A NEW WAVE EQUATION FOR LAGRANGIAN DYNAMICS: THE PLANAR ORBIT FOR ANY FORCE LAW.

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ABSTRACT

A new wave equation is developed by Lagrangian methods for applications in celestial mechanics as part of ECE theory. A method is inferred for the calculation of a planar orbit for any force between a mass m orbiting a mass M in a plane. Two methods of solution of the wave equation are given and the self consistency of the method checked in the Newtonian limit. It is shown that the Einsteinian general relativity (EGR) is one out of an infinite number of force laws that give orbital precession.

Keywords: ECE theory, celestial mechanics, Lagrangian wave equation, planar orbit for any force law.

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1. INTRODUCTION

As part of this series of two hundred and forty two papers to date {1 - 10}, Einsteinian general relativity (EGR) has been refuted in comprehensive detail and in many ways. In order to seek a replacement theory applicable for all observable orbits, a number of methods have been proposed in recent papers of this series. It has been found that orbital precession is given by an infinite number of force laws, not just the force law of Einsteinian general relativity (EGR). For example the Minkowski force gives a precession, and this process has been animated by B. Foltz on www.aias.us. Any force law proportional to r to the power n gives a precession, with the sole exception of n = 2, and this precession has again been animated on www.aias.us by B. Foltz. Essay 80 on www.aias.us lists fifty seven of the available refutations of EGR, so it is a completely refuted theory, and the preceding papers of this series severely criticise the dogmatic claims of EGR to be a precise theory.

In Section 2 a wave equation is inferred from a combination of the two classical Euler Lagrange equations of an object of mass m orbiting an object of mass M in a plane. These classical equations are used in EGR, despite its claim to be a relativistic theory. So EGR is not only incorrect in many ways, it is conceptually self inconsistent. This wave equation can be extended to the Minkowski spacetime straightforwardly, and that will be the subject of future work, but in this paper the classical limit is examined. Two solutions are given of the wave equation, one derived by computer algebra by H. Eckardt. This second solution is tested for self consistency and correctness in the Newtonian limit and applied with some simple force laws to find the orbit. As far as the authors are aware, this is the first time that a method has been devised in celestial mechanics to find the orbit for any force law. The orbit for the Hooke Newton inverse square law is an ellipse, as is well known. The precessing ellipse is given by an infinite number of other types of force law. Using this method the claims of EGR can be tested in another way.

Section 3 is a description of the numerical methods used by H. Eckardt: computer algebra used to check the hand calculations and used to solve the new wave equation.

2. WAVE EQUATION AND PLANAR ORBITAL SOLUTIONS FOR ANY FORCE.

Consider the orbit of a mass m around a mass M in a plane. The two equations of motion from a well known Lagrangian analysis {11} are:

$$m\left(\ddot{r} - r\dot{\theta}^2\right) = F(r) \quad -(1)$$

and

$$L_0 = mr^2\dot{\theta}. \quad -(2)$$

Here F is the force between m and M and L_0 is the conserved total angular momentum. The plane polar coordinates are r and θ . From Eq. (2) in Eq. (1):

$$m\left(\ddot{r} - \frac{rL_0^2}{m^2r^4}\right) = F(r) \quad -(3)$$

so:

$$\ddot{r} - \frac{L_0^2}{m^2r^3} = \frac{F(r)}{m} \quad -(4)$$

and

$$\ddot{r} - \omega^2 r = \frac{F(r)}{m}. \quad -(5)$$

This is a wave equation with structure:

$$\frac{d^2r}{dt^2} + \Omega^2 r = 0 \quad -(6)$$

where

$$\Omega^2 = -\frac{L_0^2}{m^2 r^4} - \frac{F(r)}{mr} . \quad -(7)$$

Under a well defined constraint to be discussed below, this equation is a harmonic oscillator with period:

$$T = \frac{2\pi}{\Omega} \quad -(8)$$

and apsidal angle:

$$\psi = \frac{1}{2} T \frac{d\theta}{dt} = \frac{1}{2} \omega T \quad -(9)$$

where the angular velocity is defined by Eq. (2):

$$\omega = \frac{d\theta}{dt} = \frac{L_0}{mr^2} . \quad -(10)$$

Therefore the angular coordinate is:

$$\theta = 2\psi \quad -(11)$$

and can be found for any force.

