A CRITICAL TEST OF THE QUANTUM THEORY BASED ON THE

EVANS / MORRIS SHIFTS.

by

M. W. Evans, H. Eckardt, G. J. Evans and T. Morris

Civil List, AIAS and UPITEC

(www.aias.us, www.webarchive.org.uk, www.atomicprecision.com , www.upitec.org

www.et3m.net)

ABSTRACT

An experiment is suggested for the accurate measurement of the Evans / Morris shifts in atomic hydrogen gas, using the well known Balmer series of visible frequency absorptions as an example. Simple expressions are derived for the Evans / Morris shifts in the high and low frequency limits. Foundational quantum theory immediately predicts Evans / Morris shifts using a combination of the fundamental Planck distribution and the fundamental Beer Lambert law. It is shown that this combination of two basic principles produces splittings in the lines of the Balmer series. It is shown that photon mass can be calculated from any absorption at any frequency of the electromagnetic spectrum from sub Hertzian to gamma ray. Photon mass is an ineluctable outcome of fundamental optical theory and the foundational de Broglie / Einstein equations.

Keywords: ECE theory, photon mass, test of the quantum theory using the Evans / Morris shifts.

1. INTRODUCTION

In recent papers of this series of three hundred and four papers and books to date dedicated to the development of the ECE theory {1 - 10} it has been shown that the recently discovered Evans / Morris shifts are properties of quantum mechanics at its most basic level - the Planck law and Planck distribution, and the Beer Lambert law. The Evans / Morris shifts have been observed in a series of experiments by G. J. Evans and T. Morris described on the blog of www.aias.us in comprehensive detail. In UFT300 on www.aias.us it has been shown that the Evans / Morris red shifts are due to foundational absorption theory, and that the Evans / Morris blue shifts are due to foundational emission theory. The method used to come to these conclusions relies on a straightforward combination of the Planck distribution and Beer Lambert law to produce a new fundamental equation of quantum theory. The latter shows that the Beer Lambert law produces changes in frequency as well as intensity of radiation. This is true in any part of the electromagnetic spectrum. The power absorption coefficient was calculated using well known methods and expressed in terms of the transition electric dipole moment. The Evans Morris frequency shift obeys a Beer Lambert law in the same way as the ordinary intensity shift of the law. The greater the sample path length, the greater the Evans Morris frequency shift. The greater the power absorption coefficient the greater the Evans Morris red shift. The incident frequency of a probe laser is shifted to lower frequencies in any absorption process, and the extent of the shift depends on the product of the power absorption coefficient and sample path length. This phenomenon is wholly new to science. It has been observed by Evans and Morris repeatably and reproducibly.

In Section 2 an experimental set up is sketched for the accurate observation of the Evans Morris shifts in atomic spectra, where a probe laser can be tuned accurately to an absorption line. The well known visible frequency Balmer series of atomic hydrogen (H) is

used as an example. It is shown that the Evans Morris effects produce a splitting of the absorption line in general, and that this splitting depends on the sample path length. The greater the path length the greater the splitting. This is the direct result of the quantum . theory, so the latter can be tested experimentally in a completely new way. Using these concepts it is shown that any absorption is accompanied by finite photon mass. Conversely, the photon mass is always non zero and can be calculated from any absorption process from sub Hertzian to gamma ray.

As usual this paper should be read with its background notes on www.aias.us, UFT304. Note 1 develops the time dependent Beer Lambert law and Note 2 gives details of the well known calculation of the integrated power absorption coefficient in terms of the transition dipole moment. Note 3 gives details of the well known calculation of the electric dipole moment. Note 4 gives details of the Lyman series of atomic hydrogen gas. Section 2 is based on Notes 5 to 7, which develop the high frequency approximation of the theory of UFT300 into a simple equation that is easy to use. Finally Notes 6 and 7 give details of the calculation of finite photon mass from any absorption process.

2. EXPERIMENTAL METHOD, SPLITTINGS AND PHOTON MASS.

In order to measure the Evans / Morris shifts accurately, and to test the quantum theory, an experimental design is sketched in Note 304(4), page 3. This measures the difference between the incident frequency ω_o and the shifted frequency ω using an interferometric method. The sketch is intended only to give the basics or outline of the apparatus and many other high accuracy methods could be used. This is a critical new test of the quantum theory itself as well as a suggestion for accurate measurements of the Evans / Morris shifts.

