ON THE NEED FOR THREE METRIC COMPATIBILITY EQUATIONS TO DEFINE A UNIQUELY ANTISYMMETRIC CONNECTION.

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ABSTRACT

It is shown that three metric compatibility equations are needed to define a unique antisymmetric connection. The need for an antisymmetric connection is indicated by the fundamental commutator method used to define the Riemann curvature and torsion. The use of one compatibility equation is not sufficient, and results in ambiguity.

Keywords: Unique antisymmetric connection, metric compatibility, ECE theory.

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INTRODUCTION

Recently in this series of papers and books {1 - 10} developing ECE unified field theory it has been realized that the antisymmetric geometrical connection can be obtained straightforwardly from the metric compatibility condition and that the Hooke / Newton force equation of universal gravitation can be described directly in terms of the antisymmetric connection. The latter is defined at a most fundamental level by the commutator method {11} that is used to represent a round trip in the space with torsion and curvature. The Riemann curvature cannot be defined without the Riemann torsion. Both always occur together and ineluctably. In the early days of the subject, circa 1900, Levi Civita and Ricci defined the geometrical connection arbitrarily to be symmetric in its lower two indices and used this assertion to extract a unique connection from three compatibility equations in cyclic permutation. This was a catastrophic error repeated uncritically down the entire twentieth century. The commutator method shows clearly that the geometrical connection is antisymmetric in its lower two indices, it takes the antisymmetry of the commutator itself, as do the torsion and curvature tensors. A simple proof of this has been given in the preceding paper UFT 187 (www.aias.us). The commutator equation uses the concept of connection, and does not use the concept of metric. Similarly the most fundamental Evans identity {1 - 10} uses only the connection. The metric enters the stage because it is used for a variety of purposes {11} in geometry and it is used to derive orbits.

The metric compatibility equation is an assertion without proof and may be obtained form the tetrad postulate of Cartan's geometry {1 - 11}, in which he was the first to generalize the idea of connection, due to Christoffel. The compatibility equation asserts that the covariant derivative of the metric vanishes. The metric is a rank two tensor so its covariant derivative contains two types of Christoffel connection. Using the idea of metric compatibility therefore, a link can be forged between the concept of metric, due to Riemann, and the concept of connection, due to Christoffel. The connection may be determined uniquely using three compatibility equations in cyclic permutation. It is shown in Section 2 that this procedure when used with a correctly antisymmetric connection produces a simple equation for the antisymmetric connection, and which defines the connection uniquely. In Section 3 computer algebra is used to show that the use of one compatibility equation is insufficient to define a connection, and leads to ambiguities.

2. DERIVATION OF A UNIQUELY DEFINED ANTISYMMETRIC CONNECTION

Consider three metric compatibility equations in cyclic permutation:

$$\partial_\rho g_{\mu\nu} - \Gamma^{\lambda}_{\rho\mu} g_{\lambda\nu} - \Gamma^{\lambda}_{\rho\nu} g_{\mu\lambda} = 0 \qquad -(1)$$

$$\partial_\mu g_{\nu\rho} - \Gamma^{\lambda}_{\mu\nu} g_{\lambda\rho} - \Gamma^{\lambda}_{\mu\rho} g_{\nu\lambda} = 0 \qquad -(2)$$

$$\partial_\nu g_{\rho\mu} - \Gamma^{\lambda}_{\nu\rho} g_{\lambda\mu} - \Gamma^{\lambda}_{\nu\mu} g_{\rho\lambda} = 0 \qquad -(3)$$

where $g_{\mu\nu}$ is the metric, and $\Gamma^{\lambda}_{\rho\mu}$ is the connection. Subtract the second two equations form the first to obtain:

$$\partial_\rho g_{\mu\nu} - \partial_\mu g_{\nu\rho} - \partial_\nu g_{\rho\mu} - \Gamma^{\lambda}_{\rho\mu} g_{\lambda\nu} - \Gamma^{\lambda}_{\rho\nu} g_{\mu\lambda}$$
$$+ \Gamma^{\lambda}_{\mu\nu} g_{\lambda\rho} + \Gamma^{\lambda}_{\mu\rho} g_{\nu\lambda} + \Gamma^{\lambda}_{\nu\rho} g_{\lambda\mu} + \Gamma^{\lambda}_{\nu\mu} g_{\rho\lambda} = 0 \qquad -(4)$$