Eq. (6) has the solution:

$$r = r_0 \exp(i\Omega t) \quad -(12)$$

with:

$$T = 2\pi \left(-\frac{L_0^2}{m^2 r^4} - \frac{F(r)}{mr} \right)^{-1/2} \quad -(13)$$

provided that:

$$\Omega^2 = -\left(\frac{dx}{dt} + x^2 \right) \quad -(14)$$

where:

$$x = i\left(\Omega + t\,\frac{d\Omega}{dt}\right) \quad - (15)$$

In order to prove this, note that Ω is a function of time, so:

$$\frac{d}{dt}\, e^{i\Omega t} = x\, e^{i\Omega t} \quad - (16)$$

and

$$\frac{d^2}{dt^2}\, e^{i\Omega t} = \left(\frac{dx}{dt} + x^2\right) e^{i\Omega t} \quad - (17)$$

i.e.

$$\frac{d^2 r}{dt^2} - \left(\frac{dx}{dt} + x^2\right) r = 0 \quad - (18)$$

so:

$$\Omega^2 = -\left(\frac{dx}{dt} + x^2\right) \quad - (19)$$

Q.E.D.

Eq. (19) is a second order differential equation for the time dependence of Ω and is a subsidiary or constraint equation that must be obeyed if the solution (12) be true. Eq. (19) can always be solved, analytically or numerically, so Eq. (12) is always true.

The second method of solution of Eq. (6) is due to computer algebra by H. Eckardt, giving the solution:

$$t = \frac{1}{\sqrt{2}} \int \left(-\int r\,\Omega^2\, dr - A\right)^{-1/2} dr + B \quad - (20)$$

where A and B are constants of integration which have to be found by further analysis. For any curve {11}:

$$dA_r = \frac{1}{2} r^2 \, d\theta \quad - (21)$$

where A_r is an area. Assume that in time T an area A_r is swept out. Then in time t an area $A_r t / T$ is swept out. Therefore:

$$A_r t / T = \int dA_r = \frac{1}{2} \int r^2 \, d\theta \quad - (22)$$

and in general:

$$t = \frac{T}{2A_r} \int r^2 \, d\theta . \quad - (23)$$

From Eqs. (20) and (23):

$$\frac{1}{\sqrt{2}} \int \left(- \int r \Omega^2 \, dr - A \right)^{-1/2} dr + B = \frac{T}{2A_r} \int r^2 \, d\theta . \quad - (24)$$

for any curve and any force law.

Define the function:

$$f(r) := \left(- \int \Omega^2 r \, dr - A \right)^{-1/2} \quad - (25)$$

and for simplicity assume that B is zero. Then:

$$t = \frac{1}{\sqrt{2}} \int f(r) \, dr = \frac{1}{2} \frac{T}{A_r} \int r^2 \, d\theta \quad - (26)$$

and:

$$\frac{dt}{dr} = \frac{1}{\sqrt{2}} f(r) , \quad \frac{dt}{d\theta} = \frac{1}{2} \frac{T}{A_r} r^2 . \quad - (27)$$

Using the chain rule:

$$\frac{d\theta}{dr} = \frac{d\theta}{dt} \frac{dt}{dr} = \sqrt{2} \, \frac{f(r)}{r^2} \frac{A_r}{T} \quad - (28)$$

and the angular polar coordinate is:

$$\theta = \sqrt{2}\,\frac{A_r}{T}\int \frac{f(r)}{r^2}\,dr \quad -(29)$$

and can be found for any force law, Q. E. D. For any curve in the classical Lagrangian dynamics {11} of any planar orbit:

$$dt = \frac{2m}{L_0}\,dA_r \quad -(30)$$

Integrating:

$$\int_0^T dt = \frac{2m}{L_0}\int_0^{A_r} dA_r \quad -(31)$$

so:

$$\frac{A_r}{T} = \frac{L_0}{2m} \quad -(32)$$

for all planar orbits.