The Evans / Morris shifts are described in general by:

$$\left(\frac{\omega}{\omega_0}\right)^3 \left(\frac{e^{y_0} - 1}{e^y - 1}\right) = \exp\left(-Al\right) \quad - (1)$$

in which:

$$y_0 = \frac{\hbar\omega_0}{kT}, \quad y = \frac{\hbar\omega}{kT} \quad - (2)$$

and:

$$A = \left(\frac{N}{V}\right)\frac{|\mu_{fi}|^2}{6\epsilon_0 v \hbar}. \quad - (3)$$

Here, k is the Boltzmann constant, T the temperature, $\hbar$ the reduced Planck constant, A the integrated power absorption coefficient, and l the sample path length. The transition electric dipole moment is denoted μ_{fi}, ϵ_0 is the S. I. Vacuum permittivity, v the velocity of light in the absorbing sample, N the number of molecules in a volume V of the absorbing sample. Eq. (1) is made up entirely of the fundamentals of quantum theory. On the left hand appears the Planck distribution {1 - 10} and on the right hand side the Beer Lambert law with A calculated from the fundamentals of quantum theory. Alternatively the experimentally observed A could be used. In either case the quantum theory shows immediately that there must be Evans Morris shifts.

In the visible range of the well known Balmer series of lines of atomic hydrogen (H):

$$\hbar\omega_0 \gg kT, \quad \hbar\omega \gg kT \quad - (4)$$

so:

$$\frac{e^{y_0} - 1}{e^y - 1} \rightarrow \exp(y_0 - y) = \exp\left(\frac{\hbar(\omega_0 - \omega)}{kT}\right).$$

$$- (5)$$

Therefore:

$$\left(\frac{\omega}{\omega_o}\right)^3 = exp\left(-\left(A\ell + \frac{kT}{\hbar(\omega_o-\omega)}\right)\right) = exp\left(-A\ell\right) \quad -(5)$$

to an excellent approximation. So the following simple equation is deduced:

$$\omega = \omega_o \, exp\left(-\frac{A\ell}{3}\right) \quad -(6)$$

in which the integrated power absorption coefficient across an absorption band {1 - 10} can be computed from the relevant wave functions of H using standard methods (UFT300) or observed experimentally.

The Balmer series of atomic H occurs in the visible, and is described by:

$$
\begin{array}{llll}
n = 2 \text{ to } n = 3 & \text{at} & 656 \cdot 3 \text{ nm} & \\
\quad " \quad\quad 4 & " & 486 \cdot 1 & " \\
\quad " \quad\quad 5 & " & 434 \cdot 0 & " \\
\quad " \quad\quad 6 & " & 410 \cdot 2 & " \\
\quad " \quad\quad 7 & " & 397 \cdot 0 & " \\
\quad " \quad\quad \infty & " & 365 \cdot 0 & "
\end{array} \quad -(7)
$$

The complete set of relevant quantum numbers from the Schroedinger equation are as follows:

$$
\begin{array}{llll}
n = 2, & \ell = 0, & m = 0 & \\
n = 2, & \ell = 1, & m = -1, 0, 1 & \\
n = 3, & \ell = 0, & m = 0 & \\
n = 3, & \ell = 1, & m = -1, 0, 1 & \\
n = 3, & \ell = 2, & m = -2, -1, 0, 1, 2 & \\
n = 4, & \ell = 0, & m = 0 & \\
& & \vdots &
\end{array} \quad -(8)
$$

and so on. The selection rules are the well known Laport selection rule:

$$\Delta\ell = \pm 1 \quad -(9)$$

and

$$\Delta m = 0, \pm 1. \quad - (10)$$

For linearly polarized light:

$$\Delta m = 0. \quad - (11)$$

For left circularly polarized light:

$$\Delta m = 1. \quad - (12)$$

For right circularly polarized light:

$$\Delta m = -1. \quad - (13)$$

The Grotian diagram for the 653.3 nm line is sketched as follows:
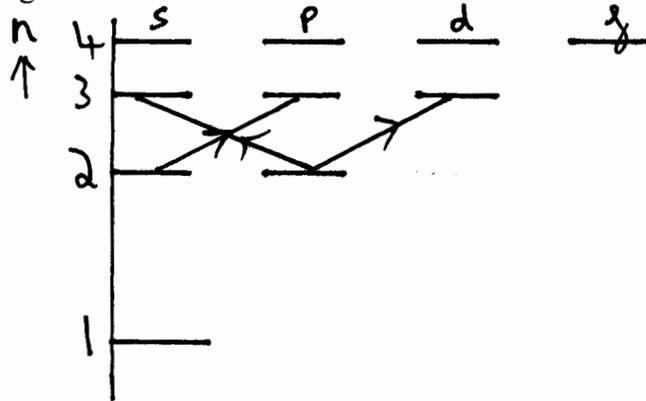
The 2s state is:

