CALCULATION OF LIGHT DEFLECTION DUE TO GRAVITATION WITH THE ECE FORCE LAW.

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ABSTRACT

The vector formulation of Cartan geometry developed recently in this series of papers is used to develop the concept of orbital force and spin force from the first Maurer Cartan structure equation and applied to the light deflection due to gravitation and precession of orbits. It is shown that both phenomena can be described with a universal spin connection, whose value is determined precisely with the experimental data. This is a torsion based cosmology that uses the rigorously correct geometry upon which to base a geneally covariant theory.

Keywords: ECE dynamics, calculation of light deflection to gravitation, precession of planetary orbits.

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1. INTRODUCTION

In recent papers of this series {1 - 10}, the Cartan geometry upon which ECE theory is based has been reduced to an easily understandable vectorial format. The original differential geometry is elegant and concise, but too abstract for application. The tensorial format is a little more transparent to the general scientist and engineer, but can be cumbersome, with hidden rules of index summation and so forth. The vector notation is however easily accessible to most scientists and engineers, and is self checking. For example the space part of the Cartan identity becomes a well known vector identity. The vector notation also makes it easier to compare the generally covariant ECE theory with well known laws of dynamics. On the most basic level the best known Newtonian law of non relativistic dynamics is the force law, with its famous equivalence of gravitational and inertial mass, and its universal gravitation. The Newtonian dynamics was brilliantly successful in many areas, but has its swell known limits which showed up in astronomy towards the end of the nineteenth century. Planetary precession could not be understood easily with the Newtonian formulation, which is valid for motions not involving rotation. So it is somewhat self contradictory to apply the formulation to planetary motion despite the fact that the inverse square force law was derived from the three Kepler laws.

The first well known attempt to make the Newton theory generally covariant was made by Einstein, in a field equation of 1915 based on the second Bianchi identity of Riemann geometry. Recently {1 - 10} papers such as UFT88 on www.aias.us have been very influential in demonstrating that the second Bianchi identity is true if and only if the Levi-Civita connection is used. This connection was defined in about 1900 to be symmetric in its lower two indices, and the first Bianchi identity was inferred in 1902 on this assumption. The general connection however has an antisymmetric component. The correct geometry was

inferred in the twenties by Cartan using two structure equations from which follow the Cartan identity, the first Bianchi identity with torsion. As in papers such as UFT 255 the second Bianchi identity follows from the first straightforwardly. The two structure equations of Cartan are always produced simultaneously by the commutator of covariant derivatives as summarized in ref. (11), a new book entitled "The Principles of ECE Theory". The commutator acts on a vector to produce the torsion multiplied by the covariant derivative of that vector plus the curvature multiplied by the same vector. The torsion is the difference of two connections, so it is immediately clear that a symmetric connection within the torsion can be produced if and only if the commutator is also symmetric. A symmetric commutator is zero, and produces a null torsion and a null curvature. So if torsion is zero, so is curvature. A null curvature produces no gravitation at all, and the Einsteinian general relativity is invalidated because a symmetric connection can never be used in general relativity.

ECE dynamics on the other hand uses the antisymmetric connection and always considers both torsion and curvature. Within the simplest possible hypotheses, ECE theory is Cartan geometry itself. So modern physics has split in to the ECE School of Thought and the obsolete Einsteinian School of Thought. Both Schools are currently very influential. Einsteinian theory is refuted experimentally simply by considering the velocity curve of a whirlpool galaxy, which it fails completely to describe as demonstrated in papers such as UFT 236. ECE is very clearly preferred experimentally to Einsteinian general relativity because it is able to describe the velocity curve of a whirlpool galaxy without use of any empiricism extraneous to Cartan geometry. The most notorious example of such empiricism is "dark matter". The apparent precision of the Einsteinian theory in the solar system is an illusion because it is refuted experimentally by whirlpool galaxies. A theory cannot be "magically" precise and at the same time totally wrong, both theoretically and experimentally.

Some other general or universal explanation is needed for the universality of light deflection due to gravitation, the famous "twice the Newtonian value", and for the precession of planetary orbits. This is given in section two after a development of dynamics with the vector formulation of Cartan geometry. The explanation is that the phenomena are given by a universal spin connection whose value can be calculated very simply using the experimental data.

