Chapter 8

Metric Compatibility Condition And Tetrad Postulate

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Abstract

The metric compatibility condition of Riemann geometry and the tetrad postulate of differential geometry are cornerstones of general relativity in respectively its Einstein Hilbert and Palatini variations. In the latter the tetrad tensor is the fundamental field, in the former the metric tensor is the fundamental field. In the Evans unified field theory the tetrad becomes the fundamental field for all types of matter and radiation, and the tetrad postulate leads to the Evans Lemma, the Evans wave equation, and to all the fundamental wave equations of physics in various well defined limits. The tetrad postulate is a fundamental requirement of differential geometry, and this is proven in this paper in seven ways. For centrally directed gravitation therefore both the metric compatibility condition and the tetrad postulate are accurate experimentally to one part in one hundred thousand.

Key words: Metric compatibility; tetrad postulate; Einstein Hilbert variation of general relativity; Palatini variation of general relativity; Evans unified field theory.

8.1 Introduction

The theory of general relativity was formulated originally in 1915 by Einstein and independently by Hilbert. It was developed for centrally directed gravitation, and was first verified by the Eddington experiment [1]. Recently [2] the precision of the Eddington experiment has been improved to one part in one hundred thousand. Therefore the basic geometrical assumptions used by Einstein and Hilbert have also been verified experimentally to one part in one hundred thousand. One of these is the metric compatibility condition [3]– [5] of Riemann geometry, a condition which asserts that the covariant derivative of the metric tensor vanishes. The metric tensor is the fundamental field in the Einstein Hilbert variation of general relativity. It is defined by:

$$g_{\mu\nu} = q^a{}_\mu q^b{}_\nu \eta_{\mu\nu} \tag{8.1}$$

where $q^a{}_\mu$ is the tetrad [3]– [5], a mixed index rank two tensor. The Latin superscript of the tetrad tensor refers to the spacetime of the tangent bundle at a point P of the base manifold indexed by the Greek subscript of the tetrad. In eqn.(8.1) η_{ab} is the Minkowski metric:

$$\eta_{ab} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{8.2}$$

The metric compatibility condition is then [3]– [5], for any spacetime:

$$D_\rho g^{\mu\nu} = D_\rho g_{\mu\nu} = 0. \tag{8.3}$$

Using the Leibnitz Theorem [3]– [5]Eq.(8.1) and (8.3) imply:

$$q^b{}_\nu D_\rho q^a{}_\mu + q^a{}_\mu D_\rho q^b{}_\nu = 0 \tag{8.4}$$

one possible solution of which is:

$$D_\rho q^a{}_\mu = D_\rho q^b{}_\nu = 0. \tag{8.5}$$

Eq.(8.5) is the tetrad postulate of the Palatini variation [3]– [8] of general relativity. In Section 8.2 it will be shown in various complementary ways that Eq.(8.5) is the unique solution of Eq.(8.4). It follows that for central gravitation, the tetrad postulate has been verified experimentally [2] to one part in one hundred thousand.

In Section 8.3 a brief discussion is given of the physical meaning of the metric compatibility condition used by Einstein and Hilbert in 1915 to describe centrally directed gravitation. In 1915 the original metric compatibility condition was supplemented by the additional assumption that the spacetime of gravitational general relativity is free of torsion:

$$T^\kappa{}_{\mu\nu} = \Gamma^\kappa{}_{\mu\nu} - \Gamma^\kappa{}_{\nu\mu} = 0 \tag{8.6}$$

where $T^{\kappa}{}_{\mu\nu}$ is the torsion tensor and where $\Gamma^{\kappa}{}_{\mu\nu}$ is the Christoffel symbol. The latter is symmetric in its lower two indices and is also known as the Levi-Civita or Riemann connection [3]– [5]. For the centrally directed gravitation of the sun these assumptions hold to one part in one hundred thousand [2]. However, the Evans unified field theory [9]– [15] has recently recognized that electromagnetism is the torsion form of differential geometry [3]– [5], gravitation being the Riemann form, and has shown how electromagnetism interacts with gravitation in a spacetime in which the torsion tensor is not in general zero. Therefore in Section 8.3 we discuss the implications for the metric compatibility condition of the 1915 theory, and summarize the conditions needed for the interaction of gravitation and electromagnetism.