In the Newtonian limit {1 - 11} the force law is:

$$F(r) = -\frac{L_0^2}{\alpha m r^2} \quad -(33)$$

where α is the half right magnitude of the elliptical orbit:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(34)$$

where ϵ is the eccentricity. It was checked by computer algebra as follows that the result (29) gives the ellipse (34) for the Newtonian force law (33). This is a rigorous test of the self consistency of the theory. The solution of Eq. (29) with the force law (33) is, by computer algebra:

$$\theta = \gamma - \frac{2mA_r}{L_0 T}\sin^{-1}\left(\frac{\frac{\alpha}{r} - 1}{\left(1-2\alpha^2 A m^2\right)^{1/2}}\right) \quad -(35)$$

where y is a constant of integration. If:

$$A = -\frac{L_0^2}{2\alpha^2 m^2}(\epsilon - 1)(\epsilon + 1) \quad -(36)$$

and

$$y = \pi \quad -(37)$$

then:

$$\theta = \frac{m A_r}{L_0 T}\left(\pi - 2\sin^{-1}\left(\frac{\frac{\alpha}{r} - 1}{((\epsilon-1)(\epsilon+1)) + 1)^{1/2}}\right)\right) \quad -(38)$$

i. e.

$$r = \alpha\left(1 + \epsilon\cos\left(\frac{L_0 T}{2 m A_r}\theta\right)\right)^{-1} \quad -(39)$$

Using Eq. (32), Eq. (39) simplifies to:

$$r = \frac{\alpha}{1 + \epsilon\cos\theta} \quad -(40)$$

which is Eq. (34), Q. E. D.

So Eq. (29) is rigorously correct for the Newtonian force. Having tested it in this way it may be applied to other forces and this is done in Section 3.

Eq. (36) can be simplified by noting that:

$$a = \frac{\alpha}{1 - \epsilon^2} = \frac{\alpha}{(1-\epsilon)(1+\epsilon)} \quad -(41)$$

where a is the semi major axis of the ellipse {11}. So:

$$A = \frac{L_0^2}{2a\alpha m^2} . \quad -(42)$$

In the Newtonian theory {11}:

$$L_0^2 = m^2 M G \alpha \quad -(43)$$

so the constant of integration x is:

$$A = \frac{MG}{2a} \quad -(44)$$

which has the correct units of the square of linear velocity.

The precessing conical section:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(45)$$

is found from the force law {1 - 11}

$$F(r) = -\frac{mMGx^2}{r^2} - \alpha(1 - x^2)\frac{mMG}{r^3} \quad -(46)$$

and computer algebra is used in Section 3 to check this result, giving a precise result for the precession constant x, a new result and an advance over previous work. EGR claims that the force law is:

$$F(r) = -\frac{mMG}{r^2} - \frac{3L_0^2 MG}{m^2 c^2 r^4} . \quad -(47)$$

This claim can be tested by using Eq. (47) in Eq. (29) to find the precession of the polar angle due to EGR, which claims that for a revolution of 2π:

$$\theta \rightarrow 2\pi(1 + x_1) \quad -(48)$$

where:

$$x_1 = \frac{3GM}{ac^2(1-\epsilon^2)} \quad -(49)$$

This claim of EGR is based on the same Lagrangian analysis that leads to Eq. (6), so Eq. (29) must produce the result (49) if EGR is correct. It is known from Essay 80 on www.aias.us that EGR is incorrect in many ways.