2. CALCULATION OF LIGHT DEFLECTION DUE TO GRAVITATION.

In vector formulation (1 - 11} the first Maurer Cartan structure equation {12} becomes two equations, respectively for orbital and spin torsion:

$$\underset{\sim}{T}^{a}(orb) = -\underline{\nabla} q_{o}^{a} - \frac{\partial \underline{q}^{a}}{\partial t} - \omega^{a}{}_{ob}\,\underline{q}^{b} + q_{o}^{b}\,\underline{\omega}^{a}{}_{b} \qquad -(1)$$

and

$$\underset{\sim}{T}^{a}(spin) = \underline{\nabla} \times \underline{q}^{a} - \underline{\omega}^{a}{}_{b} \times \underline{q}^{b} \qquad -(2)$$

The space part of the Cartan identity becomes:

$$\underline{\nabla} \cdot \underline{\omega}^{a}{}_{b} \times \underline{q}^{b} = \underset{\sim}{q}^{b} \cdot \underline{\nabla} \times \underline{\omega}^{a}{}_{b} - \underline{\omega}^{a}{}_{b} \cdot \underline{\nabla} \times \underline{q}^{b} \qquad -(3)$$

so:

$$\underline{\nabla} \cdot \underset{\sim}{T}^{a}(spin) = \underline{\omega}^{a}{}_{b} \cdot \underline{\nabla} \times \underline{q}^{b} - \underline{q}^{b} \cdot \underline{\nabla} \times \underline{\omega}^{a}{}_{b} \qquad -(4)$$

In the special case:

$$\underline{\nabla} \cdot \underset{\sim}{T}^{a}(spin) = 0 \qquad -(5)$$

the Cartan identity simplifies to:

$$\underline{\nabla} \cdot \underline{\omega}^{a}{}_{b} \times \underline{q}^{b} = 0 \quad -(6)$$

an equation which has the Beltrami structure:

$$\underline{\nabla} \times \left(\underline{\omega}^{a}{}_{b} \times \underline{q}^{b} \right) = \kappa \underline{\omega}^{a}{}_{b} \times \underline{q}^{b} \quad -(7)$$

It follows that:

$$\underline{\nabla} \times \underline{T}^{a}(spin) = \kappa \underline{T}^{a}(spin) \quad -(8)$$

$$= \underline{\nabla} \times \left(\underline{\nabla} \times \underline{q}^{a} \right) - \underline{\nabla} \times \left(\underline{\omega}^{a}{}_{b} \times \underline{q}^{b} \right)$$

From Eqs. (7) and (8):

$$\underline{\nabla} \times \left(\underline{\nabla} \times \underline{q}^{a} \right) = \kappa \underline{\nabla} \times \underline{q}^{a} \quad -(9)$$

so:

$$\underline{\nabla} \times \underline{q}^{a} = \kappa \underline{q}^{a} \quad -(10)$$

which is also a Beltrami structure. In the special case:

$$\underline{\nabla} \cdot \underline{q}^{a} = 0 \quad -(11)$$

Eqs. (10) and (11) give the Helmholtz equation:

$$\left(\nabla^{2} + \kappa^{2} \right) \underline{q}^{a} = 0. \quad -(12)$$

The tetrad postulate {1 - 12} is the invariance of the complete vector field and a very fundamental theorem of geometry. It asserts that:

$$D_\mu q^a_\nu = \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\lambda_{\mu\nu} q^a_\lambda = 0 \quad -(13)$$

where by definition:

$$\omega^a_{\mu\nu} = \omega^a_{\mu b} q^b_\nu \quad -(14)$$

$$\Gamma^a_{\mu\nu} = \Gamma^\lambda_{\mu\nu} q^a_\lambda . \quad -(15)$$

So the tetrad postulate is:

$$\partial_\mu q^a_\nu = \Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} . \quad -(16)$$

Therefore:

$$\Box q^a_\nu = \partial^\mu \partial_\mu q^a_\nu = \partial^\mu \left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \right) := -R \cdot q^a_\nu \quad -(17)$$

where R is defined by:

$$R := q^\nu_a \partial^\mu \left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu} \right) . \quad -(18)$$

So the tetrad postulate can always be written as the equally fundamental wave equation:

$$\left(\Box + R \right) q^a_\nu = 0 \quad -(19)$$

which is the basis for all the wave equations of physics and of quantum mechanics unified with general relativity {1 - 12}.