$$n = 2, \quad \ell = 0, \quad m = 0 \quad - (14)$$

The 2p state is:

$$n = 2, \quad \ell = 1, \quad m = -1, 0, 1 \quad - (15)$$

The 3s state is:

$$n = 3, \quad \ell = 0, \quad m = 0 \quad - (16)$$

The 3p state is:

$$n = 3, \quad \ell = 1, \quad m = -1, 0, 1 \quad - (17)$$

and the 3d state is:

$$n = 3, \quad \ell = 2, \quad m = -2, -1, 0, 1, 2 \quad - (18)$$

The energy levels of the 2p and 2s states are the same (n = 2). There are four states in all, and each has the same energy. So it is four fold degenerate. Similarly the energy levels of the nine n = 3 states are the same, and it is nine fold degenerate. In general for H the states are n^2 degenerate as is well known {1 - 10}. The above Grotian diagram shows the possible transitions between the n = 2 and n = 3 states at 656.3 nm. This wavelength corresponds to the initial angular frequency ω_0 of the Evans Morris shifts. The possible transitions are given by the selection rules. In linear polarization of the probe laser they are as follows.

1) 2s to 3p ($\Delta \ell = 1, \Delta m = 0$).

2) 2p to 3s ($\Delta \ell = -1, \Delta m = 0$)

3) 2p to 3d ($\Delta \ell = 1, \Delta m = 0$).

These transitions all occur at the same wavelength, so there is only one absorption line and only one emission line with its characteristic colour in the visible. This was first observed by Balmer in the late nineteenth century. However, as described in Section 2 this line is split by the Evans / Morris effects, and this is a prediction of the fundamental quantum theory: the Planck distribution combined with the Beer Lambert law as in Eq. (1). The details of this splitting are computed and graphed in Section 3. This is a fundamentally important theoretical inference and should be investigated experimentally. It is given the

appellation "absorption induced splitting of the Evans Morris effects" and is previously

unknown to science despite being a fundamental prediction of the Planck /Einstein / de

Broglie quantum theory proposed over a century ago.

Eq. (**1**) also describes the cosmological red shift without Big Bang, so can be

used to deduce the integrated absorption coefficient A of the light years between a red shifted

source and a telescope. There should also be absorption induced splittings that accompany a

red shift. Big Bang is meaningless because of neglect of torsion, as is well known and

accepted (Book of Scientometrics on www.aias.us)

The photon mass can be calculated from any absorption using well known equations

of optics combined with the de Broglie / Einstein equations (UFT158 ff. on www.aias.us).

Consider the wave of electric field strength in electromagnetic radiation:

$$E = E^{(0)} \exp\left(i\omega\left(t - \frac{z}{v} \right) \right) - (19)$$

where the velocity is defined as:

$$\frac{1}{v} = \frac{1}{c}\left(n' - i n'' \right) - (20)$$

in terms of the real n' and imaginary n'' parts of the refractive index. The imaginary part is

known in optics as the extinction coefficient. Therefore:

$$E = E^{(0)} \exp\left(i\left(\omega t - \omega \frac{z}{c} n' \right) \right) \exp\left(-\frac{\omega}{c} z n'' \right) . - (21)$$

The Beer Lambert law is therefore:

$$\frac{I}{I_0} = \frac{E E^*}{E^{(0)} E^{(0)*}} = \exp\left(-\frac{2 \omega n''}{c} z \right) - (21)$$

so the power absorption coefficient is:

$$\alpha = 2 \frac{\omega}{c} n'' \quad - (22)$$

By definition:

$$\epsilon' - i\epsilon'' = (n' - in'')^2 = n'^2 - n''^2 - 2in'n'' \quad - (23)$$

so:

$$\epsilon' = n'^2 - n''^2 \quad - (24)$$

$$\epsilon'' = 2n'n'' \quad - (25)$$

where ϵ' is the real part of the relative permittivity and ϵ'' is the imaginary part, known as the dielectric loss {1 - 10}. Therefore :

