THE ANOMALOUS ZEEMAN EFFECT IN ECE2.

by

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ABSTRACT

It is shown that the rigorously correct Fermion / Dirac equation of ECE2 special relativity leads to new hyperfine structure in the anomalous Zeeman effect, structure that can be investigated experimentally with high resolution techniques such as double resonance. It is inferred for the first time that the ninety year old Dirac approximation means that the classical hamiltonian vanishes. This unphysical result means that a lot of hyperfine structure has been missed. If it exists experimentally, it is immediately useful in analytical chemistry and medicine. If it does not exist, there is a crisis in relativistic quantum mechanics at the foundational level.

Keywords: ECE2, anomalous Zeeman effect, new hyperfine structure.

UFT 332

1. INTRODUCTION

In recent papers of this series {1 - 12} it has been shown that the Fermion / Dirac equation can be written in a space in which both curvature and torsion are non zero - within the context of ECE2 theory. As a result of this development the foundational assumptions of the original Dirac equation were re examined, resulting in the inference of new hyperfine structure in atomic and molecular spectroscopy, exemplified in the simplest type of theory by atomic hydrogen H. In the Zeeman effect for example there are splittings of the usual spectrum as described in the previous paper UFT331. There can also be shifts of the ususal spectrum from the Dirac equation. These shifts and splittings are within range of ultra high resolution double resonance spectroscopy, and the foundational assumptions of Dirac and contemporaries can be tested with great experimental precision.

This paper should be read with its accompanying notes, which give all the calculational details needed to understand the main points of this paper, UFT332 on www.aias.us . Note 332(1) proves that the ninety year old Dirac approximation means that the classical hamiltonian (H_o) vanishes, an unphysical result that brings into question the validity of the approximation. Note 332(1) gives full details of the development of the rigorously correct hamiltonian of special relativity that Dirac and his contemporaries should have used. Note 332(2) solves a version of the rigorously correct Fermion / Dirac equation to give well defined hyperfine shifts that are missing entirely from the usual Dirac type theory with its crude and unphysical approximation leading to zero H_o . Note 332(3) discusses transition rules and Note 332(4) evaluates the hamiltonian of Note 332(2). Finally Note 332(5) gives details of the anomalous Zeeman effect theory and its modification through use of the rigorously correct Fermion / Dirac equation.

Section 2 discusses the main results of these notes, and Section 3 gives sample

results by computation, using the non relativistic hydrogenic wave functions in the first approximation. More rigorously, a vast amount of new insight awaits discovery through the application of advanced computational quantum chemistry and experimental investigation using ultra high resolution techniques.

2 RIGOROUS DEVELOPMENT AND ANOMALOUS ZEEMAN EFFECT.

The classical source of relativistic quantum mechanics is the Einstein energy equation:

$$E^2 = p^2c^2 + m^2c^4 \qquad -(1)$$

which is now known to be part of ECE2 unified field theory in a space in which both curvature and torsion are identically non zero. The original Eq. (1) was written in a space in which torsion and curvature are both identically zero, Minkowski or flat spacetime. In Eq. (1), E is the total relativistic energy:

$$E = \gamma mc^2 \qquad -(2)$$

where m is the particle mass, c the speed of light in vacuo and γ the Lorentz factor:

$$\gamma = \left(1 - \frac{p_0^2}{m^2c^2}\right)^{-1/2} \qquad -(3)$$

in which $\underline{p_0}$ is the non relativistic linear momentum, defined by:

$$\underline{p_0} = m\underline{v_0} \qquad -(4)$$

where $\underline{v_0}$ is the non relativistic linear velocity. In Eq. (1) $\underline{p}$ is the relativistic momentum:

$$\underline{p} = \gamma \underline{p_0} \qquad -(5)$$

The relativistic hamiltonian is:

$$H = E + U \quad -(6)$$

and the non relativistic hamiltonian is defined as:

$$H_0 := H - mc^2 \quad -(7)$$

As shown in detail in Note 332(1), Eq. (6) can be written as:

$$H_0 = \frac{p^2}{(1+\gamma)m} + U \quad -(8)$$

which reduces correctly as follows to the classical hamiltonian:

$$H_0 \xrightarrow[\gamma \to 1]{} \frac{p_0^2}{2m} + U \quad -(9)$$

Dirac approximated Eq. (8) by assuming that:

$$H_0 = \frac{p^2c^2}{H - U + mc^2} + U \sim \frac{p^2c^2}{mc^2 - U + mc^2} + U \quad -(10)$$

to obtain:

$$H_0 \sim \frac{p^2c^2}{2mc^2 - U} + U \sim \frac{p^2}{2m}\left(1 + \frac{U}{2m^2c^2}\right) + U \quad -(11)$$