In Eq. (19):

$$q^a_\nu = \left(q^a_0 , - \underline{q}^a \right) \quad -(20)$$

and

$$\Box = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 . \quad -(21)$$

So it follows that:

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + R \right) \underline{q}V^a = \underline{0} \quad -(22)$$

The Helmholtz equation (12) is a special case of the very fundamental wave equation (22). If it assumed that:

$$\underline{q}V^a = qV^a(0)\, e^{i\omega t} \quad -(23)$$

then it follows that:

$$\left(\nabla^2 + \frac{\omega^2}{c^2} - R \right) \underline{q}V^a = \underline{0} \quad -(24)$$

Comparing Eqs. (12) and (24):

$$k^2 = \frac{\omega^2}{c^2} - R \quad -(25)$$

which becomes the Einstein energy equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(26)$$

if

$$E = \hbar\omega, \quad \underline{p} = \hbar \underline{k} \quad -(27)$$

and:

$$R = \left(\frac{mc}{\hbar} \right)^2 . \quad -(28)$$

The transformation from special to general relativity is therefore:

$$R = \left(\frac{mc}{\hbar} \right)^2 \longrightarrow \tilde{q}V_a \partial^\mu \left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu} \right) \quad -(29)$$

Now define the linear momentum tetrad:

$$p^{a}_{\mu} = p^{(0)} q^{a}_{\mu} \quad -(30)$$

which follows from the ECE postulate:

$$A^{a}_{\mu} = A^{(0)} q^{a}_{\mu} \quad -(31)$$

and the minimal prescription:

$$p^{a}_{\mu} \rightarrow p^{a}_{\mu} + e A^{a}_{\mu} . \quad -(32)$$

It follows from Eqs. (1) and (30) that the orbital force of ECE theory is:

$$\underset{\sim}{F}^{a}(orb) = -\nabla \phi^{a}_{0} - \frac{\partial \underline{p}^{a}}{\partial t} - \omega^{a}_{0b} \underline{p}^{b} + \phi^{b}_{0} \underline{\omega}^{a}{}_{b} \quad -(33)$$

and that the spin force is:

$$\underline{F}^{a}(spin) = \underline{\nabla} \times \underline{p}^{a} - \underline{\omega}^{a}{}_{b} \times \underline{p}^{b} . \quad -(34)$$

In the single polarization ECE theory {1 - 11}:

$$\underset{\sim}{F}(orb) = -\underline{\nabla} \phi - \frac{\partial \underline{p}}{\partial t} - \omega_{0} \underline{p} + \phi \underline{\omega} \quad -(35)$$

and:

$$\underline{F}(spin) = \underline{\nabla} \times \underline{p} - \underline{\omega} \times \underline{p} . \quad -(36)$$

In the non relativistic limit the spin connection vanishes and:

$$\underline{F}(orb) = -\underline{\nabla} \phi - \frac{\partial \underline{p}}{\partial t} \quad -(37)$$

and

$$\underline{F}\left(spin\right) = \underline{\nabla} \times \underline{p} \quad -(38)$$

The famous equivalence of inertial and gravitational mass is recovered from Eq. (37) using the antisymmetry law of ECE theory:

$$-\frac{\partial \underline{p}}{\partial t} = -m \underline{\nabla} \phi \quad -(39)$$

So:

$$\underline{F} = -\frac{\partial \underline{p}}{\partial t} = -m \underline{\nabla} \phi \quad -(40)$$

For Newtonian dynamics:

$$\phi = -GM/r, \quad -(41)$$

$$\underline{\nabla} \phi = \frac{GM}{r^2} \underline{e}_r \quad -(42)$$

and

$$\underline{p} = m \underline{a} \quad -(43)$$

so:

$$\underline{F} = -m \underline{a} = -\frac{mMG}{r^2} \underline{e}_r \quad -(44)$$

This powerful and precise result of ECE theory was first inferred in UFT 141. The ECE theory is therefore precise to on part in about 10^{17}, the precision of the experimental proof of the equivalence of inertial and gravitational mass. The equivalence is due to geometry.

The calculation of light deflection by gravitation proceeds by applying the ECE antisymmetry law to Eq. (35):

$$-\underline{\nabla} \phi + \underline{\omega} \phi = -\frac{d \underline{p}}{dt} - \omega_0 \underline{p} \quad -(45)$$

In which it has been assumed that:

$$\frac{d\underline{\rho}}{dt} = \frac{\partial \underline{\rho}}{\partial t} \quad -(46)$$

So:

$$\underline{F} = 2\left(\frac{-d\underline{\rho}}{dt} - \omega_0 \underline{\rho}\right) = -2\left(\underline{\nabla}\phi - \underline{\omega}\phi\right) \quad -(47)$$