Finally in this Section the Minkowski force has been shown in immediately preceding papers to be:

$$\underline{F} = \left(\gamma^4 m \frac{d^2 r}{dt^2} - \frac{\gamma^2 L_0^2}{m r^3}\right)\underline{e}_r + \frac{L_0 \gamma^4}{mc^2 r}\frac{dr}{dt}\frac{d^2 r}{dt^2}\underline{e}_\theta \quad -(50)$$

where $\underline{e}_r$ and $\underline{e}_\theta$ are the unit vectors of the plane polar coordinate system. The Minkowski force is the Newtonian force corrected for the Lorentz factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(51)$$

where the velocity appearing in the Lorentz factor is given by the Minkowski metric as:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + \left(\frac{L_0}{mr}\right)^2. \quad -(52)$$

From Eq. (20):

$$\frac{dt}{dr} = \frac{1}{\sqrt{2}} f(r) \quad -(53)$$

so:

$$\frac{dr}{dt} = \frac{\sqrt{2}}{f(r)}. \quad -(54)$$

Using the chain rule:

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \frac{L_0}{mr^2}\frac{dr}{d\theta} \quad -(55)$$

so Eq. (55) can be used to find the orbit in the form:

$$\frac{dr}{d\theta} = \frac{mr^2}{L_0}\frac{dr}{dt} = \sqrt{2}\,\frac{m}{L_0}\frac{r^2}{f(r)} \quad -(56)$$

for any force law. Using Eq. (54) in Eq. (29) the true anomaly (or orbital polar angle) can be expressed as:

$$\theta = \frac{\sqrt{2}L_0}{2m}\int \frac{f(r)\,dr}{r^2} \quad -(57)$$

where:

$$f(r) = \left(-\int r\Omega^2\,dr - A\right)^{-1/2} \quad -(58)$$

So the true anomaly becomes:

$$\theta = \frac{L_0}{m}\int \frac{dt}{r^2} \quad -(59)$$

implying that:

$$\frac{d\theta}{dt} = \frac{L_0}{mr^2} \quad -(60)$$

which is Eq. (2) Q. E. D. So the theory of this section is rigorously correct and self consistent and is a new and powerful method of celestial mechanics.

3. COMPUTATION OF ORBITS FOR SELECTED FORCE LAWS.

Section by H. Eckardt.

A new wave equation for Lagrangian dynamics: the planar orbit for any force law

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3 Computation of orbits for selected force laws

Before considering the orbits emerging from the force laws we calculate the apsidal angles for some cases. The apsidal angle is defined by Eq.(9) and can be written with aid of (7), (8) and (60):

$$\psi = \frac{\pi L_0}{m r^2 \sqrt{-\frac{{L_0}^2}{m^2 r^4} - \frac{F(r)}{m r}}}. \tag{61}$$

The results are presented in Table1. In most cases the apsidal angle depends on the radius r. To find a reasonable value for r we calculate r for the case $\psi = \pi$. The apsidal angle is constant only for the $1/r^3$ force law which gives a precessing ellipse of canonical form. For higher exponents r varies as well as for an exponential and even a constant force law. The same holds for a radially oscillating force.

In the following we apply Eq.(29) to some force laws to obtain the orbital function $\theta(r)$ in the most general way. With the general result (32) this fuction can be written

$$\theta = \sqrt{2}\frac{L_0}{2m}\int \frac{f(r)}{r^2} dr + B \tag{62}$$

with

$$f(r) = \left(-\int \Omega^2 r dr - A\right)^{-1/2} \tag{63}$$

where Ω^2 is given by Eq.(7) and A and B are constants of integration. Inserting the force law of a precessing ellipse,

$$F(r) = -\frac{m M G x_1^2}{r^2} - \frac{\alpha \left(1 - x_1^2\right) m M G}{r^3}, \tag{64}$$

