a coefficient in units of $e/\hbar$, the inverse fluxon. Therefore the inverse Faraday effect (magnetization by circularly or elliptically polarized electromagnetic radiation) observes $\boldsymbol{B}^{(3)}$ directly and is the magnetization of matter due to $\boldsymbol{B}^{(3)}$, as originally inferred by Evans in Dec. 1991 [17].

The correct homogeneous and inhomogeneous field equations of electrodynamics are found from Evans' generally covariant unified field theory by using the guidelines of differential geometry. The inhomogeneous field equation of generally covariant electrodynamics follows from the first Maurer Cartan structure relation (18.4) and is the fundamental Bianchi identity of differential geometry for spacetimes with both torsion and curvature [2]:

$$D \wedge \tau^a := R^a{}_b \wedge q^b. \tag{18.25}$$

Using the second Evans duality equation, Eq. (18.25) becomes:

$$D \wedge \tau^a := -\kappa\epsilon^a{}_{bc}\tau^c \wedge q^b. \tag{18.26}$$

Therefore the inhomogeneous field equation is a differential equation in the torsion form and can be solved analytically or numerically for any given situation in electrical and electronic engineering. Using Eq. (18.15), Eq. (18.26) becomes:

$$D \wedge F^a = \frac{-\kappa}{A^{(0)}} \epsilon^a{}_{bc} B^c \wedge A^b \tag{18.27}$$

i.e.

$$D \wedge F^a = g\epsilon^a{}_{bc} A^b \wedge B^c. \tag{18.28}$$

Eq. (18.28) replaces the familiar inhomogeneous field equation of Maxwell Heaviside field theory [19], and so replaces the Coulomb Law and the Ampere Maxwell Law.

The correct homogeneous field equation of electrodynamics is found from the following identity of differential geometry:

$$d \wedge \tau^a := R^a{}_b \wedge q^b - \omega^a{}_b \wedge \tau^b = 0 \tag{18.29}$$

where:

$$\widetilde{\tau}^a{}_{\rho\sigma} = \frac{1}{2} \epsilon_{\rho\sigma\mu\nu} \tau^{a\mu\nu} \tag{18.30}$$

is the dual of the torsion form in the base manifold. Using Eq. (18.15) the homogeneous field equation of generally covariant electrodynamics is therefore:

$$d \wedge F^a \sim 0 \tag{18.31}$$

and can again be solved analytically or numerically for any spacetime. The homogeneous equation (18.28) and the inhomogeneous equation (18.31) must be solved simultaneously for quantities of interest in practical electrical and electronic engineering, but this should be easily possible with contemporary software and hardware. Eq. (18.31) replaces the familiar homogeneous field equation of Maxwell Heaviside field theory and therefore replaces the Gauss Law and the Faraday Law of induction. Eq. (18.31) is an identity obeyed by any anti-symmetric tensor such as the torsion tensor, which in differential geometry becomes the vector valued torsion two form (vector valued because of the single index c, two form because of the two indices μ and ν, indicating an anti-symmetric tensor for each c [2]). In the older Maxwell Heaviside field theory the c index is missing and the electromagnetic field tensor is not recognized as a torsion form dual to the curvature or Riemann form. The reason for this is that the Maxwell Heaviside field theory is the archetypical theory of special relativity (flat spacetime) and historically preceded (circa 1900 to1905) the theory of general relativity (1915) (Riemann or curved spacetime but torsion form incorrectly omitted). It was first shown by Evans in 2003 [3]–[18] that field unification occurs correctly only in a spacetime with non-zero curvature correctly dual to non-zero torsion through the fundamental Evans duality equations (18.6) and (18.7) of differential geometry.

It follows that the correct equation of charge current density (J^a)in generally covariant electrodynamics is found from the right hand side of Eq. (18.28):

$$d \wedge F^a = \mu_0 j^a \tag{18.32}$$

From Eq. (18.31) it is seen that there are no physical magnetic monopoles in Evans' generally covariant unified field theory. The basic reason for this is that the covariant derivative of the dual of the torsion form is identically zero - a geometrical theorem obeyed in all spacetimes. There are, however, observable topological magnetic monopoles given by the covariant derivative [3]–[18]. These originate again in the fact that spacetime in general has curvature and torsion from differential geometry.