$$\alpha = \frac{\omega \epsilon''}{n' c} \quad - (26)$$

as used in the early Omnia Opera papers on www.aias.us. It follows that:

$$n' = \frac{\omega \epsilon''}{\alpha c} \quad - (27)$$

and:

$$n'' = \frac{\alpha c}{2\omega} \quad - (28)$$

From the fundamental optical equation (20) it follows that the concept of complex velocity is used in optical theory:

$$v = v' + iv'' \quad - (29)$$

Therefore the square modulus of v is:

$$|v|^2 = vv^* = v'^2 + v''^2 \quad -(30)$$

It follows that:

$$V = v' + iv'' = \frac{c}{n' - in''} = \frac{c(n' + in'')}{n'^2 + n''^2} \quad -(31)$$

so that:

$$\frac{v'}{c} = \frac{n'}{n'^2 + n''^2} \quad -(32)$$

and

$$\frac{v''}{c} = \frac{n''}{n'^2 + n''^2} \quad -(33)$$

where:

$$n' = \frac{\omega \epsilon''}{\alpha c} \quad -(34)$$

and:

$$n'' = \frac{\alpha c}{2\omega} \quad -(35)$$

Similarly:

$$\frac{1}{v' + iv''} = \frac{v' - iv''}{v'^2 + v''^2} = \frac{1}{c}(n' - in'') \quad -(36)$$

so that:

$$\frac{n'}{c} = \frac{v'}{v'^2 + v''^2} = \frac{v'}{|v|^2} \quad -(37)$$

and

$$\frac{n''}{c} = \frac{v''}{v'^2 + v''^2} = \frac{v''}{|v|^2} \cdot \qquad -(38)$$

From these equations, computer algebra shows that:

$$\epsilon'' = \frac{\alpha c^2}{2\omega^2 v'}\left(\left(\omega^2 - \alpha^2 v'^2\right)^{1/2} + \omega\right) - (39)$$

Using Eq. (26) it is found that:

$$n' = \frac{c}{2\omega v'}\left(\left(\omega^2 - \alpha^2 v'^2\right)^{1/2} + \omega\right) - (40)$$

which is the equation for the real part of the refractive index, n' , in terms of the real part of

the photon velocity, v' , and the power absorption coefficient of any absorption process.

Note that in the limit:

$$\alpha \rightarrow 0 \qquad -(41)$$

then:

$$n' \rightarrow \frac{c}{v'} \qquad -(42)$$

which is the usual expression for the refractive index in terms of velocity. Note carefully that

Eq. (42) is true if and only if the power absorption coefficient is zero. In the presence of

absorption it is no longer true.

The power absorption coefficient integrated over a complete absorption band is:

$$\alpha = \frac{d}{(|v|^2)^{1/2}} \qquad -(43)$$

where d is defined by

$$d = \left(\frac{N}{V}\right) \frac{|\mu_{fi}|^2}{6\epsilon_0 \hbar} \quad -(44)$$

In Eq. (43) v is defined by:

$$v := \left(|v^2|\right)^{1/2} = \frac{d}{\alpha} \quad -(45)$$

and v can be used as a definition of the photon velocity:

$$v^2 = |v^2| = v'^2 + v''^2 \quad -(46)$$

There appears to be not enough information in optical theory to find v completely in terms of α from the quantum theory alone. However v can be found from:

$$v = \frac{d \, (\text{theoretical})}{\alpha \, (\text{experimental})} \quad -(47)$$

if α is measured experimentally at a given frequency ω_0 and if d is computed theoretically. This procedure gives the photon velocity v for any absorption.

Having found the photon velocity v the photon mass is determined from the wave number κ defined by the phase in Eq. (19):

$$\kappa = \kappa' + i\kappa'' = \frac{\omega}{c}\left(n' - in''\right), \quad -(48)$$

$$\kappa^2 = \kappa\kappa^* = (\omega/c)^2\left(n'^2 + n''^2\right) = (\omega/v)^2 .$$

Therefore the Einstein / de Broglie momentum equation gives:

$$\hbar\kappa = \gamma m v \quad -(49)$$

where γ is the Lorentz factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(50)$$

and m can be found from v, Q. E. D. Some properties of m are plotted in Section 3 for the

Balmer transitions, assuming a model for the experimental α. More accurately α can

be measured experimentally.