So Dirac assumed that:

$$E = \gamma mc^2 = H - U \sim mc^2 - U \quad -(12)$$

i.e. that the Lorentz factor can be approximated by:

$$\gamma \sim 1 - \frac{U}{mc^2} \quad -(13)$$

However, the correct Lorentz factor is Eq. (3). In the limit:

$$v_0 \ll c \quad -(14)$$

the correct Lorentz factor can be approximated by:

$$\gamma \sim 1 + \frac{1}{2}\frac{p_0^2}{m^2c^2} . \quad -(15)$$

Comparing Eqs. (13) and (15):

$$\frac{p_0^2}{2m} = -U \quad -(16)$$

which means that the classical hamiltonian vanishes:

$$H_0 = \frac{p_0^2}{2m} + U = 0 . \quad -(17)$$

Despite its uncritical use for almost ninety years, the Dirac approximation is therefore highly restrictive, and as shown in UFT331 and this paper, loses a great deal of hyperfine spectral structure of great potential utility. This hyperfine structure is also a severe new experimental test of the foundations of relativistic quantum mechanics.

The usual Dirac approximation gives the hamiltonian:

$$H = \frac{p^2}{2m} + \frac{U}{4m^2c^2}p^2 + U \quad -(18)$$

and as shown in detail in Note 332(1) leads to the spin orbit hamiltonian:

$$\mathrm{Re}H_{so}\psi = -\frac{\hbar e^2}{16\pi\epsilon_0 m^2c^2r^3}\underline{\sigma}\cdot\underline{L}\psi \quad -(19)$$

(Note that there are some sign changes in Note 331(1) given on the blog of www.aias.us. The correct signs are given in this paper.) In Eq. (19), $\underline{L}$ is the relativistic angular momentum

$$\underline{L} = \gamma \underline{L}_0 \quad -(20)$$

where $\underline{L}_0$ is the non relativistic angular momentum, $\hbar$ is the reduced Planck constant, e the charge on the proton, ϵ_0 the S. I. vacuum permittivity m the mass of the electron, c the vacuum speed of light and $\underline{r}$ the distance between the proton and electron of the H atom. The

spin quantum operator is:

$$\underline{\hat{S}} = \frac{\hbar}{2}\underline{\hat{\sigma}} \qquad -(21)$$

where $\underline{\hat{\sigma}}$ is the Pauli matrix operator.

In the usual ninety year old development the non relativistic angular momentum is used, but the relativistic angular momentum should be used. The correct development as in Note 332(1) leads to:

$$\langle R_e H_{so} \rangle = \frac{-e^2}{16\pi\epsilon_0 m^2 c^2}\left(\frac{J(J+1)-L(L+1)-S(S+1)}{a_0^3 n^3 L\left(L+\frac{1}{2}\right)(L+1)}\right)\left(1+\frac{1}{mc^2}\left\langle\frac{p_0^2}{2m}\right\rangle\right)$$

in which:

$$\frac{1}{mc^2}\left\langle\frac{p_0^2}{2m}\right\rangle = \frac{1}{2}\left(\frac{\lambda_c}{a_0}\right)\frac{\alpha}{n^2} = \frac{2.662567\times10^{-5}}{n^2} \qquad -(22)\ -(23)$$

where λ_c is the Compton wavelength of the electron, a_0 is the Bohr radius, α the fine structure constant and n the principal quantum number. Here J is the total angular momentum quantum number, L the angular and S the spin quantum numbers. Remarkably, this is the first time that Eq. (22) has been inferred, and this development follows that of UFT331.