From the commutator equation any non-zero connection must be antisymmetric:

$$\Gamma^{\lambda}_{\mu\nu} = -\Gamma^{\lambda}_{\nu\mu} \qquad -(5)$$

and the metric is defined to be symmetric {11}:

$$g_{\mu\nu} = g_{\nu\mu} . \qquad -(6)$$

Therefore:

$$\Gamma^{\lambda}_{\mu\nu} g_{\lambda\rho} + \Gamma^{\lambda}_{\nu\mu} g_{\rho\lambda} = 0. \quad -(7)$$

For simplicity of development and without loss of generalization consider a diagonal metric:

$$\mu = \nu \quad -(8)$$

so

$$\Gamma^{\lambda}_{\mu\rho} g_{\nu\lambda} + \Gamma^{\lambda}_{\nu\rho} g_{\lambda\mu} = 0. \quad -(9)$$

Therefore Eq. (4) simplifies to:

$$\underset{\mu = \nu.}{\partial_\rho g_{\mu\nu} - \partial_\mu g_{\nu\rho} - \partial_\nu g_{\rho\mu}} = \Gamma^{\lambda}_{\rho\mu} g_{\lambda\nu} + \Gamma^{\lambda}_{\rho\nu} g_{\mu\lambda}. \quad -(10)$$

Let:

$$\mu = \nu = \alpha \quad -(11)$$

then:

$$\partial_\rho g_{\alpha\alpha} - \partial_\alpha g_{\alpha\rho} - \partial_\alpha g_{\rho\alpha} = 2\Gamma^{\lambda}_{\rho\alpha} g_{\lambda\alpha}. \quad -(12)$$

Multiply both sides by $g^{\lambda\alpha}$, the inverse metric, then:

$$g^{\lambda\alpha}\left(\partial_\rho g_{\alpha\alpha} - \partial_\alpha g_{\alpha\rho} - \partial_\alpha g_{\rho\alpha}\right) = 2 g^{\lambda\alpha} \Gamma^{\lambda}_{\rho\alpha} g_{\lambda\alpha} = 2\Gamma^{\lambda}_{\rho\alpha} \quad -(13)$$

Therefore:

$$\Gamma^{\lambda}_{\rho\alpha} = \frac{1}{2} g^{\lambda\alpha}\left(\partial_\rho g_{\alpha\alpha} - \partial_\alpha g_{\alpha\rho} - \partial_\alpha g_{\rho\alpha}\right). \quad -(14)$$

The metric must be diagonal in this development, so:

$$\Gamma^{\alpha}_{\rho\alpha} = \frac{1}{2} g^{\alpha\alpha} \left(\partial_\rho g_{\alpha\alpha} - \partial_\alpha g_{\alpha\rho} - \partial_\alpha g_{\rho\alpha} \right) \quad -(15)$$

because:

$$\alpha = \lambda . \quad -(16)$$

The connection must be antisymmetric, so:

$$\rho \neq \alpha \quad -(17)$$

in which case:

$$g_{\alpha\rho} = g_{\rho\alpha} = 0 \quad -(18)$$

and

$$\Gamma^{\alpha}_{\rho\alpha} = \frac{1}{2} g^{\alpha\alpha} \partial_\rho g_{\alpha\alpha} . \quad -(19)$$

This is a simple equation in which summation over repeated indices is not implied. Therefore for example, if

$$\alpha = 0, \rho = 1 \quad -(20)$$

then

$$\Gamma^{0}_{10} = \frac{1}{2} \frac{1}{g_{00}} \partial_1 g_{00} . \quad -(21)$$

If

$$\alpha = 2, \rho = 1 \quad -(22)$$

then:

$$\Gamma^{2}_{12} = \frac{1}{2} \frac{1}{g_{22}} \partial_1 g_{22} \quad -(23)$$

If

$$\alpha = 3, \rho = 1 \qquad -(24)$$

then

$$\Gamma^3_{13} = \frac{1}{2} \frac{1}{g_{33}} \partial_1 g_{33} \qquad -(25)$$

and if

$$\alpha = 3, \rho = 2 \qquad -(26)$$

then

$$\Gamma^3_{23} = \frac{1}{2} \frac{1}{g_{33}} \partial_2 g_{33} \qquad -(27)$$

Note that the index:

$$\alpha = 1 \qquad -(28)$$

is not used because this produces a symmetric connection, which is disallowed at the outset.