From Eq. (38):

$$\frac{n''}{c} = \frac{v''}{v^2} = \frac{\alpha}{2\omega} = \frac{d}{2v\omega} \quad - (51)$$

so:

$$\frac{v''}{v} = \frac{d}{2\omega} \quad - (52)$$

The ratio of v'' to v can be found for any d, i.e. for any trasnition dipole moment of any

absorption spectrum. This ratio is plotted in Section 3 for the Balmer series of H. By

definition:

$$\left(\frac{v'}{v}\right)^2 = 1 - \left(\frac{v''}{v}\right)^2 = 1 - \left(\frac{d}{2\omega}\right)^2 \quad - (53)$$

so the ratio of v' to v can also be expressed in terms of d, and this is also plotted in Section 3.

In the absence of absorption:

$$v = v' \quad - (54)$$

so the absorptive part of the velocity, v'', measures the deviation from Eq. (54). If v is

found using:

$$v = \frac{d(\text{theory})}{d(\text{experimental})} \quad - (55)$$

then both v' and v'' can be determined. These define the dispersive photon mass:

$$\left. \begin{array}{l} \hbar \kappa' = \gamma' m' v', \\ \kappa' = \frac{\omega}{c} n' = \frac{\omega}{c} \frac{v'}{v^2}, \\ \gamma' = \left(1 - \frac{v'^2}{v^2}\right)^{-1/2} \end{array} \right\} \quad - (56)$$

and the absorptive photon mass:

$$\hbar \kappa'' = \gamma'' m'' v'' \, , \quad \kappa'' = \frac{\omega}{c} \hbar'' = \frac{\omega}{c} \frac{v''}{v^2} \, , \quad] \quad -(57)$$

$$\gamma'' = \left(1 - v''^2 / c^2\right)^{-1/2}$$

and some plots of these are given in Section 3.

ACKNOWLEDGMENTS

REFERENCES.

{1} M .W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "The Principles of ECE Theory" (UFT281 to UFT288 on www.aias.us and book format, in prep., UFT281 translated into Spanish by Alex Hill, with illustrations)

{2} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (www.cisp-publishing.com, CISP, and open source on www.aias.us as UFT301).

{3} H .Eckardt, "The ECE Engineering Model" (open source on www.aias.us as UFT303).

{4} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis 2007, and open source on www.aias.us as UFT302, translated into Spanish by Alex Hill).

{5} M. W. Evans, Ed. "Definitive Refutations of the Einsteinian General Relativity" (CISP 2012 and open source on www.aias.us).

{6} M. W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis 2005 to 2011 and open source on www.aias.us) in seven volumes.

{7} M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3)

Field" (World Scientific, 2001, and open source on www.aias.us).

{8} M .W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley Interscience, New York, 1992, 1993, 1997 and 2001), in six volumes and two editions.

{9} M .W. Evans and J.-P Vigier, "The Enigmatic Photon" (Kluwer 1994 to 2002 and open source on www.aias.us) five volumes hardback, five volumes softback.

{10} M . W. Evans and A. A Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific 1994).

A critical test of the quantum theory based on the Evans/Morris shifts

M. W. Evans,[*] H. Eckardt,[†] G. J. Evans, T. Morris
Civil List, A.I.A.S. and UPITEC

(www.webarchive.org.uk, www.aias.us,
www.atomicprecision.com, www.upitec.org)

3 Numerical analysis and discussion

The energy loss of light being absorbed in matter on a path l was given by Eqs.(6) and (3) in the optical limit by

$$\omega = \omega_0 \exp\left(-\frac{A\,l}{3}\right) \tag{58}$$

with the integrated power absorption coefficient defined by quantum mechanical quantities:

$$A = \frac{N}{V}\frac{|\mu_{fi}|^2}{6\epsilon_0 v \hbar}. \tag{59}$$

The equations have been evaluated numerically. For determination of N and V Avogadro's constant and the mole volume of Hydrogen (22.4 Litres) was used. The dipole matrix element μ_{fi} of transitions in atomic Hydrogen can

