VECTOR FORMAT OF THE FIRST AND SECOND EVANS IDENTITIES.

by

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ABSTRACT

The first and second Evans identities of tensor analysis are developed into several new vector relations, valid in free space and in field matter interaction. Similarities in structure appear with the ECE equations of charge / current density in the vacuum. The first Evans identity was discovered in UFT109 and is an exact torsion identity that is also given by the Jacobi identity as shown in UFT313.

Keywords: First and Second Evans identities, ECE theory, vector format.

UFT 314

1. INTRODUCTION

In the preceding paper of this series {1 - 10}, UFT313, the second Bianchi identity of 1902 was corrected for torsion. In so doing five new identities of tensor analysis were discovered: the Bianchi Cartan Evans (BCE) identity; the Jacobi Cartan Evans (JCE) identity, the Ricci Evans identity, and first and second Evans identities. The first Evans identity was first discovered in UFT109 and shown to be an exact new identity of tensor analysis valid in any space of any dimension. All of the identities of UFT313 emerge from the Jacobi identity, including the Evans identity, and UFT313 is a development of a series of papers that correct the second Bianchi identity for torsion, notably UFT88, UFT255, UFT281 and UFT313. The second Evans identity is valid in four dimensions only, and is the first Evans identity with Hodge duals replacing torsion tensors.

As usual this paper should be read with its background notes accompanying UFT314 on www.aias.us. Note 314(1) derives novel field equations from the Evans identity using vector analysis. The result is equivalent to but more transparent than tensor notation and is useful for non specialist physicists, chemists and engineers. Note 314(2) derives a new identity of electric and magnetic fields in vector notation. Given the ECE hypotheses this is a direct transcription of the first Evans identity from tensor to vector notation. The latter is much more transparent in this case. The identity is checked with plane waves. Note 314(3) is a development of the field tensor in a new format, a tensor valued one form of differential geometry. The note develops its mathematical properties and applies the result to the field equations discovered in Note 314(1). Note 314(4) develops the second Evans identity into field equations in vector format. The final field equations are closely similar to equations involving the spin connection in free space summarized in the Engineering Model, UFT303. Note 314(5) gives a proof of tensor algebra in the presence of a double index summation, and

Note 314(6) develops the First Evans Identity for use with polarization and magnetization, scalar and vector potentials, and spin connections.

The main results of these notes are summarized in Section 2.

2. SOME VECTOR EQUATIONS FROM THE EVANS IDENTITIES.

Consider the first Evans identity:

$$T^{\lambda}_{\mu\nu} T^{\alpha}_{\rho\lambda} + T^{\lambda}_{\rho\mu} T^{\alpha}_{\nu\lambda} + T^{\lambda}_{\nu\rho} T^{\alpha}_{\mu\lambda} := 0 \quad -(1)$$

first derived in UFT109 and shown in UFT313 to be part of the Jacobi identity. Replace the λ indices by a indices of Cartan geometry {1 - 10}:

$$T^{a}_{\mu\nu} T^{\alpha}_{\rho a} + T^{a}_{\rho\mu} T^{\alpha}_{\nu a} + T^{a}_{\nu\rho} T^{\alpha}_{\mu a} := 0 \quad -(2)$$

and use

$$T^{\alpha}_{\rho\lambda} = T^{a}_{\rho\lambda} q^{\alpha}_{a} \quad -(3)$$

where q^{α}_{a} is an inverse Cartan tetrad. It follows that:

$$\left(T^{a}_{\mu\nu} T^{b}_{\rho a} + T^{a}_{\rho\mu} T^{b}_{\nu a} + T^{a}_{\nu\rho} T^{b}_{\mu a} \right) q^{\alpha}_{b} := 0 \quad -(4)$$

A possible solution of this equation is:

$$T^{a}_{\mu\nu} T^{b}_{\rho a} + T^{a}_{\rho\mu} T^{b}_{\nu a} + T^{a}_{\nu\rho} T^{b}_{\mu a} = 0. \quad -(5)$$

In the notation of differential geometry Eq. (5) is a wedge product:

$$T^{b}_{\rho a} \wedge T^{a}_{\mu\nu} = 0 \quad -(6)$$

between a tensor valued one form $T^{b}_{\rho a}$, and a vector valued two form $T^{a}_{\mu\nu}$.

In order to transform the geometry to electrodynamics use the ECE hypothesis as follows:

$$F^{b}_{\rho a} = A^{(0)} T^{b}_{\rho a}, \quad F^{a}_{\mu\nu} = A^{(0)} T^{a}_{\mu\nu} \qquad -(7)$$

in order to obtain a new equation of electrodynamics:

$$F^{b}_{\rho a} \wedge F^{a}_{\mu\nu} = 0. \qquad -(8)$$

Similarly, new equations can be obtained for gravitation and mixed gravitation / electromagnetism, giving a large number of new results.