The new hyperfine structure depends on the way in which the rigorous hamiltonian (8) is developed. This is another new inference that affects the whole of relativistic quantum mechanics. Using:

$$p^2 = \gamma^2 p_0^2 \qquad -(24)$$

the hamiltonian (8) can be written as:

$$H_0 = \left(\frac{\gamma^2}{1+\gamma}\right)\frac{p_0^2}{2m} + u \qquad -(25)$$

and in the SU(2) basis (Note 332(2)) it can be written in three ways:

$$H_0 = \frac{1}{m}\underline{\sigma}\cdot\underline{p}_0\frac{\gamma^2}{1+\gamma}\underline{\sigma}\cdot\underline{p}_0 + U \quad -(26)$$

or

$$H_0 = \frac{\gamma}{m}\underline{\sigma}\cdot\underline{p}_0\frac{\gamma}{1+\gamma}\underline{\sigma}\cdot\underline{p}_0 + U \quad -(27)$$

or

$$H_0 = \frac{1}{m}\left(\frac{\gamma^2}{1+\gamma}\right)\underline{\sigma}\cdot\underline{p}_0\,\underline{\sigma}\cdot\underline{p}_0 + U \quad -(28)$$

These three different expressions of the same hamiltonian give rise to three different patterns of hyperfine structure. So the foundations of relativistic quantum mechanics are incompletely defined. The correct development of the theory can be found only by comparison with experimental data using ultra a high resolution spectroscopy.

As shown in Note 332(2):

$$\frac{\gamma^2}{1+\gamma} \doteq \frac{1}{2}\left(1 - \frac{U}{2mc^2} + \frac{1}{mc^2}\left(\frac{H_0}{2} + \frac{p_0^2}{2m}\right)\right) \quad -(29)$$

where:

$$\left\langle \frac{p_0^2}{2m^2c^2} \right\rangle = \frac{1}{2}\left(\frac{\lambda_c}{a_0}\right)\frac{\alpha}{n^2} \quad -(30)$$

and:

$$\left\langle \frac{H_0}{2mc^2} \right\rangle = -\frac{1}{4}\left(\frac{\lambda_c}{a_0}\right)\frac{\alpha}{n^2} \quad -(31)$$

So if the quantized hamiltonian is chosen to be:

$$H_0\psi = \left(\frac{1}{m}\underline{\sigma}\cdot\underline{p}_0\frac{\gamma^2}{1+\gamma}\underline{\sigma}\cdot\underline{p}_0 + U\right)\psi \quad -(32)$$

it can be developed as in Note 332(2) as:

$$H_0\psi = \frac{p_0^2}{2m}\left(1+\frac{1}{mc^2}\left(\frac{H_0}{2}+\frac{p_0^2}{2m}\right)\right)\psi - \frac{1}{4m^2c^2}\underline{\sigma}\cdot\underline{p}_0\, U\, \underline{\sigma}\cdot\underline{p}_0\,\psi \quad -(33)$$

The second term on the right hand side of this equation gives the usual spin orbit fine structure, and the first term on the right hand side is a hitherto unknown hyperfine structure. Note 332(3) gives the transition rules needed and in Section 3 a table of shifts is given. The transition rules in spin orbit fine structure are:

$$\Delta J = 0, \pm 1\ ;\ J=0 \not\to J=0 \quad -(34)$$

with

$$m_J = -J, \ldots, J \quad -(35)$$

and

$$\Delta m_J = 0, \pm 1. \quad -(36)$$

The usual spin orbit energy levels are:

$$E_{so} = -\frac{e^2}{16\pi\epsilon_0 m^2c^2}\left(\frac{J(J+1)-L(L+1)-S(S+1)}{a_0^3 n^3 L\left(L+\frac{1}{2}\right)(L+1)}\right) \quad -(37)$$

but the correct levels according to the choice (26) of hamiltonian are:

$$E_{so1} = E_{so}\left(1+\frac{2.662567\times10^{-5}}{n^2}\right). \quad -(38)$$

A table of shifts due to the correction (38) is given in Section 3. In the presence of a magnetic field a very richly structured hyperfine spectrum is obtained as follows:

$$E_{SO2} = E_{SO1} - \frac{e\hbar}{2m} g_J m_J B_Z \quad - (34)$$

where g_J is the Lande factor:

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \quad - (35/40)$$

and B_Z is the magnetic flux density applied in the Z axis.