This result has been derived by using:

$$\phi^a_\mu = \phi^{(0)} q^a_\mu \quad -(48)$$

The factor 2 in Eq. (47) can be eliminated without affecting the physics by assuming that:

$$\phi^a_\mu = \frac{\phi^{(0)}}{2} q^a_\mu \quad -(49)$$

So the orbital force is:

$$\underline{F} = -\frac{d\underline{\rho}}{dt} - \omega_0 \underline{\rho} = -\underline{\nabla}\phi + \underline{\omega}\phi \quad -(50)$$

For vanishing spin connection this immediately gives the equivalence principle:

$$\underline{F} = -\frac{d\underline{\rho}}{dt} = -\underline{\nabla}\phi \quad -(51)$$

Now use the experimentally observed fact that all planar orbits precess, inside and outside the solar system. As in previous work such as UFT 215 and UFT 216 this precessional motion is described by the precessing conical section equation:

$$r = \frac{\alpha}{1 + \epsilon \cos(\varkappa\theta)} \quad -(52)$$

with the constraint:

$$\varkappa \sim 1 \quad -(53)$$

imposed again by experimental observation. Clearly the factor x must be related to the spin connection because both concepts represent a departure from Newtonian dynamics. Eq. (52) is a description of precessing orbits and can be used in the following general equation {1 - 11}, valid for any orbit in which the angular momentum L is conserved.

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{L^2}F(r) \quad -(54)$$

The force from Eqs. (52) and (54) is:

$$F(r) = -\frac{kx^2}{r^2} - \frac{k(1-x^2)\alpha}{r^3} \quad -(55)$$

where k is a constant. If:

$$\underline{\rho} = \rho_r \underline{e}_r, \quad -(56)$$

$$\underline{\omega} = \omega_r \underline{e}_r, \quad -(57)$$

then:

$$F = -\frac{\partial\phi}{\partial r} + \phi\omega_r = -\frac{kx^2}{r^2} - \frac{k(1-x^2)\alpha}{r^3} \quad -(58)$$

For small deviations from a Newtonian orbit as in planetary precession, or even in binary pulsar precession:

$$-\frac{\partial\phi}{\partial r} = -\frac{kx^2}{r^2} \quad -(59)$$

i.e.

$$x \sim 1 \quad -(60)$$

to an excellent approximation. Eq. (59) is the equivalence principle as argued already.

So from Eqs. (58) and (59):

$$\phi\omega_r = -k\left(1-\alpha^2\right)\frac{\alpha}{r^3} \quad -(61)$$

to an excellent, almost Newtonian, approximation. In this approximation:

$$\phi = -\frac{k}{r} \quad -(62)$$

so the spin connection is:

$$\omega_r = \left(1-\alpha^2\right)\frac{\alpha}{r^2} \quad -(63)$$

where:

$$\alpha = \frac{b^2}{a} \quad -(64)$$

Here α is the half right latitude, a and b are the major and minor semi axes, and ϵ the eccentricity. In grazing light deflection by the sun and almost all objects, the orbit is a hyperbola. As argued in UFT 216 the total deflection of the light in a hyperbolic orbit is:

$$\Delta\phi = 2\sin^{-1}\frac{1}{\epsilon} \quad -(65)$$

As shown in UFT 216, for small angles of deflection at closest approach R_o :

$$\sin\phi \sim \phi = \frac{1}{\epsilon} = \left[\frac{m^2\alpha R_o}{\alpha^2 L^2}\left(v^2 - \frac{L^2}{m^2}\left(\frac{\alpha^2-1}{R_o^2}\right)\right) - 1\right]^{-1} \quad -(66)$$

where v is the velocity of a mass m orbiting a mass M. For light m is the photon mass.

In the Newtonian limit this equation reduces to:

$$\sin\phi \sim \phi = \frac{1}{\epsilon} = \left(\frac{R_o v^2}{Mb} - 1\right)^{-1} \quad -(67)$$

and for a photon of tiny but non zero mass m:

$$v \to c \qquad -(68)$$

so:

$$2\phi = \frac{2MG}{R_o c^2} \qquad -(69)$$

This is the so called "Newtonian value" for light deflection by gravitation. However by precise contemporary experimental observation the value of light deflection by any mass M seems always to be:

$$2\phi = \frac{4MG}{R_o c^2} \qquad -(70)$$

The correction needed to produce Eq. (70) from Eq. (69) is:

$$\frac{R_o c^2}{MG} \to \frac{R_o c^2}{MG} + \frac{\alpha}{R_o}\left(\frac{1-x^2}{x^2}\right) \qquad -(71)$$