Having summarized the basic concepts the rest of this Section illustrates in detail the derivation of O(3) electrodynamics [3]–[18], which is now known to be an example of the more general Evans unified field theory.

From Eq. (18.15) it is seen that the following generally covariant gauge field is part of the general definition of gauge field:

$$G^c{}_{\mu\nu} = G^{(0)} \left(q^a{}_\mu q^b{}_\nu - q^a{}_\nu q^b{}_\mu \right). \tag{18.33}$$

Here $G^{(0)}$ is a scaling factor and $q^a{}_\mu$ and $q^b{}_\nu$ are tetrads. The magnetic field components from Eq (18.33) are:

$$B^c{}_{ij} = B^{(0)} \left(q^a{}_i q^b{}_j - q^a{}_j q^b{}_i \right),$$
$$i, j, k = 1, 2, 3 \tag{18.34}$$

and the electric field components are:

$$E^c{}_{0i} = E^{(0)} \left(q^a{}_0 q^b{}_i - q^a{}_i q^b{}_0 \right). \tag{18.35}$$

In the complex circular basis these equations become:

$$B^{(1)*}{}_{ij} = -iB^{(0)} \left(q^{(2)}{}_i q^{(3)}{}_j - q^{(2)}{}_j q^{(3)}{}_i \right) \tag{18.36}$$

$$B^{(2)*}{}_{ij} = -iB^{(0)} \left(q^{(3)}{}_i q^{(1)}{}_j - q^{(3)}{}_j q^{(1)}{}_i \right) \tag{18.37}$$

$$B^{(3)*}{}_{ij} = -iB^{(0)} \left(q^{(1)}{}_i q^{(2)}{}_j - q^{(1)}{}_j q^{(2)}{}_i \right) \tag{18.38}$$

$$E^{(1)*}{}_{0i} = -iE^{(0)} \left(q^{(0)}{}_0 q^{(2)}{}_i - q^{(0)}{}_i q^{(2)}{}_0 \right) \tag{18.39}$$

$$E^{(2)*}{}_{0i} = iE^{(0)} \left(q^{(0)}{}_0 q^{(1)}{}_i - q^{(0)}{}_i q^{(1)}{}_0 \right) \tag{18.40}$$

$$E^{(3)*}{}_{0i} = -iE^{(0)} \left(q^{(0)}{}_0 q^{(3)}{}_i - q^{(0)}{}_i q^{(3)}{}_0 \right). \tag{18.41}$$

The tetrad is defined by the Evans wave equation for the electromagnetic potential field:

$$(\Box + kT) A^a{}_\mu = 0. \tag{18.42}$$

The tetrad components appropriate to circularly polarized electromagnetic radiation uninfluenced by gravitation are as follows:

$$q^{(1)}{}_1 = -q^{(1)}{}_x = -ie^{i\phi}/\sqrt{2}, \tag{18.43}$$

$$q^{(1)}{}_2 = -q^{(1)}{}_y = -ie^{i\phi}/\sqrt{2}, \tag{18.44}$$

$$q^{(2)}{}_1 = -q^{(2)}{}_x = ie^{i\phi}/\sqrt{2}, \tag{18.45}$$

$$q^{(2)}{}_2 = -q^{(2)}{}_y = -ie^{i\phi}/\sqrt{2}, \tag{18.46}$$

$$q^{(0)}{}_0 = -q^{(3)}{}_z = 1. \tag{18.47}$$

The electric field components are therefore defined by:

$$
\begin{aligned}
E^{(2)}{}_{01} &= E^{(1)*}{}_{01} = -iE^{(0)}q^{(0)}{}_0 q^{(2)}{}_1 \\
&= -E^{(2)}{}_1 = E^{(2)}{}_x = E^{(0)}e^{-i\phi}/\sqrt{2}, \\
E^{(2)}{}_{02} &= E^{(1)*}{}_{02} = -iE^{(0)}q^{(0)}{}_0 q^{(2)}{}_2 \\
&= -E^{(2)}{}_2 = E^{(2)}{}_y = iE^{(0)}e^{-i\phi}/\sqrt{2}, \\
E^{(1)}{}_{01} &= E^{(2)*}{}_{01} = iE^{(0)}q^{(0)}{}_0 q^{(1)}{}_1 \\
&= -E^{(1)}{}_1 = E^{(1)}{}_x = E^{(0)}e^{i\phi}/\sqrt{2}, \\
E^{(1)}{}_{02} &= E^{(2)*}{}_{02} = iE^{(0)}q^{(0)}{}_0 q^{(1)}{}_2 \\
&= -E^{(1)}{}_2 = E^{(1)}{}_y = -iE^{(0)}e^{i\phi}/\sqrt{2}, \\
E^{(3)}{}_{03} &= -iE^{(0)} = -E^{(3)}{}_3 = E^{(3)}{}_z,
\end{aligned}
\tag{18.48}
$$