Note 332(4) gives further details of the evaluation of the hamiltonian (~~28~~ 33), and Note 332(5) develops the hamiltonian in the presence of a magnetic field, giving some details of how the Lande factor is derived. It is shown in Note 332(5) that the correct hamiltonian of the anomalous Zeeman effect is:

$$\langle H_{AZE} \rangle = -\frac{mc^2\alpha}{n^2}\left(1 - \frac{1}{4}\frac{\lambda_c}{a_0}\frac{\alpha}{n^2}\right) - \frac{e\hbar}{2m} g_J m_J B_Z \quad - (36/41)$$

The H alpha line of atomic hydrogen for example is split into six lines by the anomalous Zeeman effect, the three lines of the normal Zeman effect being further split into three pairs. The rigorously correct hamiltonian (~~36~~ 41) produces hitherto unknown hyperfine shifts of the anomalous Zeeman effect as discussed in Section 3.

3. COMPUTATIONAL ANALYSIS OF THE NEW HYPERFINE SHIFTS

Section by Dr. Horst Eckardt

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3 Computational analysis of the new hyperfine shifts

The expectation value of the potential energy can be calculated from the relation

$$\langle U \rangle = -2 \langle T \rangle \tag{42}$$

with the non-relativistic kinetic energy

$$\langle T \rangle = \left\langle -\frac{\hbar \nabla^2}{2m} \right\rangle = \left\langle \frac{p_0^2}{2m} \right\rangle = m c^2 \frac{\alpha^2}{2 n^2} \tag{43}$$

as was derived in section 2. The relativistic energy levels of Dirac theory (without an external magnetic field) are given by

$$E_D = \langle U \rangle + \frac{m c^2 \alpha^2}{2 n^2} + \frac{\alpha^2}{4 m a_0 n^3} \frac{J(J+1) - L(L+1) - S((S+1)}{L(L+1/2)(L+1)} \tag{44}$$

while the new hyperfine structure leads to the result:

$$\begin{aligned} E =& \langle U \rangle + \frac{m c^2 \alpha^2}{2 n^2} \left(1 + \frac{\alpha^2}{4 n^2} \right) \\ &+ \frac{\alpha^2}{4 m a_0 n^3} \left(1 + \frac{\alpha^2}{2 n^2} \right) \frac{J(J+1) - L(L+1) - S((S+1)}{L(L+1/2)(L+1)} \end{aligned} \tag{45}$$

The term schema of transitions will not change since the new hyperfine structure does not depend on the m_J quantum numbers. Therefore there are no additional splittings for the Zeeman effect, only a small shift. The energies of both cases are compared for the first levels of atomic Hydrogen in Table 1. There is always a lifting of energies due to the correct solution of the fermion/Dirac equation presented in this paper. In addition, the energies are shown as term schemes in Figs. 1-6.

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Term	Non-rel.	Dirac	ECE2 theory	Diff. ECE2-Dirac
1 $s_{1/2}$	-0.5	-0.5	-0.4999933435807	$6.656 \cdot 10^{-6}$
2 $s_{1/2}$	-0.125	-0.125	-0.1249995839738	$4.160 \cdot 10^{-7}$
2 $p_{1/2}$	-0.125	-0.1250001386754	-0.1249997226492	$4.160 \cdot 10^{-7}$
2 $p_{3/2}$	-0.125	-0.1250001386754	-0.1249997226492	$4.160 \cdot 10^{-7}$
3 $s_{1/2}$	-0.05555555555555	-0.0555555555556	-0.0555554733775	$8.218 \cdot 10^{-8}$
3 $p_{1/2}$	-0.05555555555555	-0.0555555677301	-0.0555554855521	$8.218 \cdot 10^{-8}$
3 $p_{3/2}$	-0.05555555555555	-0.0555555494683	-0.0555554672903	$8.218 \cdot 10^{-8}$
3 $d_{3/2}$	-0.05555555555555	-0.0555555592079	-0.0555554770299	$8.218 \cdot 10^{-8}$
3 $d_{5/2}$	-0.05555555555555	-0.0555555531207	-0.0555554709426	$8.218 \cdot 10^{-8}$

Table 1: Energies (in Hartree units) of atomic Hydrogen with spin-orbit splitting (Dirac theory and this work).