[*]email: emyrone@aol.com
[†]email: mail@horst-eckardt.de

transition	μ_{fi}	numerical value (SI)
2s → 3p	$\dfrac{2^{\frac{21}{2}}\,3^{\frac{7}{2}}\,a_0\,e}{15625\,\sqrt{6}\,Z}$	$1.50022 \cdot 10^{-29}$
2p → 3s	$\dfrac{16 \cdot 6^{\frac{7}{2}}\,a_0\,e}{15625\,Z}$	$4.59347 \cdot 10^{-30}$
2p → 3d, $m_l = 0$	$\dfrac{8192 \cdot 3^{\frac{9}{2}}\,a_0\,e}{5^{\frac{13}{2}}\,\sqrt{6}\,\sqrt{30}\,Z}$	$2.07877 \cdot 10^{-29}$
2p → 3d, $m_l = \pm 1$	$\dfrac{4096 \cdot 3^{\frac{9}{2}}\,\sqrt{15}\,a_0\,e}{78125\,\sqrt{6}\,\sqrt{30}\,Z}$	$1.80026 \cdot 10^{-29}$

Table 1: Dipole matrix elements μ_{fi} of hydrogenic wave functions.

be computed by the analytically known wave functions. For the transitions relevant for the Balmer series the matrix elements are listed in Table 1. The numerical values (in SI units) are quite small, but the denominator of Eq.(59) is also small by appearance of Planck's quantum. For an approximation of the velocity parameter the speed of light was used. For incident light with wavelength 656.3 nm then the frequency ω in dependence of the sample path length l can be computed. The results are shown in Fig. 1. Because the four transitions listed in Table 1 lead to different transition probabilities, the energy loss depends on the type of transition. For the second transition 2p $\rightarrow$ 3s the dipole matrix element is smallest, leading to a longer-range decrease. The result is a transition-specific splitting which appears without spin-orbit splittings responsible for spectroscopic line splittings in Hydrogen elsewhere. The frequencies fall to zero after some centimeters, with exeption of the low-probability transition. In Fig. 2 the same diagram is shown in the range of some millimeters. There the decrease is linear. All splittings should be observable by optical spectroscopy.

Next we compute the numerical values for the real and imaginary part of the photon velocity, v' and v'', as well as their absolute value v. We use the parameter

$$d = \frac{N}{V} \frac{|\mu_{fi}|^2}{6\epsilon_0 \hbar} \tag{60}$$

so that the integrated power absorption coefficient is

$$\alpha = \frac{d}{v} \tag{61}$$

and the velocity of light v can be computed by

$$v = \frac{d}{\alpha}. \tag{62}$$

α is used as a parameter so that one has to look up the curves as soon as such an experimental value for H and the considered transitions is available. A problem could be that the calculations are for mono-atimc H and the gas normally is in H_2 form which gives different orbitals. From section 2 we have the equations

$$v' = \frac{d}{\alpha} \sqrt{1 - \frac{d^2}{4\omega^2}}, \tag{63}$$

$$v'' = \frac{d^2}{2 \, \alpha \, \omega}. \tag{64}$$

The components v' and v'' (for dielectric permittivity and dielectric loss) have been graphed in a logarithmic plot in Fig. 3. Both v components show the splitting by μ_{fi} and depend on α essentially in hyperbolic form $(1/\alpha)$. For small α they exceed luminal velocity. In general we have

$$v'' << v' \tag{65}$$

i.e. dielectric loss is small and the modulus of velocity is

$$v \approx v'. \tag{66}$$

2

The photon masses were calculated according to Eqs.(56-57). In principle they can be evaluated for v, v' and v'' separately. Using v, the mass is given by

$$m = m' = \hbar\omega \frac{\alpha^2}{d^2} \sqrt{1 - \frac{d^2}{\alpha^2 \, c^2}}. \tag{67}$$

Because of $v \approx v'$, the mass values for the absolute value and dielectric permittivity are identical. They are shown in Fig. 4 as function of the absorption coefficient. For the most probable transitions the mass is below 10^{-32} kg, but for the transition with lowest probability it is in the range of the electron mass, similar as found for Compton scattering. The mass is defined only above a certain threshold. From (67) follows that for the square root to be real the condition

$$1 - \frac{d^2}{\alpha^2 \, c^2} > 0 \tag{68}$$

or, respectively,

$$\alpha \, c > d \tag{69}$$

has to hold. This means that α has to be larger than $1000/\text{m}$ in order to give a real-valued γ factor. This is still in the superluminal range for v'.