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Force law	Apsidal angle	Radius for $\psi = \pi$
$F(r) = -\frac{m\,G\,M}{r^2}$		
$= -\frac{L_0^2}{\alpha m r^2}$	$\psi = \frac{\pi}{\sqrt{\frac{r}{\alpha}-1}}$	$r = 2\,\alpha$
$F(r) = -\frac{\beta}{r^3}$	$\psi = \frac{\pi\,L_0}{\sqrt{\beta\,m - {L_0}^2}}$	any
$F(r) = -\frac{\beta}{r^4}$	$\psi = \frac{\pi\,\sqrt{r}\,L_0}{\sqrt{\beta\,m - r\,{L_0}^2}}$	$r = \frac{\beta\,m}{2\,{L_0}^2}$
$F(r) = -F_0\,e^{-\beta\,r}$	$\psi = \frac{\pi\,e^{\frac{\beta\,r}{2}}\,L_0}{\sqrt{m\,r^3\,F_0 - e^{\beta\,r}\,{L_0}^2}}$	$r^3\,e^{-\beta\,r} = \frac{2\,{L_0}^2}{m\,F_0}$
$F(r) = -F_0$	$\psi = \frac{\pi\,L_0}{\sqrt{m\,r^3\,F_0 - {L_0}^2}}$	$r = \frac{2^{\frac{1}{3}}\,{L_0}^{\frac{2}{3}}}{m^{\frac{1}{3}}\,{F_0}^{\frac{1}{3}}}$
$F(r) = F_0 \sin(\kappa\,r)$	$\psi = \frac{\pi\,L_0}{\sqrt{m\,r^3\,\sin(\kappa\,r)\,F_0 - {L_0}^2}}$	$r^3 = \frac{2\,{L_0}^2}{F_0\,m\,\sin(\kappa\,r)}$

Table 1: Apsidal angles ψ and radii for $\psi = \pi$ for several force laws.

leads (by computer algebra) to the orbit relation

$$r = \frac{\alpha}{-\epsilon \sin(x_1(\theta + B)) + 1} \tag{65}$$

with

$$\begin{aligned} \epsilon &= \frac{L_0^2\,x_1^2 - 2\,\alpha^2\,A\,m^2}{L_0\,x_1\,\sqrt{L_0^2\,x_1^2 - 2\,\alpha^2\,A\,m^2}} \\ &= \frac{1}{L_0\,x_1}\sqrt{L_0^2\,x_1^2 - 2\,\alpha^2\,A\,m^2}. \end{aligned} \tag{66}$$

By choosing the constant

$$B = \frac{\pi}{2\,x_1} \tag{67}$$

we can utilize the relation

$$\sin(\beta - \frac{\pi}{2}) = -\cos\beta \tag{68}$$

so that we arrive at the standard orbit of a precessing ellipse:

$$r = \frac{\alpha}{\epsilon\cos(x_1\theta) + 1}. \tag{69}$$

The remaining constant A is related to the physical parameters via Eq.(66):

$$A = \frac{L_0^2\,x_1^2\,\left(1 - \epsilon^2\right)}{2\,\alpha^2\,m^2}. \tag{70}$$

The constraint that the square root in ϵ has to be real valued, leads to the restriction

$$L_0^2\,x_1^2 - 2\,\alpha^2\,A\,m^2 \geq 0 \tag{71}$$

or

$$A \leq \frac{L_0^2\,x_1^2}{2\,\alpha^2\,m^2}. \tag{72}$$

In case of equality we have a circular orbit. The limit of a parabola ($\epsilon = 1$) is reached for $A = 0$, and the orbit is hyperbolic for $A < 0$.

The orbit of the precessing ellipse is graphed in form $\theta(r)$ and $d\theta/dr$ in Fig. 1 for comparison with other force laws. The vertical tangents at the return points of r are significant.

Next we inspect the Einstein force law

$$F(r) = -\frac{m\,M\,G}{r^2} - \frac{3\,L_0^2\,M\,G}{m^2\,c^2\,r^4}. \tag{73}$$

It follows from Eq.(62) that the integral is not solveable analytically, it must be evaluated numerically. The orbit is given by

$$\theta = L_0\,c\,\sqrt{m}\int \frac{1}{\sqrt{r}\,\sqrt{\left(2\,c^2\,m^3\,r^2 + 2\,L_0^2\right)\,G\,M - 2\,c^2\,A\,m^3\,r^3 - L_0^2\,c^2\,m\,r}}dr + B. \tag{74}$$

The Minkowski force law is

$$F(r) = -\frac{\gamma(r)^4\,m\,M\,G}{r^2} - \frac{\gamma(r)^2\,\alpha\,m\,M}{r^3}\left(1 - \gamma(r)^2\right). \tag{75}$$