3.. THE NON VANISHING ANTISYMMETRIC CONNECTIONS OF A SPHERICALLY SYMMETRIC SPACETIME.

As shown in the Appendix the use of three metric compatibility equations in cyclic permutation leads to the fundamental theorem (A11) linking the antisymmetric connection and the metric for any mathematical space of any dimension. The fundamental theorem is the metric compatibility condition for a connection defined to be antisymmetric. Conversely any antisymmetric metric is metric compatible by definition. The use of three metric compatibility equations leads back again to one metric compatibility equation. The latter is therefore necessary and sufficient to define the antisymmetric connection given a metric.

It is well known {11} that the metric of a spherically symmetric spacetime is:

$$g_{00} = -m(r,t),\ g_{11} = n(r,t),\ g_{22} = r^2,\ g_{33} = r^2 \sin^2 \phi \quad -(29)$$

where $m(r,t)$ and $n(r,t)$ are functions of time t and the radial coordinate r. In this case the allowed antisymmetric connections are given in the following table.

Table 1: The Antisymmetric Connections of a Spherical Spacetime

μ	ν	$\Gamma^{\lambda}_{\mu\nu} = -\Gamma^{\lambda}_{\nu\mu}$
0	1	Γ^{0}_{10}
	2	Γ^{0}_{20}
	3	Γ^{0}_{30}
1	0	Γ^{1}_{01}
	2	Γ^{1}_{21}
	3	Γ^{1}_{31}
2	0	Γ^{2}_{02}
	1	Γ^{2}_{12}
	3	Γ^{2}_{32}
3	0	Γ^{3}_{03}
	1	Γ^{3}_{13}
	2	Γ^{3}_{23}

For a metric of type (29) there are five antisymmetric connections defined as follows:

$$\Gamma^{0}_{10} = -\Gamma^{0}_{01} = \frac{1}{2m}\frac{\partial m}{\partial r} \quad -(30)$$

$$\Gamma^{1}_{01} = -\Gamma^{1}_{10} = \frac{1}{2n}\frac{1}{c}\frac{\partial n}{\partial t} \quad -(31)$$

$$\Gamma^{2}_{12} = -\Gamma^{2}_{21} = \frac{1}{r} \quad -(32)$$

$$\Gamma^{3}_{13} = -\Gamma^{3}_{31} = \frac{1}{r} \quad -(33)$$

$$\Gamma^{3}_{23} = -\Gamma^{3}_{32} = \frac{\cos\phi}{\sin\phi}. \quad -(34)$$

These are constrained by the Evans identity {1 - 10}

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa\ \ \mu\nu}_{\ \mu} \quad -(35)$$

The allowed elements of the torsion tensor for a spherical spacetime described by metric (29) are simply twice the allowed connection elements:

$$T^{\lambda}_{\mu\nu} = 2\Gamma^{\lambda}_{\mu\nu}. \quad -(36)$$

The curvature tensor is defined by

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}. \quad -(37)$$

For use in Eq. (35) indices are raised and lowered with the metric elements as usual.

With the collapse of Einsteinian cosmology, it is necessary to attempt the development of a new cosmology based on the antisymmetric connection. A spherical spacetime in which there is no dependence of r on t produces the metric (29) and the antisymmetric connections (30) to (34). The Evans identity (35) can be used to attempt to define m and n. Finally the equation of orbits is obtained from the infinitesimal line element:

$$ds^2 = -c^2 m(r,t)\,dt^2 + n(r,t)\,dr^2 + r^2 d\Omega^2 \quad -(38)$$

The challenge is to describe all orbits without the use of dark matter. There are many types of orbits in astronomy, ranging from those in the solar system (the relativistic Kepler orbits) to those of spiral galaxies. It has been known for half a century that the obsolete Einsteinian cosmology was completely unable to describe the orbits of a spiral galaxy, so cosmology fell back on pure empiricism by introducing unobservable dark matter. At that stage cosmology ceased to be a scientific subject. The only way of putting it back on course is through use of the correct geometry as outlined in this and other papers of this series {1 - 10}.

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APPENDIX TO UFT 188: THEOREM OF THE ANTISYMMETRIC CONNECTION.