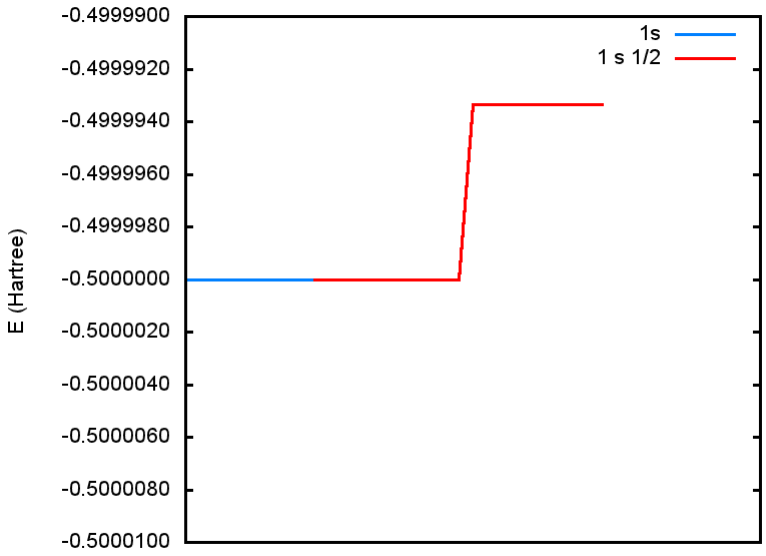

Figure 1: Energy schema for 1s states (non-rel., Dirac theory, ECE2 theory).

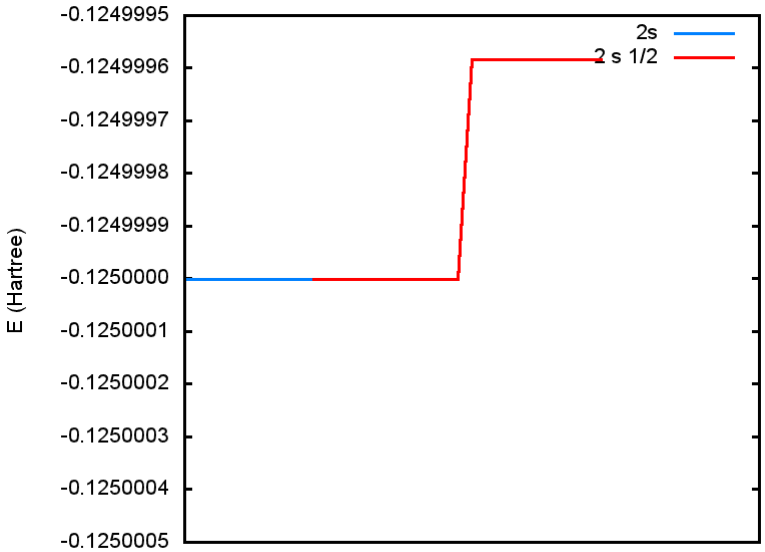

Figure 2: Energy schema for 2s states (non-rel., Dirac theory, ECE2 theory).

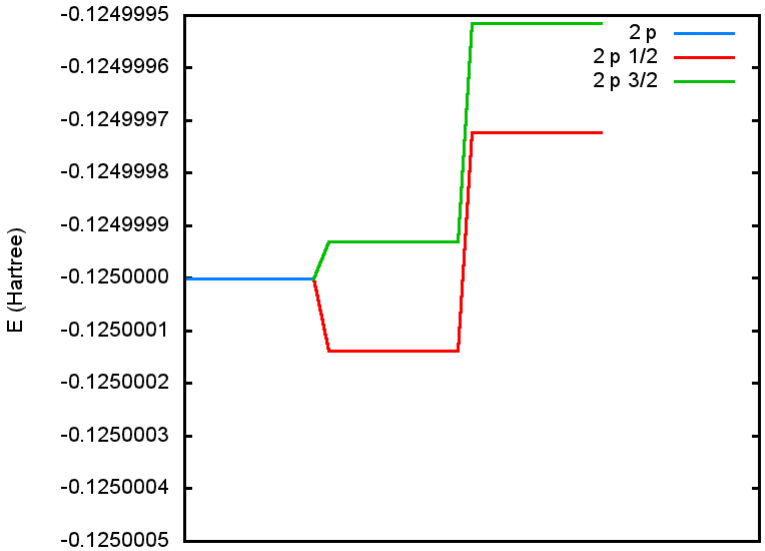

Figure 3: Energy schema for 2p states (non-rel., Dirac theory, ECE2 theory).

