

THE MAXWELLIAN LIMIT OF THE EINSTEIN / DE BROGLIE THEORY  
OF ELECTROMAGNETIC RADIATION.

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ABSTRACT

In the Einstein / de Broglie theory of electromagnetic radiation, the photon has a rest mass of  $10^{-36}$  kgm (the Einstein photon mass). Within the framework of this theory, it is shown that there exist longitudinal electromagnetic fields which are analytically related to the corresponding transverse components, implying that electric and magnetic fields in vacuo are four-vectors, as first proposed by Einstein and de Broglie. The observation of these longitudinal fields would support the Einstein / de Broglie Theory, and experimental arrangements are proposed.

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## INTRODUCTION

It is almost universally asserted in the contemporary literature that the photon is massless, and that the range of the electromagnetic field is infinite. However, Einstein {1-5} has proposed that the mass of the photon is about  $10^{-68}$  kgm, an estimate based on the Hubble constant. In consequence, the range of electromagnetic radiation is about  $10^{26}$ m, (several tens of thousands of millions of light years, but finite). The finite photon mass is the basis of the Einstein / de Broglie theory of light {6} in which the Copenhagen interpretation of Bohr and others is rejected in favor of light being constituted by real Maxwellian waves coexisting with photons in Minkowski spacetime. This is a causal, stochastic, model of electromagnetic radiation. In the Copenhagen interpretation, on the other hand, light is made up of waves of probability, which can never co-exist with photons in spacetime. A recent experiment by Mizobuchi and Ohtake {7} contradicts the Copenhagen interpretation but can be interpreted straightforwardly {8} with the Einstein / de Broglie Theory. The experiment demonstrates that electromagnetic waves and photons co-exist.

In this Letter, it is shown that the Einstein / de Broglie Theory of light allows longitudinal electromagnetic fields in vacuo, fields which are related analytically to the corresponding transverse components through the equation recently derived by Evans {9-11}:

$$\begin{aligned} B^{(3)} &= \frac{E^{(1)} \times E^{(2)}}{E^{(0)} c i} \\ &= \frac{B^{(1)} \times B^{(2)}}{B^{(0)} i} \\ &= B^{(0)} k \end{aligned} \tag{1}$$

Here  $B^{(0)}$  is the longitudinal magnetic field,  $E^{(1)}$  and  $E^{(2)}$  are transverse components of the electric field,  $\frac{E^{(0)}}{\sqrt{2}}$  is the electric field's scalar amplitude,  $c$  the speed of light in vacuo,  $B^{(1)}$

and  $B^{(2)}$  are transverse magnetic field components; and  $\frac{B^{(0)}}{\sqrt{2}}$  their scalar amplitude. Farahi

and Evans {12} have shown that a non-zero  $B^{(0)}$  implies a non-zero  $E^{(0)}$ , i.e. a longitudinal electric field travelling with the photon in vacuo.

A simple demonstration is given of the existence of  $B^{(0)}$  for a finite photon mass, and of the fact that eqn. (1) is the Maxwellian (zero photon mass) limit of the Einstein / de Broglie Theory. Experimental observation of  $B^{(0)}$  and  $E^{(0)}$  would therefore provide support for this theory, which implies {13} that electric and magnetic fields in vacuo are four-vectors, with physically meaningful spacelike and timelike components. This deduction is also the foundation for manifestly covariant electrodynamics, recently proposed by Evans {14} on the basis of eqn. (1).

In the conventional contemporary theory of electromagnetic radiation {15} the longitudinal spacelike and timelike polarizations are rejected as unphysical, an arbitrary and self-contradictory procedure {16}, because the d'Alembert equation, and its quantised counterpart, the Gupta Bleuler condition {17} produce four polarizations. Recent work {14} has shown that the existence of four physically meaningful polarizations can be reconciled straightforwardly with two helicities, coming from the theory of the Poincaré group. Even in the massless limit, therefore, the existence of four field (photon) polarizations is rigorously

supported by fundamental considerations. Eqn. (1) shows clearly that the notion (which has gained acceptance) of arbitrarily rejecting the longitudinal fields as meaningless is untenable, because the longitudinal (3) component is directly proportional to the vector product of the transverse (1) and (2) components, the timelike component (0) being associated with the scalar field amplitudes in vacuo.

