LOW ENERGY NUCLEAR REACTION: QUANTUM TUNNELLING AND SPACETIME ABSORPTION.

by

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ABSTRACT

The conditions under which low energy nuclear reaction occurs can be optimized by a straightforward application of the Schroedinger equation with a realistic model of the internuclear potential. Starting from the ECE wave equation, the effect of spacetime absorption can be considered. The conditions for low energy reaction are defined as total energy E of the incoming atom much less than the potential energy V of interaction. Quantum tunnelling is optimized when the transmission coefficient T is maximized. For a Coulomb barrier it is demonstrated that T is maximized for E << V when the mass of the incoming atom is maximized. A more realistic potential is considered, made up of a combination of Coulomb repulsion force between nuclear protons of two different atoms, and a strong nuclear attraction force.

Keywords: ECE wave equation, optimal condition for low energy nuclear reaction.

UFT 229

1 INTRODUCTION

In recent papers of this series of 229 papers to date {1 - 10} the theory of low energy nuclear reaction (LENR) has been considered in detail. In this paper the optimal conditions for LENR are defined using a straightforward procedure based on the Schroedinger equation with a realistic model potential. The optimal conditions for LENR are defined by maximum T for $E \ll V$, where T is the transmission coefficient of quantum tunnelling, E is the total energy and V the potential energy of the Schroedinger equation, the non relativistic quantum limit of the ECE wave equation {1 - 10}. In addition, the effect of wave absorption is considered on the LENR process. In some working devices {11} a phonon wave is applied to the reaction. In ECE theory this is a wave of spacetime within a proportionality factor, and ECE theory also considers the absorption of momentum. It has been shown in previous papers of this series (notably UFT 158 ff. on www.aias.us) that conventional Compton scattering, absorption and Raman scattering theory collapses without correct consideration of momentum transfer.

In Section 2 a realistic model potential is defined which consists of Coulombic repulsion {12} between the protons of the nuclei of two interacting atoms 1 and 2, and strong nuclear attraction {13, 14} with the Woods Saxon mean field model. As atom 1 approaches atom 2 it first meets the Coulomb barrier. It is shown in Section 3, using computer algebra, that it can quantum tunnel effectively through this barrier when $E \ll V$. The coefficient T is maximized when the mass of the incoming atom is maximized. The Coulomb repulsion inside the fused entity defined in Section 2 is modelled as in conventional theory of nuclear fusion as the Coulombic repulsion inside a sphere. In the fused entity there is also a strong nuclear attraction between nucleons, both protons and neutrons. This is modelled with the well known Woods Saxon mean field potential. The complete potential is the sum of the

Coulombic repulsion between protons and the strong attraction between protons and neutrons. As atom 1 approaches atom 2, the complete potential goes through a positive maximum before decreasing to a negative minimum in the fused entity made up of atoms 1 and 2 combined. The entity decomposes almost immediately to give the products of the fusion reaction.

In order for low energy nuclear reaction to occur at low total energy E, the fused entity must be formed by quantum tunnelling of atom 1 into atom 2. The well known WKB approximation {12} used in the previous paper UFT228 of this series (www.aias.us) is extended to the complete potential and the transmission coefficient of quantum tunnelling evaluated numerically in Section 3. The effect of absorption is developed in Section 2 from the ECE wave equation {1 - 10}, which is a well known and generally covariant generalization to unified field theory of the Schroedinger equation.

2. ABSORPTION AND QUANTUM TUNNELLING IN LENR.

Consider the ECE wave equation:

$$(\square + R)\, q^a_\mu = 0 \quad -(1)$$

which can be expanded as:

$$(\square + R)\begin{bmatrix} \phi^R_1 & \phi^R_2 \\ \phi^L_1 & \phi^L_2 \end{bmatrix} = 0. \quad -(2)$$

Here q^a_μ is the Cartan tetrad, and R is defined {1 - 10} by geometry. Eq. (2) can be reduced to the Klein Gordon equation:

$$\left(\square + \left(\frac{mc}{\hbar}\right)^2\right)\phi = 0 \quad -(3)$$

where m is the particle mass, c the speed of light and $\hbar$ the reduced Planck constant. Eq.