Now express Eq. (8) as:

$$F^{b}_{\mu a} \tilde{F}^{a\mu\nu} = 0 \qquad -(9)$$

where the tilde denotes Hodge duality. Eq. (9) is valid in four dimensions only. As in UFT303 the free space field tensors are:

$$\tilde{F}^{a\mu\nu} = \begin{bmatrix} 0 & -cB_X^a & -cB_Y^a & -cB_Z^a \\ cB_X^a & 0 & E_Z^a & -E_Y^a \\ cB_Y^a & -E_Z^a & 0 & E_X^a \\ cB_Z^a & E_Y^a & -E_X^a & 0 \end{bmatrix} -(10a); \quad F^{b}_{\mu\nu} = \begin{bmatrix} 0 & E_X^b & E_Y^b & E_Z^b \\ -E_X^b & 0 & -cB_Z^b & cB_Y^b \\ -E_Y^b & cB_Z^b & 0 & -cB_X^b \\ -E_Z^b & -cB_Y^b & cB_X^b & 0 \end{bmatrix} \qquad -(10b)$$

where $\underline{E}$ is the electric field strength and $\underline{B}$ the magnetic flux density. The new type of field tensor $F^{b}_{\mu a}$ is a tensor valued one form:

$$F^{b}_{\mu a} = \left(F^{b}_{0a}, -\underline{F}^{b}_{a} \right). \qquad -(11)$$

The tensor equation (9) splits into two new vector equations of electrodynamics:

$$\underline{F}^b{}_a \cdot \underline{B}^a = 0 \quad -(12)$$

$$c F^b{}_{0a} \underline{B}^a = \underline{F}^b{}_a \times \underline{E}^b . \quad -(13)$$

There are also equivalent field equations of gravitation and mixed electromagnetism and gravitation.

Now use:

$$F^b{}_{\mu a} = F^b{}_{\mu\nu} q^{\nu}{}_a \quad -(14)$$

in eq. (9) to obtain:

$$q^{\nu}{}_a F^b{}_{\mu\nu} \tilde{F}^{a\mu\nu} = 0. \quad -(15)$$

As shown in all detail in Note 314(5), a possible solution of Eq. (15) is:

$$F^b{}_{\mu\nu} \tilde{F}^{a\mu\nu} = 0 \quad -(16)$$

which is the second format of the first Evans identity. Using the field tensors (10a) and (10b) gives the result:

$$\underline{E}^b \cdot \underline{B}^a + \underline{B}^b \cdot \underline{E}^a = 0 \quad -(17)$$

which is a new and generally valid equation of electrodynamics. In note 314(2) it is shown to be valid for plane waves, but it is valid for any fields.

Using the equation:

$$F^b{}_{\mu\nu} = q^a{}_{\nu} F^b{}_{\mu a} \quad -(18)$$

it follows that:

$$F^{b}_{\mu\nu} = A^{a}_{\nu} T^{b}_{\mu a} \qquad (19)$$

and this is a new relation between the field and potential in electrodynamics. It follows that the first Evans identity gives a new structure equation of differential geometry:

$$T^{b}_{\mu\nu} = q^{a}_{\nu} T^{b}_{\mu a} \qquad (20)$$

i.e. a new relation between the torsion and the tetrad. Note 314(3) systematically develops the properties of the new field tensor ($F^{a}_{\mu b}$), and shows that one possible solution is:

$$\underline{F}^{b}{}_{a} \times \underline{E}^{a} = \underline{0} . \qquad (21)$$

The second Evans identity is:

$$\tilde{T}^{\lambda}_{\mu\nu} \tilde{T}^{\alpha}_{\rho\lambda} + \tilde{T}^{\lambda}_{\rho\mu} \tilde{T}^{\alpha}_{\nu\lambda} + \tilde{T}^{\lambda}_{\nu\rho} \tilde{T}^{\alpha}_{\mu\lambda} := 0 \qquad (22)$$

and is valid in four dimensions. The tilde denotes Hodge dual. In four dimensions {1 - 10} the Hodge dual of a two form is another two form, so in four dimensions Eq. (22) is an example of Eq. (1). Using Hodge duality Eq. (22) may be written as:

$$\tilde{T}^{b}_{\mu\nu} T^{a\mu\nu} = 0 \qquad (23)$$

which is the second format of the second Evans identity of tensor analysis in four dimensions.