Equation (1) is therefore the fundamental link between physically meaningful longitudinal and transverse components of electromagnetic radiation, and provides new insight into the Einstein / de Broglie Theory. The equation was first derived {9-11} using the Maxwell equations, equivalent to zero photon mass, but it is shown in this Letter to be valid for finite photon mass. The most important consequence of eqn. (1), however, is that it implies four physically meaningful electromagnetic field polarizations, and this is also implied {13} by the Einstein / de Broglie Theory. In the contemporary theory of electrodynamics, however, the notion of abandoning two polarizations (either of the classical field or the photon) has been accepted uncritically. This notion must be questioned in view of equation (1), which is consistent with the Einstein / de Broglie Theory. The question arises immediately of whether or not eqn. (1) is consistent or inconsistent with the Copenhagen interpretation, and the best way of answering this is by reference to the Mizobuchi / Ohtake experiment {7} as interpreted by Vigier {7}. Thus, even if the Copenhagen interpretation can be made to satisfy eqn. (1) theoretically, it would still be in contradiction with experimental data, implying that it is better from the outset to work within the framework of the Einstein / de Broglie Theory. Significantly, it was shown by de Broglie {18} and by Schrödinger {19} that this theory allows longitudinal as well as transverse waves in vacuo, and thus

longitudinal and transverse photon polarizations, which coexist with the waves, but neither author appears to have realized the existence of equation (1). The latter rigorously links together transverse and longitudinal polarizations, and shows that the longitudinal polarization is independent of the phase of the wave, and thus satisfies {8} the Gauss Theorem in vacuo.

### 1. EQUATION (1) FOR FINITE PHOTON MASS.

One of the fundamental equations of the Einstein / de Broglie theory of light is

$$\square \psi_{\mu} = 2\mu^2 \psi_{\mu} \quad (2)$$

where  $\psi_{\mu}$  is a complex vector wave {7}. As we have mentioned, this equation was shown by de Broglie and Schrödinger to have longitudinal and transverse components. In a theory structured in Maxwell's framework, eqn. (2) can be written as a d'Alembert equation with a finite right hand side term in vacuo:

$$\square A = -\xi^2 A \quad (3)$$

where  $\xi$  is a constant, and the d'Alembertian, as usual, is

$$\square \equiv -\nabla^2 + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \quad (4)$$

We use the transformations

$$\nabla \rightarrow \frac{i}{\hbar} \mathbf{p}; \quad \frac{\partial}{\partial t} \rightarrow -\frac{i}{\hbar} E \quad (5)$$

into quantum mechanics {20}, where, as usual,  $\mathbf{p}$  and  $E$  denote momentum and energy respectively, of a particle associated with the wave equation (3). This particle is the photon with finite Einstein mass:

$$\frac{1}{\hbar^2}(-p^2 + \frac{E^2}{c^2})A = \xi^2 A \quad (6)$$

and since  $A$  is a wave quantity which loses significance {20} in a particulate context:

$$E^2 = p^2 c^2 + m^2 c^4 \quad (7)$$

This is Einstein's relativistic equation linking mass and energy, with a photon mass of

$$m = \frac{\hbar}{c} \xi \quad (8)$$

This mass is  $10^{-68}$  kgm {7}, calculated from the Hubble constant. Eqn. (8) gives the constant

$$\xi = \frac{mc}{\hbar} \doteq 10^{-26} m^{-1} \quad (9)$$

and the finite range of the electromagnetic field:

$$\frac{1}{\xi} = \frac{\hbar}{mc} \doteq 10^{26} m \quad (10)$$

(some tens of thousands of millions of light years, a cosmic but finite dimension of the order of the radius of the universe). The d'Alembert equation (3) is therefore:

$$\square A \doteq 10^{-52} A \quad (11)$$

For practical purposes the right hand side is so small as to be essentially zero, and eqn (11) reduces to the standard d'Alembert equation in vacuo:

$$\square A = 0 \quad (12)$$

It is clear therefore that the Einstein / de Broglie theory of light approximates closely the Maxwell equations in the classical regime described by the wave equation (11). It is also clear that the solutions of eqn. (11) coexist with those of eqn. (7), for photons of finite mass. Eqn. (1) also holds to an excellent approximation in the Maxwellian description of the Einstein / de Broglie Theory, (eqn. (11)), because eqn. (1) is consistent with the d'Alembert equation (12).

Furthermore, eqn. (3) implies the following equation in magnetic flux density in vacuo:

$$\nabla^2 \mathbf{B} = \xi^2 \mathbf{B} \quad (13)$$

whose physical solution {3} is an exponentially decaying longitudinal field in vacuo:

$$\mathbf{B}^{(3)} = B^{(0)} \exp(-\xi Z) \mathbf{k} \quad (14)$$

(Using the relation

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (15)$$

in eqn. (13) implies

$$\nabla^2 (\nabla \times \mathbf{A}) = \xi^2 \nabla \times \mathbf{A}; \quad \text{i.e. } \nabla^2 \mathbf{A} = \xi^2 \mathbf{A}; \quad (16)$$

whose Lorentz covariant form is obtained {20} by replacing the laplacian by the d'alembertian. Thus eqn. (3) implies eqn. (13) and vice versa.)

For all practical purposes, eqn. (14) is:

$$\mathbf{B}^{(3)} = B^{(0)} \mathbf{k} \quad (17)$$

which is the left hand side of eqn. (1). It is therefore straightforward to show that eqn. (1) corresponds to the zero photon mass, Maxwellian, form of the Einstein / de Broglie theory of light.



## 2. DISCUSSION AND EXPERIMENTAL CONSEQUENCES

It is important to note that eqn. (1) is implied in the work of Moles and Vigier {21} and that of Bass and Schrödinger {22}. This has become clear through the following comments by Vigier {23}. The  $E^L$  field of Moles and Vigier {21} is parallel to the  $B^{(3)}$  field of equation (1). The three vectors  $E_i^T$  and  $B_i^T$  are then defined by

$$E_i^T = \frac{1}{2} \epsilon_{ijk} F^{jk}; \quad B_i^T = \frac{1}{2} \epsilon_{ijk} F^{jk} \quad (18)$$

in the notation of Moles and Vigier {21}. Equation (1) of this paper can then be written in that notation as

$$B_k^{(3)} = \frac{1}{2} \epsilon_{ijk} E_i^T \frac{E_j^{T*}}{(B_0 c t)} = \frac{1}{2} \epsilon_{ijk} B_i^T \frac{B_j^{T*}}{(B_0 t)} \quad (19)$$

so that  $B_k$  is parallel to  $k_k$  and  $B_0^2 \propto (J_0)^2$  keeping to the notation of ref. {21}. Since all field amplitudes are multiplied in that notation by  $\exp(-i(kX - \omega t))$ , i.e.  $k_x = -i\hbar(A_x^* \partial_x A_x - \text{c.c.})$  and  $E^T$ ,  $H^T$  and  $k$  are mutually perpendicular, we have {21}:

$$B^{(3)} = B^0 k \text{ and } E^L \perp k \quad (20)$$

which is equation (1) of this paper.

Therefore equation (1) is straightforwardly implied by the equations of Moles and Vigier {21} in a paper which discusses the possible physical consequences of the existence of a

non zero photon mass in the interaction of light and matter. It becomes clear that the  $B^{(0)}$  and  $E^{(0)}$  fields are the Maxwellian limiting forms of the longitudinal fields implied by the Einstein / de Broglie theory of light, and for all practical purposes, are indistinguishable (because  $\xi$  in eqn. (14) is of the order  $10^{-26}m^{-1}$ ). Thus, experimental evidence for  $B^{(0)}$  and  $E^{(0)}$  would be evidence for finite photon mass, because it would be consistent both with the Einstein / de Broglie theory and with OTHER experimental evidence for finite photon mass reviewed by Vigier {8}. There is no experimental evidence for zero photon mass.