Finally the photon mass for v'' is graphed in Fig. 5 according to the formula

$$m'' = \frac{\hbar \, \omega}{v''^2} \sqrt{1 - \frac{v''^2}{c^2}} = 4 \, \hbar \omega^3 \frac{\alpha^2}{d^4} \sqrt{1 - \frac{d^4}{4 \, \alpha^2 \, c^2 \, \omega^2}}. \tag{70}$$

Because v'' is orders of magnitude smaller than v', the photon mass appears to be very large, even larger than the mass of the free electron. The threshold is quite small now. The mass values are significantly lowered near to the threshold. The deviation from the free-particle photon mass ($\approx 10^{-35}$kg) is similar as for the effective electron mass in molecular and solid state physics where even negative masses can be found.

3
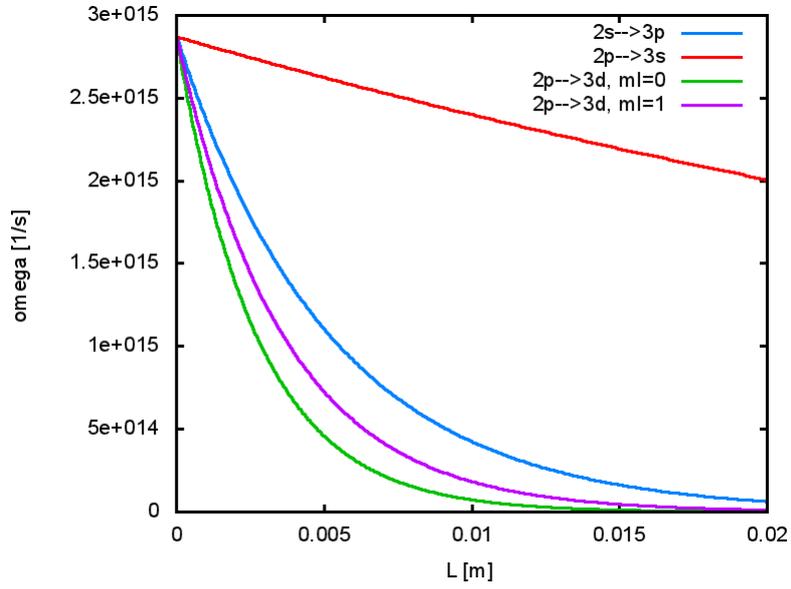
Figure 1: Frequency shift by absorption for several transitions of the Balmer series.
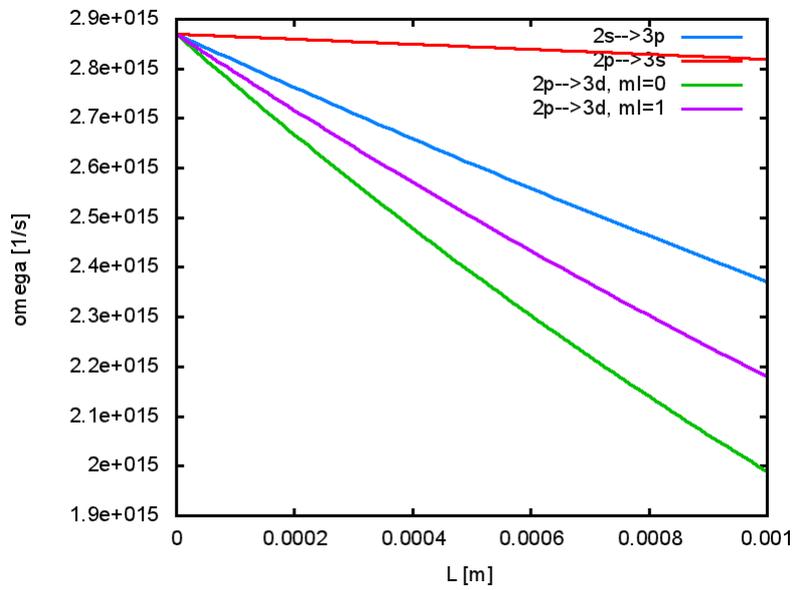
Figure 2: Same as Fig. 1, initial range of L.
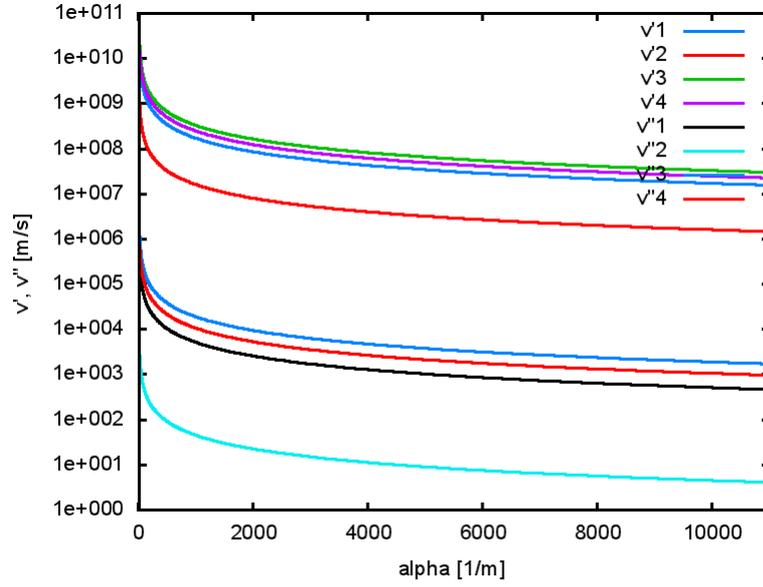
Figure 3: dielectric permittivity and loss velocities v', v'' for 4 Balmer series transitions (Table 1).
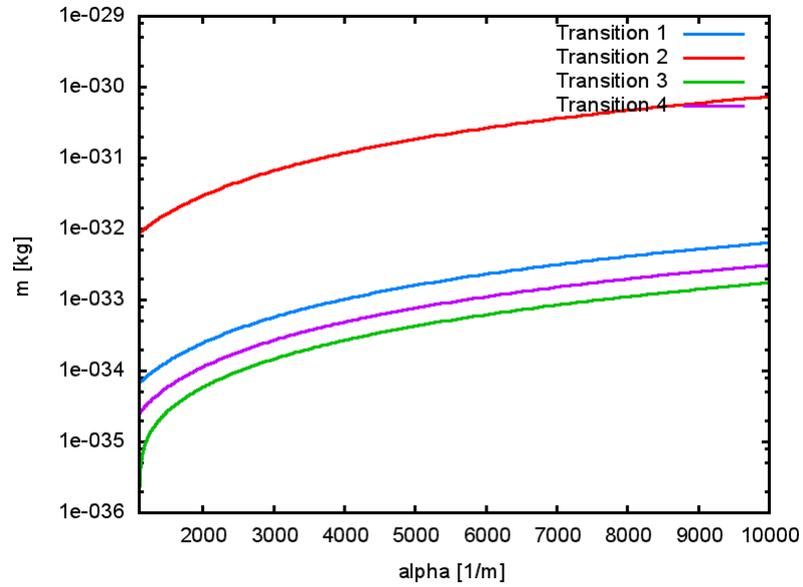
Figure 4: Permittivity photon mass of v' (identical with v).