i.e

$$
\begin{aligned}
E^{(2)}{}_{01} &= -E^{(2)}{}_1 \\
E^{(2)}{}_{02} &= -E^{(2)}{}_2 \\
E^{(1)}{}_{01} &= -E^{(1)}{}_1 \\
E^{(1)}{}_{02} &= -E^{(1)}{}_2 \\
E^{(3)}{}_{03} &= -E^{(3)}{}_3.
\end{aligned}
\tag{18.49}
$$

and the magnetic field components by:

$$B^{(3)*}{}_{12} = B^{(3)}{}_{12} = B^{(3)}{}_{z} = -B^{(3)}{}_{3}$$
$$= -iB^{(0)}\left(q^{(1)}{}_{1}q^{(2)}{}_{2} - q^{(1)}{}_{2}q^{(2)}{}_{1}\right)$$
$$= B^{(0)},$$

$$B^{(1)*}{}_{23} = B^{(2)}{}_{23} = B^{(2)}{}_{x} = -B^{(2)}{}_{1}$$
$$= -iB^{(0)}\left(q^{(2)}{}_{2}q^{(3)}{}_{3} - q^{(2)}{}_{3}q^{(3)}{}_{2}\right)$$
$$= -iB^{(0)}e^{-i\phi}/\sqrt{2},$$

$$B^{(1)*}{}_{31} = B^{(2)}{}_{31} = -B^{(2)}{}_{y} = B^{(2)}{}_{2}$$
$$= -iB^{(0)}\left(q^{(2)}{}_{3}q^{(3)}{}_{1} - q^{(2)}{}_{1}q^{(3)}{}_{3}\right) \qquad (18.50)$$
$$= -B^{(0)}e^{-i\phi}/\sqrt{2},$$

$$B^{(1)*}{}_{13} = B^{(2)}{}_{13} = B^{(2)}{}_{y} = -B^{(2)}{}_{2}$$
$$= -iB^{(0)}\left(q^{(2)}{}_{1}q^{(3)}{}_{3} - q^{(2)}{}_{3}q^{(3)}{}_{1}\right)$$
$$= B^{(0)}e^{-i\phi}/\sqrt{2},$$

$$B^{(1)*}{}_{32} = B^{(2)}{}_{32} = -B^{(2)}{}_{x} = B^{(2)}{}_{1}$$
$$= -iB^{(0)}\left(q^{(2)}{}_{3}q^{(3)}{}_{2} - q^{(2)}{}_{2}q^{(3)}{}_{3}\right)$$
$$= iB^{(0)}e^{-i\phi}/\sqrt{2},$$

i.e.

$$B^{(3)}{}_{12} = -B^{(3)}{}_{21} = -B^{(3)}{}_{3},$$
$$B^{(2)}{}_{23} = -B^{(2)}{}_{32} = -B^{(2)}{}_{1}, \qquad (18.51)$$
$$B^{(2)}{}_{13} = -B^{(2)}{}_{31} = -B^{(2)}{}_{2},$$

and

$$B^{(1)}{}_{i} = -\frac{1}{2}\epsilon_{ijk}B^{(1)}{}_{jk},$$
$$B^{(2)}{}_{i} = -\frac{1}{2}\epsilon_{ijk}B^{(2)}{}_{jk}, \qquad (18.52)$$
$$B^{(3)}{}_{i} = -\frac{1}{2}\epsilon_{ijk}B^{(3)}{}_{jk}.$$

There are three sets of equations which give the correctly covariant form of the familiar Maxwell Heaviside field equations, to which we refer now only because of historical context. In other words the Maxwell Heaviside field equations must be regarded now as particular special cases of the more general Evans field equations defined as follows. The equations of index (1)

are deduced from the particular geometrical relations implied by using the complex circular basis:

$$\boldsymbol{B}^{(1)} = i\frac{\boldsymbol{E}^{(1)}}{c},$$
$$\boldsymbol{E}^{(1)} = -ic\boldsymbol{B}^{(1)}. \tag{18.53}$$