The following proposed experiments, if positive, can therefore be considered {23} as evidence for finite photon mass as well as for the longitudinal fields  $B^{(0)}$  and  $E^{(0)}$ ; the exchange of longitudinal photons; the action of longitudinal photons on matter, and for the fact that sources emit longitudinal as well as transverse photons in THREE polarisation states, not two as in the conventional interpretation of light.

The major experimental consequence of eqn (1), (or eqn. (17)), is that there exist longitudinal fields  $B^{(0)}$  and  $E^{(0)}$  which are proportional to the square root of light intensity in vacuo {9-11}. In the context of  $B^{(0)}$ , for example, the field has all the attributes of a magnetic flux density {9-11,14} and should therefore produce optical effects in analogy with effects due to a conventional magnetic field. Examples have been proposed and discussed in some detail in the literature {9-11,14} and include the following, all proportional to the square root of light intensity ( $watt\ m^{-2}$ ) of a circularly polarized laser pulse:

- a) An inverse Faraday effect (magnetization).
- b) An optical Faraday effect (azimuth rotation).
- c) An optical Zeeman effect (spectral splitting).

- d) Optically induced shifts in ESR and NMR.
- e) Optically induced Cotton Mouton and Majorana effects.
- f) Optically induced forward backward birefringence effects.
- g) — Extra effects in Compton scattering.
- h) Other magnetic effects.

Additionally, the longitudinal field  $E^{(0)}$  should produce similar optically induced effects such as Stark shifts, which depend on  $E^{(0)}$  to first order.

Furthermore, it has been shown {9-11,14} that conventional interpretations of such well known phenomena as simple absorption, ellipticity, circular dichroism, the Kerr effect, antisymmetric scattering, and well known parameters such as those of Stokes {15}, can be developed in terms of  $E^{(0)}$  and/or  $B^{(0)}$  with equal validity as the conventional interpretation in terms of  $E^{(1)}$ ,  $E^{(2)}$ ,  $B^{(1)}$  and  $B^{(2)}$ , the oscillating, transverse fields.

We can therefore conclude that the Einstein / de Broglie theory, which has been shown to be experimentally verifiable by recent work {7} produces physically meaningful longitudinal magnetic and electric fields which are linked to the transverse fields by eqn. (1), and which are expected to produce new effects {9-11,14} as summarised in this Section. It is interesting to note that there is a growing literature {23} on new, physically meaningful, solutions of Maxwell's equations in vacuo. Risset, for example, has shown {24} that there is a class of non-diffracting solutions which emerge as a superposition of circularly polarized evanescent waves, called "lip waves". They appear as right and left circularly polarized, non-diffracting, TEM waves, and two alternative interpretations have been proposed by Risset {24}, namely that in these solutions, the electromagnetic field progresses, as a whole,

along its axis with a phase velocity  $c$ , or that the field spins, without apparent propagation,

around this axis, with angular velocity  $\epsilon \frac{\omega}{2}$ , where  $\epsilon = \pm 1$ , and  $\omega$  is the angular

frequency. Applying eqn. (1) to these solutions, it becomes clear that lip waves also imply a longitudinal photon polarisation, given in the notation of Risset {24} by

$$B^{(3)} = \frac{B \times B^*}{B_0^i} = 2B_0 \delta^4 \frac{(x^2 - y^2 + 4xy^2)^2}{(x^2 + y^2)^4} k \quad (21)$$

where  $\delta$  is a normalization constant for length {24}. A schematic of the lip waves is given by Risset on page 1057 of ref. {24}, to which should be added the longitudinal fields generated as above by his novel solutions of Maxwell's equations.

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