(3) can be deduced from the Einstein energy equation:

$$E^2 = p^2c^2 + m^2c^4 \quad -(4)$$

using the Schroedinger postulates:

$$E = i\hbar\frac{\partial}{\partial t}, \quad \underline{p} = -i\hbar\underline{\nabla}, \quad -(5)$$

so it follows that:

$$\left(\frac{mc}{\hbar}\right)^2 = \left(\frac{\omega}{c}\right)^2 - \kappa^2 \quad -(6)$$

and

$$\left(\Box + \frac{\omega^2}{c^2} - \kappa^2\right)\phi = 0. \quad -(7)$$

Eq. (7) can also be written as:

$$\left(E^2 - c^2p^2\right)\phi = m^2c^4\phi = \hbar^2\left(\omega^2 - \kappa^2c^2\right)\phi \quad -(8)$$

which is an example of wave particle dualism. It can be linearized using:

$$E^2 - m^2c^4 = \left(E - mc^2\right)\left(E + mc^2\right) - (9)$$

so:

$$\left(E - mc^2\right)\phi = \left(\frac{c^2p^2}{E + mc^2}\right)\phi. \quad -(10)$$

In the non relativistic approximation:

$$E \rightarrow mc^2 \quad -(11)$$

so:

$$\left(E - mc^2\right)\phi = \frac{p^2}{2m}\phi \quad -(12)$$

Eq. (12) is the free particle Schroedinger equation, usually written as:

$$\frac{p^2}{2m}\phi = -\frac{\hbar^2}{2m}\nabla^2\phi = E\phi \quad -(13)$$

i.e. the total energy, written E in the Schroedinger equation, is the total relativistic energy of the free particle minus its rest energy:

$$E = T = (\gamma - 1)mc^2 \quad -(14)$$

where T is the relativistic kinetic energy of the free particle. The free particle Schroedinger equation may be written therefore as:

$$\nabla^2\phi = -\frac{2mE}{\hbar^2}\phi \quad -(15)$$

where the mass is:

$$m = \frac{\hbar}{c^2}\left(\omega^2 - \kappa^2 c^2\right)^{1/2} \quad -(16)$$

If the de Broglie Einstein postulates are assumed then:

$$E = \hbar\omega = \gamma mc^2 \quad -(17)$$

$$\underline{p} = \hbar\underline{\kappa} = \gamma m\underline{v} \quad -(18)$$

where:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(19)$$

is the Lorentz factor. Here $\underline{v}$ is the velocity of the free particle, ω is its angular frequency and $\underline{\kappa}$ its wavenumber.

The process of absorption of spacetime energy and momentum can be described as a change of mass:

$$m \rightarrow m + m_1 = \frac{\hbar}{c^2}\left[\left(\omega^2 - \kappa^2 c^2\right)^{1/2} + \left(\omega_1^2 - \kappa_1^2 c^2\right)^{1/2}\right] \quad -(20)$$

so the effective mass of the particle increases. The same conclusion holds for the reduced mass of two interacting masses, m_1 and m_2 :

$$\mu = \frac{m_1 m_2}{m_1 + m_2} . \quad -(21)$$

In the presence of a potential energy V the free particle Schroedinger equation becomes:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\phi = E\phi \quad -(22)$$

i.e. :

$$\nabla^2\phi = -\frac{2m}{\hbar^2}\left(E - V\right)\phi . \quad -(23)$$

In the well known {12} WKB approximation the transmission coefficient for quantum tunnelling from Eq. (23) is {12}:

$$T = \frac{4}{\left(2\theta + \frac{1}{2\theta}\right)^2} \quad -(24)$$

where:

$$\theta = \exp\left(\left(\frac{2\mu}{\hbar}\right)^{1/2}\int_a^b \left(V(r) - E\right)^{1/2} dr\right) \quad -(25)$$

where the integral is evaluated between two points on the potential. In general, this quantum tunnelling process may be accompanied by quantum absorption described as follows:

$$\mu \rightarrow \mu + \frac{\hbar}{c^2}\left(\omega_1^2 - \kappa_1^2 c^2\right)^{1/2} \quad -(26)$$