The duality relations (18.53) are obeyed by the following set of index (1) equations:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B}^{(1)} = 0, \ \boldsymbol{\nabla} \cdot \boldsymbol{E}^{(1)} = 0,$$
$$\frac{\partial \boldsymbol{B}^{(1)}}{\partial t} + \boldsymbol{\nabla} \times \boldsymbol{E}^{(1)} = 0, \tag{18.54}$$
$$\boldsymbol{\nabla} \times \boldsymbol{B}^{(1)} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}^{(1)}}{\partial t} = 0.$$

These are the O(3) electrodynamical field equations of index (1) [3]–[18]. The electric and magnetic fields for index (1) can be expressed as the well known transverse plane waves

$$\boldsymbol{E}^{(1)} = E^{(0)}(\boldsymbol{i} - i\boldsymbol{j}) e^{i\phi}/\sqrt{2},$$
$$\boldsymbol{B}^{(1)} = B^{(0)}(i\boldsymbol{i} + \boldsymbol{j}) e^{i\phi}/\sqrt{2}. \tag{18.55}$$

From elementary vector analysis:

$$\frac{\partial \boldsymbol{B}^{(1)}}{\partial t} = -\omega B^{(0)}(\boldsymbol{i} - i\boldsymbol{j}) e^{i\phi}/\sqrt{2}, \tag{18.56}$$

and the curl is defined as:

$$\boldsymbol{\nabla} \times \boldsymbol{E}^{(1)} = \frac{E^{(0)}}{\sqrt{2}} \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial X} & \frac{\partial}{\partial Y} & \frac{\partial}{\partial Z} \\ e^{i\phi} & -ie^{i\phi} & 0 \end{vmatrix}$$
$$= \kappa E^{(0)}(\boldsymbol{i} - i\boldsymbol{j}) e^{i\phi}/\sqrt{2}. \tag{18.57}$$

This verifies eq. (18.54).

Similarly:

$$\frac{\partial \boldsymbol{E}^{(1)}}{\partial t} = \omega E^{(0)}(i\boldsymbol{i} + \boldsymbol{j}) e^{i\phi}/\sqrt{2} \tag{18.58}$$

and

$$\boldsymbol{\nabla} \times \boldsymbol{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}} \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial X} & \frac{\partial}{\partial Y} & \frac{\partial}{\partial Z} \\ ie^{i\phi} & e^{i\phi} & 0 \end{vmatrix}$$
$$= \omega E^{(0)}(i\boldsymbol{i} + \boldsymbol{j}) e^{i\phi}/\left(\sqrt{2}c^2\right) \tag{18.59}$$

thus verifying Eq. (18.54).

The O(3) field equations of index (2) are built up from the geometrical duality:

$$\boldsymbol{B}^{(2)} = -i\frac{\boldsymbol{E}^{(2)}}{c},$$
$$\boldsymbol{E}^{(2)} = ic\boldsymbol{B}^{(2)}, \tag{18.60}$$

which obey the O(3) electrodynamical field equations of index (2):

$$\boldsymbol{\nabla} \cdot \boldsymbol{B}^{(2)} = 0, \ \ \boldsymbol{\nabla} \cdot \boldsymbol{E}^{(2)} = 0,$$
$$\frac{\partial \boldsymbol{B}^{(2)}}{\partial t} + \boldsymbol{\nabla} \times \boldsymbol{E}^{(2)} = \boldsymbol{0}, \tag{18.61}$$
$$\boldsymbol{\nabla} \times \boldsymbol{B}^{(2)} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}^{(2)}}{\partial t} = \boldsymbol{0}.$$

The electric and magnetic fields form Eqs. (18.61) are found to be the plane waves:

$$\boldsymbol{E}^{(2)} = E^{(0)}\left(\boldsymbol{i} + i\boldsymbol{j}\right)e^{-i\phi}/\sqrt{2},$$
$$\boldsymbol{B}^{(2)} = B^{(0)}\left(-i\boldsymbol{i} + \boldsymbol{j}\right)e^{-i\phi}/\sqrt{2}. \tag{18.62}$$

These are complex conjugates of the plane waves (18.55).