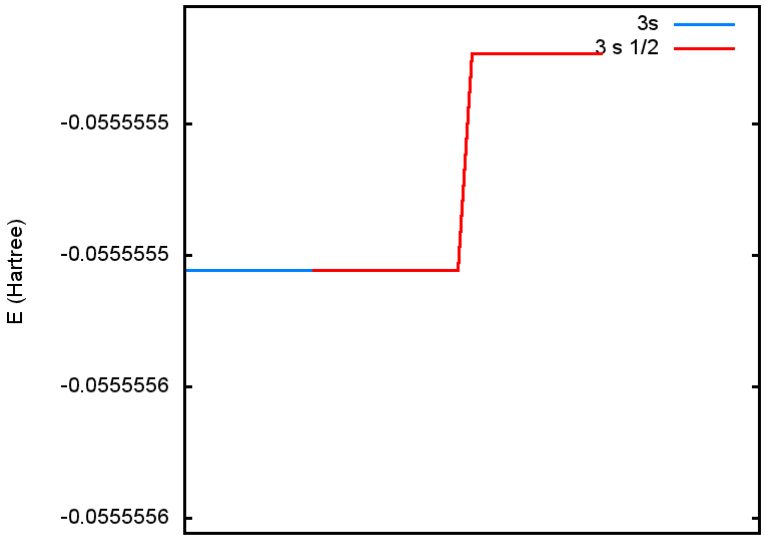

Figure 4: Energy schema for 3s states (non-rel., Dirac theory, ECE2 theory).

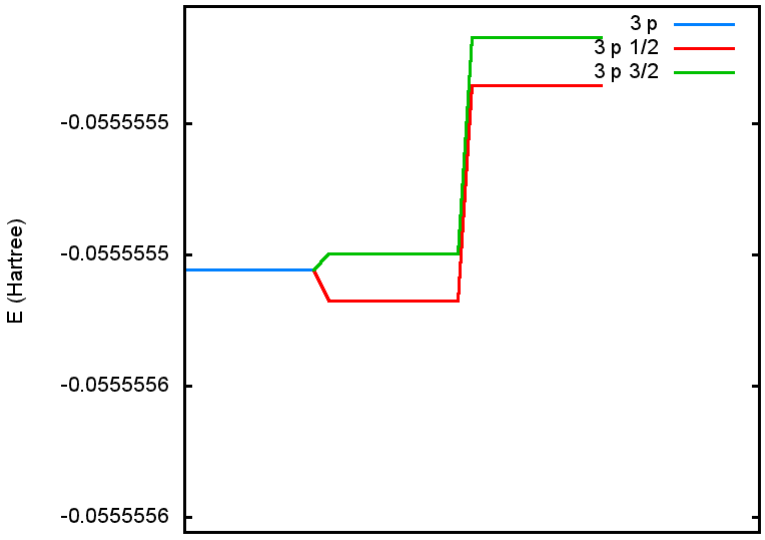

Figure 5: Energy schema for 3p states (non-rel., Dirac theory, ECE2 theory).

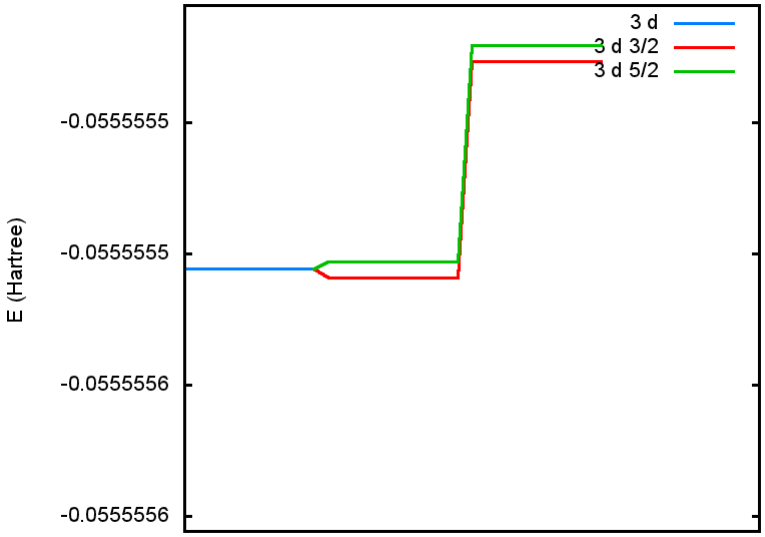

Figure 6: Energy schema for 3d states (non-rel., Dirac theory, ECE2 theory).

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