The orbit can be calculated first in the approximation of a constant relativistic γ factor which is defined by

$$\gamma(r) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{76}$$

with v being the absolute value of the orbital velocity. Assuming a near-circular orbit may justify the approximation $v \approx const$. This results in an orbit

$$r = -\frac{\gamma^4\,\alpha\,m^2\,M - \gamma^2\,\alpha\,m^2\,M + L_0^2}{\epsilon\sin\left(\frac{(\theta-B)\,\sqrt{(\gamma^4-\gamma^2)\,\alpha\,m^2\,M + L_0^2}}{L_0}\right) - \gamma^4\,m^2\,G\,M} \tag{77}$$

with

$$\epsilon = m\,\sqrt{\gamma^8\,m^2\,G^2\,M^2 + \left(2\,\gamma^2 - 2\,\gamma^4\right)\,\alpha\,A\,m^2\,M - 2\,L_0^2\,A}. \tag{78}$$

The result is a precessing ellipse which is "deformed" compared to the canonical case of Eq.(65), see Fig. 2 in comparison to Fig. 1.

When the correct form of γ is to be taken into account, we have to use an expression for the velocity. Here we took the approximation from the notes of paper 238:

$$\gamma(r) = \frac{1}{\sqrt{1 - \frac{M\,G}{c^2}\,\left(\frac{2}{r} - \frac{1}{a}\right)}} \tag{79}$$

so that we have a radial dependence only. Then the function $f(r)$ can be evaluated. The structure of constants is so complicated that we only give the expression used for the plot, the integrand of Eq.(29):

$$\frac{\sqrt{2}\,L_0}{2\,m}\,\frac{\mathrm{f}(r)}{r^2} = \frac{1}{\sqrt{2}\,r^2\,\sqrt{-\frac{3\log(2\,r-2)}{4} + \frac{3\log(r)}{4} - \frac{4\,r^2-4}{8\,r^3-8\,r^2} + 0.45}}. \tag{80}$$

This integrand represents $d\theta/dr$ and is shown in Fig. 3 as the red curve. This function has been integrated numerically to give the blue curve. The graphics can directly be compared to Fig. 2 for the constant gamma case. One sees that $\theta(r)$ behaves differently at the lower r boundary, a clear relativistic effect.

It should be noted that the parameters in the formulas were experimentally adopted for the graphs in such a way that the results were comparable. In addition they were chosen so that bound curves came out (no hyperbolas etc.). It would take a greater effort e.g. to set the integration constants in a way that the same physical system is described in all cases.

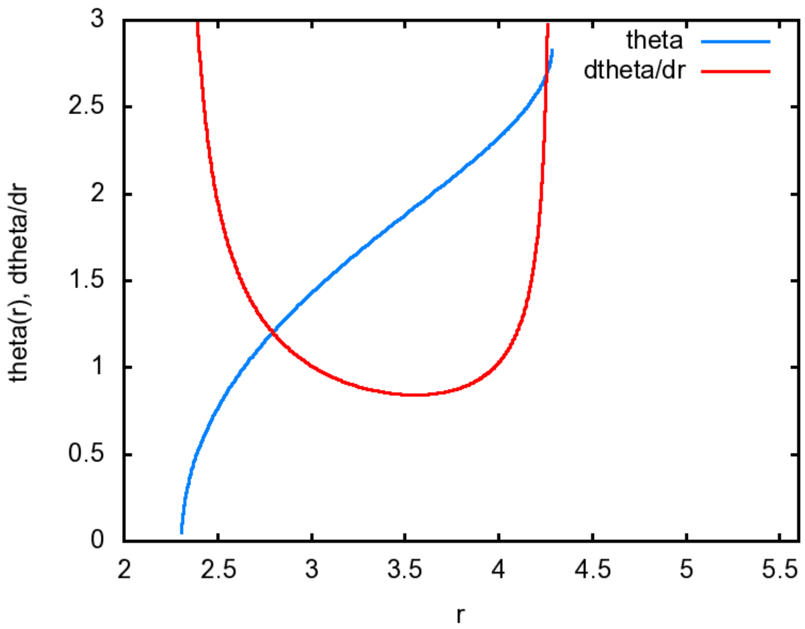

Figure 1: $\theta(r)$ and $d\theta/dr$ for a precessing ellipse with $\epsilon = 0.3, \alpha = 3, x = 1.1$.