Consider three metric compatibility conditions in cyclic permutation:

$$\partial_\rho g_{\mu\nu} - \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} - \Gamma^\lambda_{\rho\nu} g_{\mu\lambda} = 0 \quad -(A1)$$

$$\partial_\mu g_{\nu\rho} - \Gamma^\lambda_{\mu\nu} g_{\lambda\rho} - \Gamma^\lambda_{\mu\rho} g_{\nu\lambda} = 0 \quad -(A2)$$

$$\partial_\nu g_{\rho\mu} - \Gamma^\lambda_{\nu\rho} g_{\lambda\mu} - \Gamma^\lambda_{\nu\mu} g_{\rho\lambda} = 0. \quad -(A3)$$

Subtract Eqs. (2) and (3) from Eq. (1):

$$\partial_\rho g_{\mu\nu} - \partial_\mu g_{\nu\rho} - \partial_\nu g_{\rho\mu} - \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} - \Gamma^\lambda_{\rho\nu} g_{\mu\lambda} + \Gamma^\lambda_{\mu\nu} g_{\lambda\rho} + \Gamma^\lambda_{\mu\rho} g_{\nu\lambda} + \Gamma^\lambda_{\nu\rho} g_{\lambda\mu} + \Gamma^\lambda_{\nu\mu} g_{\rho\lambda} = 0 \quad -(A4)$$

Subtract Eqs. (1) and (2) from Eq. (3):

$$\partial_\nu g_{\rho\mu} - \partial_\mu g_{\nu\rho} - \partial_\rho g_{\mu\nu} - \Gamma^\lambda_{\nu\rho} g_{\lambda\mu} - \Gamma^\lambda_{\nu\mu} g_{\rho\lambda} + \Gamma^\lambda_{\mu\nu} g_{\lambda\rho} + \Gamma^\lambda_{\mu\rho} g_{\nu\lambda} + \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} + \Gamma^\lambda_{\rho\nu} g_{\mu\lambda} = 0 \quad -(A5)$$

Subtract Eqs. (1) and (3) from Eq. (2):

$$\partial_\mu g_{\nu\rho} - \partial_\rho g_{\mu\nu} - \partial_\nu g_{\rho\mu} - \Gamma^\lambda_{\mu\nu} g_{\lambda\rho} - \Gamma^\lambda_{\mu\rho} g_{\nu\lambda} + \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} + \Gamma^\lambda_{\rho\nu} g_{\mu\lambda} + \Gamma^\lambda_{\nu\rho} g_{\lambda\mu} + \Gamma^\lambda_{\nu\mu} g_{\rho\lambda} = 0 \quad -(A6)$$

Now apply antisymetry:

$$\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu} \quad -(A7)$$

to obtain:

$$\partial_\rho g_{\mu\nu} - \partial_\mu g_{\nu\rho} - \partial_\nu g_{\rho\mu} = 2\left(\Gamma^\lambda_{\rho\mu} g_{\lambda\nu} + \Gamma^\lambda_{\rho\nu} g_{\lambda\mu}\right) \quad -(A8)$$

$$\partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} - \partial_\mu g_{\nu\rho} = 2\left(\Gamma^\lambda_{\nu\rho} g_{\lambda\mu} + \Gamma^\lambda_{\nu\mu} g_{\lambda\rho}\right) \quad -(A9)$$

$$\partial_\mu g_{\nu\rho} - \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} = 2\left(\Gamma^\lambda_{\mu\nu} g_{\lambda\rho} + \Gamma^\lambda_{\mu\rho} g_{\lambda\nu}\right). \quad -(A10)$$

Add Eqs. (8) and (10):

$$\partial_\nu g_{\rho\mu} = -\left(\Gamma^\lambda_{\rho\nu} g_{\lambda\mu} + \Gamma^\lambda_{\mu\nu} g_{\lambda\rho}\right) \quad -(A11)$$

This equation relates the general metric to the antisymmetric connection.

For a diagonal metric:

$$\rho = \mu \quad -(A12)$$

so

$$\Gamma^\mu_{\nu\mu} = \frac{1}{2 g_{\mu\mu}} \partial_\nu g_{\mu\mu} \quad -(A13)$$

$$\nu \neq \mu \quad -(A14)$$

This result is given the appellation "Theorem of the Antisymmetric Connection".