In conventional nuclear fusion theory the potential in one dimension is:

$$V = -\frac{V_0}{1+\exp\left(\frac{r-R}{a_N}\right)} + V_c \quad -(27)$$

which is the sum of Coulombic repulsion and strong nuclear attraction. This potential is sketched in Fig. 1

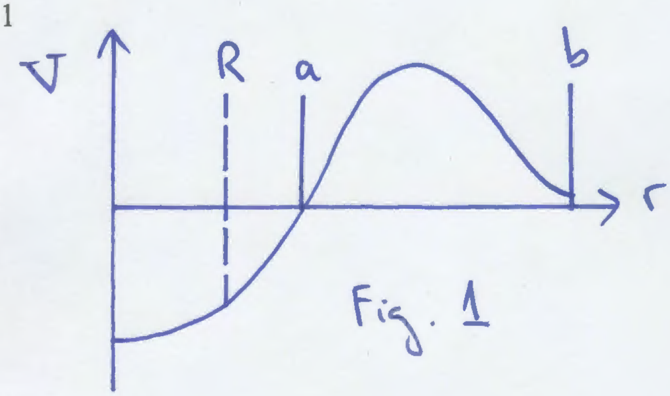

Fig. 1

The Coulombic repulsion in conventional fusion theory is:

$$V_c = Z_1 Z_2 e^2 / r, \quad r > R, \quad -(28)$$

$$V_c = \frac{Z_1 Z_2 e^2}{R}\left(3 - \left(\frac{r}{R}\right)^2\right), \quad r < R, \quad -(29)$$

where there are Z_1 protons in atom 1 and Z_2 protons in atom 2. The region r less than R defines the interior of the fused entity of radius R. Therefore the interior is modelled as a charged sphere of radius R. Outside the fused entity in the region defined by r greater than R the usual Coulomb law is used conventionally. On the ECE level {1 - 10} the Coulomb law is changed.

The potential due to the strong nuclear force between protons and neutrons is modelled conventionally in the mean field approximation by the Woods Saxon potential:

$$V = -\frac{V_0}{1+\exp\left(\frac{r-R}{a_N}\right)}. \quad -(30)$$

The minus sign means that the strong force is a force of attraction between both protons and neutrons, i.e. between all nucleons, either in the separate nuclei 1 or 2, or in the fused nucleus. Here V_0 is the well depth, and a_N is the surface thickness of the nucleus.

Therefore the transmission coefficient is worked out in general using the combined potential (27). In the region where the repulsive part dominates the potential reduces to:

$$V \rightarrow Z_1 Z_2 e^2 / r \quad -(31)$$

and as shown in Section 3 the transmission coefficient in this case can be expressed as:

$$T = \frac{16y}{16y^2 + 8y + 1} \quad -(32)$$

where:

$$y = \exp\left(\frac{(2mE)^{1/2}}{\hbar}\pi a\right). \quad -(33)$$

Maximum transmission occurs at exactly:

$$y = \frac{1}{4}, \quad -(34)$$

and at this value:

$$mE = \frac{(\log_e 4)^2 \hbar^2}{2\pi^2 a^2}. \quad -(35)$$

Since a varies very slowly with Z, then:

$$E \propto \frac{1}{m} \quad -(36)$$

for optimal transmission, the heavier the element the less the required energy.

3. RESULTS AND DISCUSSION

Section by Dr. Horst Eckardt and Dr. Douglas Lindstrom

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Low Energy Nuclear Reaction: Quantum Tunneling and Spacetime Absorption