From elementary vector analysis:

$$\frac{\partial \boldsymbol{B}^{(2)}}{\partial t} = \omega B^{(0)}\left(-\boldsymbol{i} - i\boldsymbol{j}\right)e^{-i\phi}/\sqrt{2} \tag{18.63}$$

and the curl is:

$$\boldsymbol{\nabla} \times \boldsymbol{E}^{(2)} = \frac{E^{(0)}}{\sqrt{2}}\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial X} & \frac{\partial}{\partial Y} & \frac{\partial}{\partial Z} \\ e^{-i\phi} & ie^{-i\phi} & 0 \end{vmatrix}$$
$$= \kappa E^{(0)}\left(\boldsymbol{i} + i\boldsymbol{j}\right)e^{-i\phi}/\sqrt{2} \tag{18.64}$$

thus verifying Eq. (18.61). Similarly

$$\frac{\partial \boldsymbol{E}^{(2)}}{\partial t} = -i\omega E^{(0)}\left(\boldsymbol{i} + i\boldsymbol{j}\right)e^{-i\phi}/\sqrt{2} \tag{18.65}$$

and

$$\boldsymbol{\nabla} \times \boldsymbol{B}^{(2)} = \frac{B^{(0)}}{\sqrt{2}}\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial X} & \frac{\partial}{\partial Y} & \frac{\partial}{\partial Z} \\ -ie^{-i\phi} & e^{-i\phi} & 0 \end{vmatrix}$$
$$= \kappa B^{(0)}\left(-i\boldsymbol{i} + \boldsymbol{j}\right)e^{-i\phi}/\sqrt{2} \tag{18.66}$$

thus verifying Eq. (18.61).

The field equations of index (3) are fond from the geometrical duality:

$$\boldsymbol{B}^{(3)} = i\frac{\boldsymbol{E}^{(3)}}{c},$$
$$\boldsymbol{E}^{(3)} = -ic\boldsymbol{B}^{(3)}, \tag{18.67}$$

and are:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B}^{(3)} = 0, \ \ \boldsymbol{\nabla} \cdot \boldsymbol{E}^{(3)} = 0,$$
$$\frac{\partial \boldsymbol{B}^{(3)}}{\partial t} + \boldsymbol{\nabla} \times \boldsymbol{E}^{(3)} = \boldsymbol{0}, \tag{18.68}$$
$$\boldsymbol{\nabla} \times \boldsymbol{B}^{(3)} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}^{(3)}}{\partial t} = \boldsymbol{0}.$$

The fields for index (3) are missing entirely from Maxwell Heaviside field theory (spacetime with no torsion) and are the fundamental spin fields of general relativity (spacetime with torsion):

$$\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{k},$$
$$\boldsymbol{E}^{(3)} = -ic\boldsymbol{B}^{(3)}, \tag{18.69}$$
$$Re\left(\boldsymbol{E}^{(3)}\right) = \boldsymbol{0}.$$

These duality relations, field equations and fields of O(3) electrodynamics all follow from the fundamental definition (18.32), which is part of the more general definition (18.18).

The older Maxwell Heaviside field equations have the structure of Eq. (18.54), (18.61) and (18.68) but there are no indices (1), (2) and (3), and no spin field $\boldsymbol{B}^{(3)}$. The fundamental reason for this is now known to be the fact that the Maxwell Heaviside field theory is not correctly (i.e. generally) covariant. The Evans field theory is correctly covariant and is also a unified field theory which contains much more information about for example electricity and magnetism or electronics, computing and communications devices than the older Maxwell Heaviside field theory. From the Bianchi identity (18.25) and the Evans duality equations, it is clear that the Evans theory also contains information about the way in which electromagnetism influences gravitation, and this information is of importance in space propulsion engineering for example. Another example of the practical usefulness of the Evans theory stems from the fact that the Evans theory shwons that the electromagnetic field is a property of spacetime torsion. So in theory, it is possible to obtain energy from spacetime with torsion, an exceedingly important goal of energy engineering [20]. Historically, the Evans field theory was gradually inferred from the Evans spin field $\boldsymbol{B}^{(3)}$, which was in turn inferred from the experimental inverse Faraday effect.