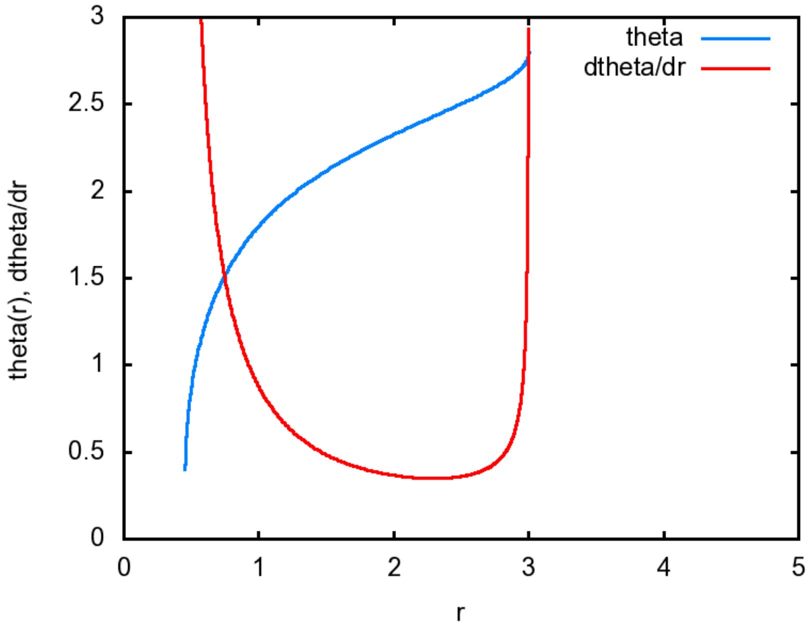

Figure 2: $\theta(r)$ and $d\theta/dr$ for the Minkowski force with adopted parameters, $\gamma = const.$

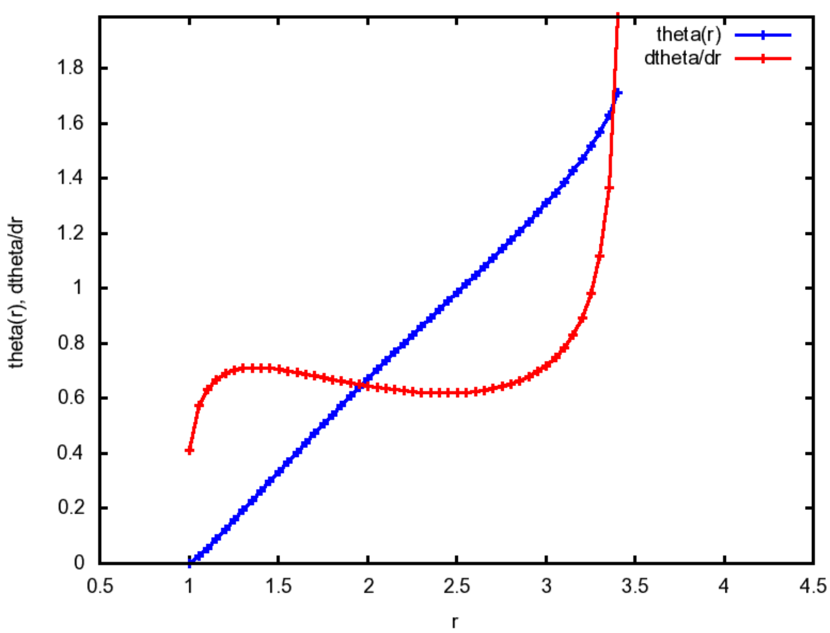

Figure 3: $\theta(r)$ and $d\theta/dr$ for the Minkowski force with adopted parameters, numerical solution.

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