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3 Results and discussion

In this section we compute examples for the transmission coefficient. In WKB approximation this was given by Eqs.(24) and (25). By variable sunbstitution of

$$\theta^2 = y, \tag{37}$$

Eq.(24) can be written as

$$T = \frac{16y}{16y^2 + 8y + 1}. \tag{38}$$

This function is graphed in Fig.2. By setting

$$\frac{dT}{dy} = 0 \tag{39}$$

it is found that the maximum is at $y = 1/4$. This is a general result and holds for all forms of y respectively θ defined by Eq.(25). The wave vector κ is defined in general by

$$\kappa = \frac{\sqrt{2\mu}}{\hbar}\sqrt{V(r) - E}. \tag{40}$$

3.1 Rectangular potential barrier

In case of a free particle we have

$$V(r) = V_0 = const. \tag{41}$$

and the energy dependence of κ is of the form

$$\kappa = \frac{\sqrt{2\mu}}{\hbar}\sqrt{V_0 - E}, \tag{42}$$

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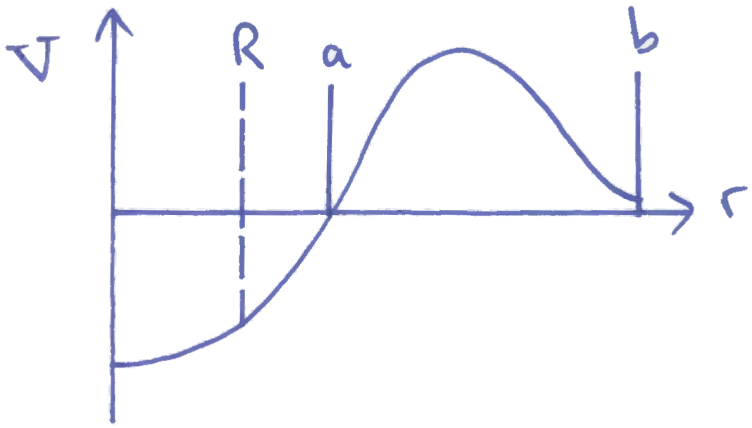

Figure 1: Schematic represantation of potential well consisting of Woods Saxon and Coulomb potential.

leading to a θ value of

$$\theta = \exp\left(\frac{\sqrt{2\mu(V_0 - E)}}{\hbar}(b-a)\right) \tag{43}$$

according to Eq.(25), where μ represents the reduced mass and the potential well reaches from position a to b. The corresponding transmission coefficient is graphed in Fig.3 for three values of μ. It can be seen that transmission is at maximum when E reaches the potential well V_0 where classical transmission sets on. Then we have $\theta = 1$ from Eq.(25) and the transmission coefficient takes the value

$$T_{max} = \frac{4}{(2+1/2)^2} = 0.64. \tag{44}$$

Consequently all curves in Fig.4 are in a range below this value.

3.2 Coulomb potential

In the case of a Coulombic barrier we have according to Eq.(28):

$$V_C = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0\ r}. \tag{45}$$

At the point $r = b$ it is

$$E = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0\ b}. \tag{46}$$

Inserting this into κ leads to the simplified expression

$$\kappa = \frac{\sqrt{2\mu E}}{\hbar}\sqrt{\frac{b}{r} - 1} \tag{47}$$

and from (25) we obtain

$$\theta = \exp\left(\frac{\sqrt{2\mu E}}{\hbar}\int_0^b \sqrt{\frac{b}{r}-1}\;dr\right). \tag{48}$$

The integral evaluates to a simple value:

$$\int_0^b \sqrt{\frac{b}{r}-1}\;dr = \frac{\pi b}{2}, \tag{49}$$

hence:

$$\kappa = \frac{\sqrt{2\mu E}}{\hbar}\frac{\pi b}{2}, \tag{50}$$

$$\theta = \exp\left(\frac{\sqrt{2\mu E}}{\hbar}\frac{\pi b}{2}\right). \tag{51}$$

Because of (46), b depends on E. The full E dependence of θ is therefore

$$\theta = \exp\left(\frac{Z_1\,Z_2\,e^2}{2^{\frac{5}{2}}\,\epsilon_0\,\hbar}\sqrt{\frac{\mu}{E}}\right). \tag{52}$$

The transmission coefficient $T(E)$ resulting from this function is shwon in Fig.4 for three mass values (other constants set to unity). It can be seen that transmission raises to maximum with increasing energy, but with inverse slope as for the constant potential well (Fig.3).