Now use the result:

$$\alpha = R_o(1+\epsilon) \qquad -(72)$$

{13, 14} to find that:

$$2\phi = \frac{2R_o c^2}{MG} + 2(1+\epsilon)\left(\frac{1-x^2}{x^2}\right) \qquad -(73)$$

Experimentally:

$$(1+\epsilon)\left(\frac{1-x^2}{x^2}\right) = \frac{R_o c^2}{MG} \qquad -(74)$$

The eccentricity is given by the observed angle of deflection in Eq. (65):

$$\frac{1}{\epsilon} = \sin\left(\frac{\Delta\phi}{2}\right) \qquad -(75)$$

For small deflections such as the arc seconds of light deflected by the sun:

$$\frac{1}{\epsilon} \sim \frac{\Delta\psi}{2} \quad -(76)$$

so:

$$\left(1+\frac{2}{\Delta\psi}\right)\left(\frac{1-x^2}{x^2}\right) = \frac{R_o c^2}{mG} \quad -(77)$$

For small deflections:

$$x \sim 1 \quad -(78)$$

to an excellent approximation so:

$$1-x^2 = \frac{R_o c^2}{mG}\left(1+\frac{2}{\Delta\psi}\right)^{-1} \quad -(79)$$

However by experiment:

$$\Delta\psi = \frac{4R_o c^2}{mG} \quad -(80)$$

so using Eq. (63):

$$\omega_r = \frac{\Delta\psi}{4}\left(1+\frac{2}{\Delta\psi}\right)^{-1}\frac{\alpha}{r^2} \quad -(81)$$

From Eq. (72):

$$\alpha = R_o(1+\epsilon) = R_o\left(1+\frac{2}{\Delta\psi}\right) \quad -(82)$$

and from Eqs. (81) and (82):

$$\omega_r = \frac{\Delta\psi}{4}\frac{R_o}{r^2} \quad -(83)$$

At closest approach:

$$r = R_0 \qquad -(84)$$

so:

$$\boxed{\omega_r = \frac{\Delta\phi}{4R_0}} \qquad -(85)$$

and the spin connection can be calculated for each object in the universe for which light deflection by gravitation is observed. The results can be tabulated in tables of astronomy.

This is a universal result because it is found experimentally that light deflection due to gravitation is always given by Eq. (70) to high precision, so is always given by the spin connection (85). In an excellent approximation the latter is determined experimentally by the angle of deflection and by the distance of closest approach. This is of course a correct theory of light deflection whereas as shown in the very influential UFT 150B and its concomitant essays on www.aias.us the Einstein theory is riddled with obscurities and outright errors. Vankov, for example {14} has independently and severely criticised the Einstein field equation which is of course incorrect due to its neglect of torsion.

The parameter x is used to explain both light deflection due to gravitation and orbital precession in one self consistent ECE theory that also derives the equivalence principle from geometry with great precision.

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REFERENCES

{1} M. W. Evans, Ed. Journal of Foundations of Physics and Chemistry, (Cambridge International Science Publishing, CISP, 2011 onwards).

{2} M .W. Evans, Ed., "Definitive Refutations of the Einsteinian General Relativity" (CISP 2012).

{3} M .W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (CISP, 2011).

{4} M. W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis, 2005 to 2011) in seven volumes.

{5} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis, 2007).

{6} M .W. Evans, H. Eckardt and D. W. Lindstrom, papers and plenary in the Serbian Academy of Sciences and in Found. Phys. Lett., Physica B and Acta Physica Polonica.

{7} M .W. Evans and S. Kielich, "Modern Nonlinear Optics" (Wiley 1992, 1993, 1997, 2001) in two editions and six volumes.

{8} M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific 2001).

{9} M .W. Evans and J.- P. Vigier, "The Enigmatic Photon" (Kluwer, Dordrecht, 1994 to 2002) in five volumes hardback and softback.

{10} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific, 1994).

{11} M. W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "The Principles of ECE Theory" (in prep., open source on www.aias.us and book and ipod format).

{12} S. M. Carroll, "Spacetime and Geometry: an Introduction to General Relativity" (Addison-Wesley, New York, 2004, and online notes).

{13} J. B. Marion and S. T. Thornton, "Classical Dynamics of Particles and Systems"

(Harcourt Brace College Publishing, New York, 1988, 3rd. Edition).

{14} UFT215 and UFT216 on www.aias.us.