5
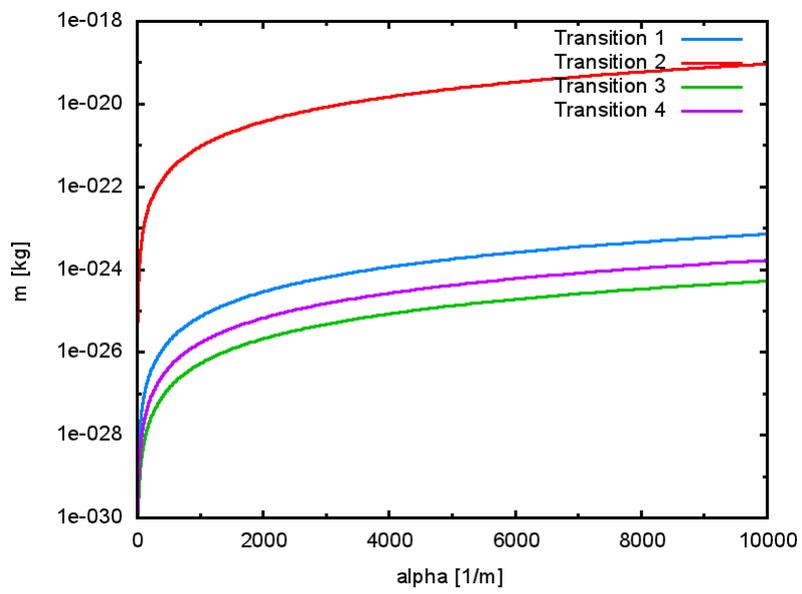
Figure 5: Dielectric loss photon mass of v''.