From the general result that y of Eq.(38) is at maximum for $y = 1/4$ we obtain with (52):

$$\theta^2_{\max} = 1/2 \tag{53}$$

or by taking the logarithm this equation:

$$\frac{\mu}{E} = \frac{32\log{(2)}^2\,\epsilon_0^2\,\hbar^2}{Z_1^2\,Z_2^2\,e^4}. \tag{54}$$

For T to become maximal, μ and E must stay in a fixed ratio. For higher ordinal numbers of the nuclei, the constant becomes smaller. Since μ gets larger on the left hand side in this case, this means that this has to be compensated by enlarging the energy. In other words, heavier masses require higher energies for the optimum transmission coefficient. From (54) the optimal energy for a combination of two masses m_1 and m_2 is given by

$$E_{\mathrm{opt}} = \frac{Z_1^2\,Z_2^2\,e^4\;\mu}{32\log{(2)}^2\,\epsilon_0^2\,\hbar^2} \tag{55}$$

with

$$\mu = \frac{m_1 m_2}{m_1+m_2}. \tag{56}$$

$V_0 = 93.89\ MeV$	depth of potential function
$r_a = 1.18 \cdot 10^{-15}\ m$	single charge radius in target nucleus
$R = r_a \cdot (A_p^{1/3} + A_t^{1/3})$	Radius of central potential function
$a_r = .454 \cdot 10^{-15}\ m$	diffuseness of the potential
$A_t = 15.9994$	atomic mass of oxygen, the target
$A_p = 12.011$	atomic mass of carbon, the projectile
$V_{WS} = -V_0/(1 + \exp[(r-R)/a_r])$	Woods-Saxon Potential
$k = 8.99 \cdot 10^9$	$1/(4\pi\epsilon)$
$Z_t = 8$	number of protons in target
$Z_p = 6$	number of protons in projectile

Table 1: Data of Woods Saxon potential from Hamada [1].

3.3 Woods Saxon and Coulomb potential

Realistic potential wells can be modeled by assuming a Woods Saxon potential inside the nucleus and a Coulomb potential everywhere. The latter has to be chosen differently inside and outside of the nucleus, see Eqs.(27-30). Inserting this combined potential into Eq.(25) gives no analytical solution for the integral, therefore κ must be determined by numerical integration. For the upper integration limit, the results of the preceding section remain valid, i.e. the integration limit b is dependent on E according to Eq.(46). The same holds for the lower integration limit a. Both can be found by solving

$$V_{WS}(r) + V_C(r) - E = 0 \tag{57}$$

with Woods Saxon potential (30) and Coulomb potential (29). This equation cannot be solved analytically, it has to be solvede numerically. The region of the positive potential is graphed in Fig.5 as a 3D plot where the relative axes are defined by

$$\eta = \frac{r}{R}, \tag{58}$$

$$\lambda = \frac{E}{V_0}. \tag{59}$$

The parameters of the Woods Saxon potential for the Carbon-Oxygen system are listed in Table 1. V_0 is the potential depth at the nuclear centre.

Fig.6 shows the transmission coefficient calculated for the Carbon-Oxygen system. It is sufficiently high to explain nuclear processes when E is in the range of some percents of V_0.