Acknowledgments

References

1. A. Einstein, *The Meaning of Relativity*, (Princeton Univ. Press, 1921).
2. S. M. Carroll, *Lecture Notes in General Relativity*, (Univ. California Santa Barbara, graduate course), arXiv:gr-gq/9712019 v1 3 Dec, 1997.
3. M. W. Evans, *Found. Phys. Lett.*, **16**, 367 (2003).
4. M. W. Evans, *Found. Phys. Lett.*, **16**, 507 (2003).
5. M. W. Evans, *Found. Phys. Lett.*, **17**, 25 (2004).
6. M. W. Evans, *Found. Phys. Lett.*, **17**, 149 (2004).
7. M. W. Evans, *Found. Phys. Lett.*, **17**, 267 (2004).
8. M. W. Evans, *Found. Phys. Lett.*, **17**, 301 (2004).
9. M. W. Evans, *Found. Phys. Lett.*, **17**, 393 (2004).
10. M. W. Evans, The Evans Lemma of Differential Geometry, *Found. Phys. Lett.*, **17** (2004), in press, preprint on www.aias.us.
11. M. W. Evans, Derivation of the Evans Wave Equation form the Lagrangian and Action, Origin of the Planck Constant in General Relativity, Found., Phys. Lett., **17** (2004), in press, preprint on www.aias.us.
12. M. W. Evans and AIAS Author Group, Development of the Evans Wave Equation in the Weak Field Limit, the Electrogravitic Equation, *Found. Phys. Lett.*, **17** (2004), in press, preprint on www.aias.us.
13. M. W. Evans, New Concepts form the Evans Unified Field Theory, Part One, The Evolution of Curvature, Oscillatory Universe without Singularity, Causal Quantum Mechanics and General Force and Field Equations, *Found. Phys,. Lett*, **17** (2004), submitted, preprint on www.aias.us.
14. ibid., part two: Derivation of the Heisenberg Equation and Replacement of the Heisenberg Uncertainty Principle, *Found. Phys. Lett.*, **17** (2004), submitted, preprint on www.aias.us.
15. M. W. Evans, Derivation of the Lorentz Boost from the Evans Wave Equation, *Found. Phys. Lett.*, **17** (2004), submitted, preprint on www.aias.us.
16. M. W. Evans, *Generally Covariant Unified Field Theory, the Geometrization of Physics*, (in press, 2005); L. Felker and M. W. Evans, *The Evans Equations of Unified Field Theory* (World Scientific, Singapore, in prep., 2004 / 2005); *The Collected Scientific Papers of Myron Wyn Evans*, (World Scientific , Singapore, from 2004, circa fifteen to twenty volumes), vols. 1 and 2, 2000-2004; M. W. Evans (ed.), *Modern Nonlinear optics*, a special topical issue of I. Prigogine and AS. A Rice, (series eds.), *Advances in Chemical Physics*, (Wiley Interscience,

New York, 2001, 2nd and e book eds.), vols.119(1) to 119(3), ibid., vol. 85(1) to 85(3), first ed. (Wiley Interscience, New York, 1992, 1993, 1997, hardback and softback); M. W. Evans and L. B. Crowell, *Classical and Quantum Electrodynamics and the $B^{(3)}$ Field* (World Scientific, Singapore, 2001); M. W. Evans, J,.-P, Vigier et al., *The Enigmatic Photon* (Kluwer Dordrecht, 1994 to 2002, hardback and softback), in five volumes; M. W. Evans and A. A Hasanein, *The Photomagneton in Quantum Field Theory* (World Scientific, Singapore, 1994).

17. M. W. Evans, Physica B, **182**, 227, 237 (1992).

18. P. Pershan, J. P. van der Ziel, and L. D. Malmstrom, Phys. Rev., **143**, 574 (1966); J .Deschamps, M. Fitaire and M. Lagoutte, *Phys. Rev. Lett.*, **25**, 1330 (1970); M. Tatarikis et al., Measurements of the Inverse Faraday Effect in High Intensity Laser Induced Plasma, (Ann. Rep., Rutherford Appleton Lab., 1998/1999).

19. M. W. Evans, reviews in vols. 119(2) and 119(3) of Modern Nonlinear Optics (2nd edition), see these and other entries of .ref. (16).

20. T. E. Bearden et al, ref (19), vol 119(2) and www.cheniere org.