The field tensors in this case are:

$$F^{\mu\nu a} = \begin{bmatrix} 0 & -E^{a}_{X} & -E^{a}_{Y} & -E^{a}_{Z} \\ E^{a}_{X} & 0 & -cB^{a}_{Z} & cB^{a}_{Y} \\ E^{a}_{Y} & cB^{a}_{Z} & 0 & -cB^{a}_{X} \\ E^{a}_{Z} & -cB^{a}_{Y} & cB^{a}_{X} & 0 \end{bmatrix}; \quad \tilde{F}^{b}_{\mu\nu} = \begin{bmatrix} 0 & cB^{b}_{X} & cB^{b}_{Y} & cB^{b}_{Z} \\ -cB^{b}_{X} & 0 & E^{b}_{Z} & -E^{b}_{Y} \\ -cB^{b}_{Y} & -E^{b}_{Z} & 0 & E^{b}_{X} \\ -cB^{b}_{Z} & E^{b}_{Y} & -E^{b}_{X} & 0 \end{bmatrix} \qquad (24)$$

Now define:

$$\tilde{F}^{b}{}_{\mu a} = \left(\tilde{F}^{b}{}_{0a}, -\underline{\tilde{F}}^{b}{}_{a}\right) \quad -(25)$$

to obtain two additional field equations of electrodynamics in vector format:

$$\underline{\tilde{F}}^{a}{}_{b} \cdot \underline{E}^{b} = 0 \quad -(26)$$

and

$$c\,\underline{\tilde{F}}^{a}{}_{b} \times \underline{B}^{b} + \tilde{F}^{a}{}_{0b}\,\underline{E}^{b} = \underline{0} \quad -(27)$$

Eq. (23) gives:

$$\underline{E}^{b} \cdot \underline{B}^{a} + \underline{B}^{b} \cdot \underline{E}^{a} = 0 \quad -(28)$$

again. Therefore the complete set of equations is:

$$\underline{F}^{b}{}_{a} \cdot \underline{B}^{a} = 0 \quad -(29)$$

$$c\,F^{b}{}_{0a}\,\underline{B}^{a} = \underline{F}^{b}{}_{a} \times \underline{E}^{a} \quad -(30)$$

$$\underline{\tilde{F}}^{b}{}_{a} \cdot \underline{E}^{a} = 0 \quad -(31)$$

$$\tilde{F}^{a}{}_{0b}\,\underline{E}^{b} + c\,\underline{\tilde{F}}^{a}{}_{b} \times \underline{B}^{b} = \underline{0} \quad -(32)$$

and from both identities:

$$\underline{E}^{b} \cdot \underline{B}^{a} + \underline{B}^{b} \cdot \underline{E}^{a} = 0 \quad -(33)$$

Eqs. (29) to (32) are identical in structure to the following set of free space equations of The ECE Engineering Model, UFT303:

$$\underline{\omega}^a{}_b \cdot \underline{B}^b = 0 \quad - (34)$$

$$c\,\omega^a{}_{0b}\underline{B}^b = \underline{\omega}^a{}_b \times \underline{E}^b \quad - (35)$$

$$\underline{\omega}^a{}_b \cdot \underline{E}^b = 0 \quad - (36)$$

$$c\,\underline{\omega}^a{}_b \times \underline{B}^b + \omega^a{}_{0b}\underline{E}^b = \underline{0} \quad - (37)$$

where the spin connection is defined as:

$$\omega^a{}_{\mu b} = \left(\omega^a{}_{0b}, -\underline{\omega}^a{}_b\right) \quad - (38)$$

Finally Note 314(6) gives full details of the development of Eq. (33) for field / matter interaction, where:

$$\underline{D}^a = \epsilon_0 \underline{E}^a + \underline{P}^a \quad - (39)$$

$$\underline{B}^a = \mu_0 \left(\underline{H}^a + \underline{M}^a\right) \quad - (40)$$

Here $\underline{D}^a$ is the electric displacement, $\underline{P}^a$ the polarization, $\underline{H}^a$ the magnetic field strength, $\underline{M}^a$ the magnetization, ϵ_0 the vacuum permittivity and μ_0 the vacuum permeability. The identity (33) can also be developed using:

$$\underline{E}^a = -\underline{\nabla}\phi^a - \frac{\partial \underline{A}^a}{\partial t} - \omega^a{}_{0b}\underline{A}^b + \phi^b \underline{\omega}^a{}_b \quad - (41)$$

and

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad - (42)$$

in terms of the scalar potential ϕ^a, the vector potential $\underline{A}^a$, and the spin connection components of Eq. (38).

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