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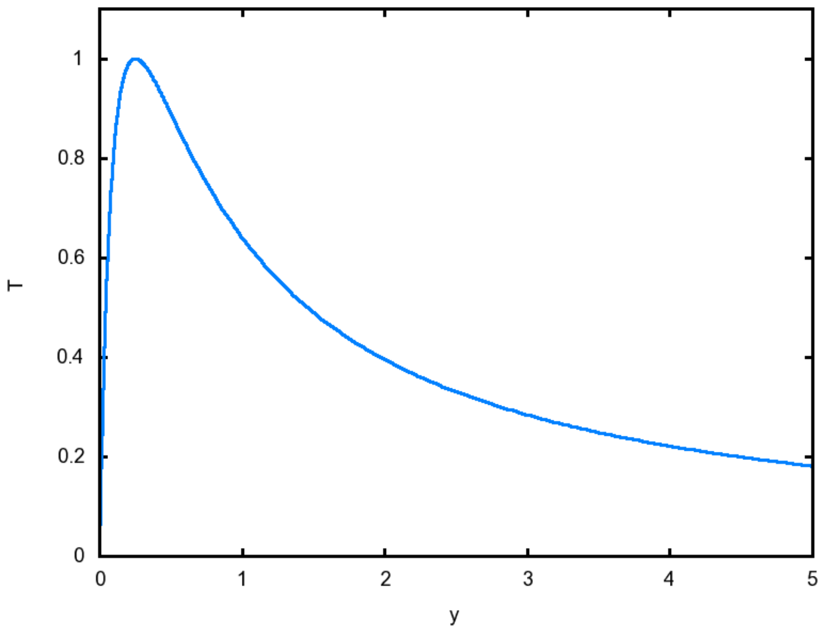

Figure 2: Transmission coefficient $T(y)$.

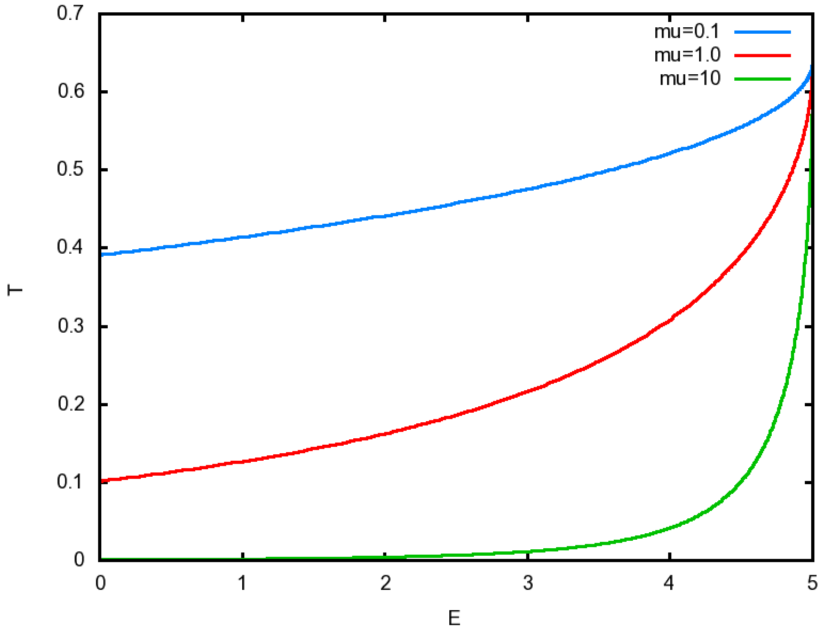

Figure 3: Transmission coefficient $T(E)$ for three values of μ for a constant potential barrier $V_0 = 5$.

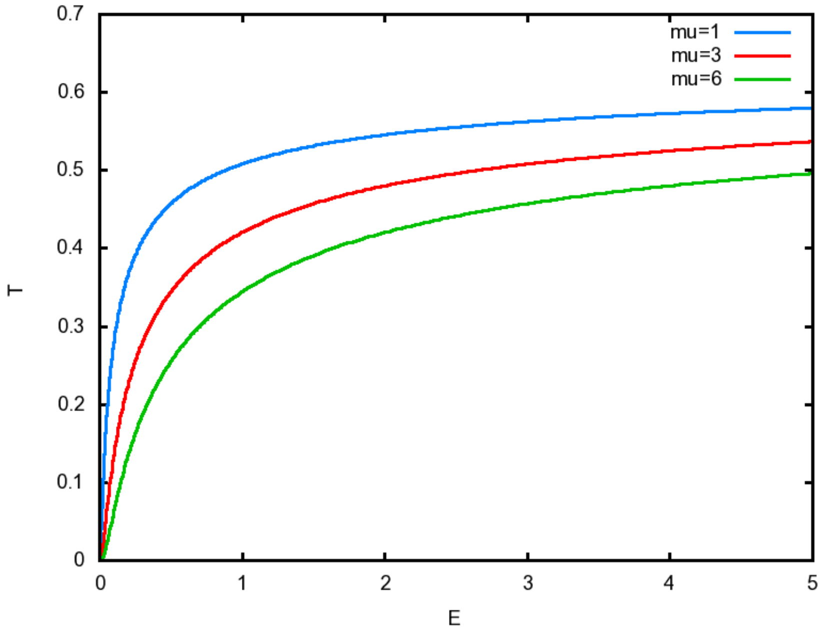

Figure 4: Transmission coefficient $T(E)$ for three values of μ for a Coulomb barrier.

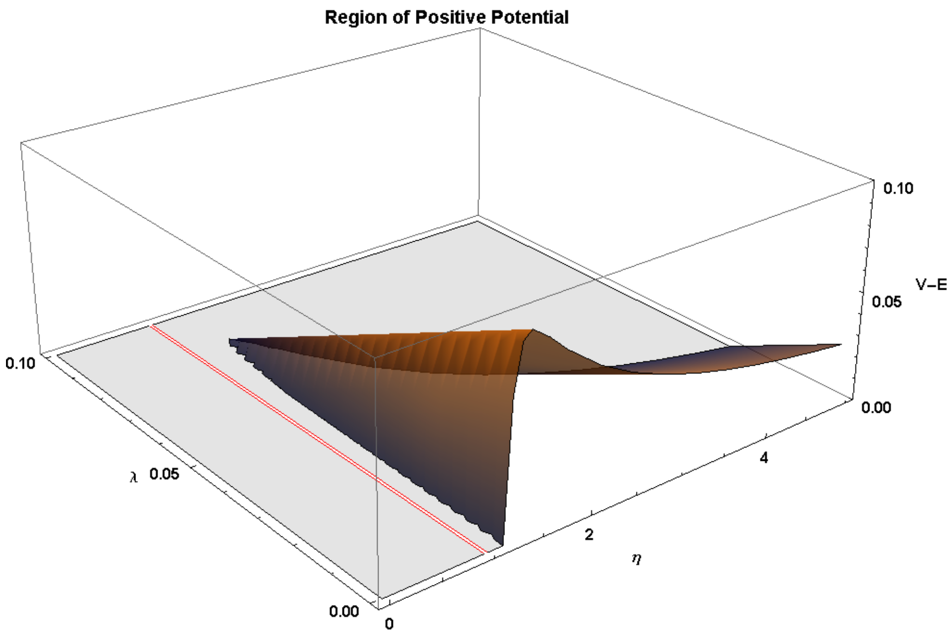

Figure 5: Transmission coefficient $T(E)$ for three values of μ for a constant potential barrier $V_0 = 5$.

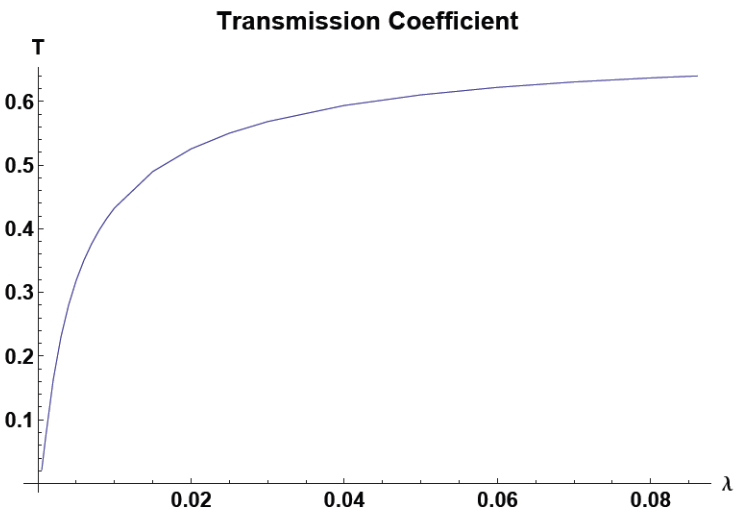

Figure 6: Transmission coefficient $T(E)$ for three values of μ for a Coulomb barrier.

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