8.2   Seven Proofs Of The Tetrad Postulate

It has been shown in the introduction that for any spacetime (whether torsion free or not) the tetrad postulate is a possible solution of the metric compatibility condition. In this section it is shown in seven ways that it is the unique solution.

1. Proof from Fundamental Matrix Invertibility.

   Consider the following basic properties of the tetrad tensor [3]– [5]:

   $$q^{b}{}_{\nu}q^{\nu}{}_{b} = 1 \tag{8.7}$$
   $$q^{a}{}_{\mu}q^{\mu}{}_{a} = 1 \tag{8.8}$$
   $$q^{\mu}{}_{a}q^{a}{}_{\nu} = \delta^{\mu}{}_{\nu} \tag{8.9}$$
   $$q^{a}{}_{\mu}q^{\mu}{}_{b} = \delta^{a}{}_{b} \tag{8.10}$$

   where $\delta^{\mu}{}_{\nu}$ and $\delta^{a}{}_{b}$ are Kronecker delta functions. Differentiate Eqs.(8.7) to (8.10) covariantly with the Leibnitz Theorem:

   $$q^{\nu}{}_{b}D_{\rho}q^{b}{}_{\nu} + q^{b}{}_{\nu}D_{\rho}q^{\nu}{}_{b} = 0 \tag{8.11}$$
   $$q^{a}{}_{\mu}D_{\rho}q^{\mu}{}_{a} + q^{\mu}{}_{a}D_{\rho}q^{a}{}_{\mu} = 0 \tag{8.12}$$
   $$q^{\mu}{}_{a}D_{\rho}q^{a}{}_{\nu} + q^{a}{}_{\nu}D_{\rho}q^{\mu}{}_{a} = 0 \tag{8.13}$$
   $$q^{a}{}_{\mu}D_{\rho}q^{\mu}{}_{b} + q^{\mu}{}_{b}D_{\rho}q^{a}{}_{\mu} = 0. \tag{8.14}$$

   Rearranging dummy indices in Eq(8.11) $(a \to b, \mu \to \nu)$:

   $$q^{\mu}{}_{a}D_{\rho}q^{a}{}_{\mu} + q^{b}{}_{\nu}D_{\rho}q^{\nu}{}_{b} = 0. \tag{8.15}$$

   Rearranging dummy indices in Eq.(8.14) $(\mu \to \nu)$:

   $$q^{\mu}{}_{b}D_{\rho}q^{a}{}_{\mu} + q^{a}{}_{\nu}D_{\rho}q^{\nu}{}_{b} = 0. \tag{8.16}$$

   Multiply Eq.(8.15) by $q^{a}{}_{\mu}$:

   $$D_{\rho}q^{a}{}_{\mu} + q^{a}{}_{\mu}q^{b}{}_{\nu}D_{\rho}q^{\nu}{}_{b} = 0. \tag{8.17}$$

Multiply Eq.(8.16) by $q^b{}_\mu$:

$$D_\rho q^a{}_\mu + q^b{}_\mu q^a{}_\nu D_\rho q^\nu{}_b = 0. \qquad (8.18)$$

It is seen that Eq.(8.17) is of the form:

$$x + ay = 0 \qquad (8.19)$$

and Eq.(8.18) is of the form:

$$x + by = 0 \qquad (8.20)$$

where

$$a \neq b. \qquad (8.21)$$

The only possible solution is:

$$x = y = 0. \qquad (8.22)$$

This gives the tetrad postulate, Q.E.D.:

$$D_\rho q^a{}_\mu = D_\rho q^\nu{}_b = 0, \qquad (8.23)$$

which is therefore the unique solution of Eq.(8.4). Note the tetrad postulate is true for any connection, whether torsion free or not.

2. Proof from Coordinate Independence of Tensors.

A tensor of any kind is independent of the way it is written [3]– [5]. Consider the covariant derivative of any tensor X in two different bases 1 and 2. It follows that:

$$(DX)_1 = (DX)_2. \qquad (8.24)$$

In the coordinate basis [3]:

$$\begin{aligned}(DX)_1 &= (D_\mu X^\nu)dx^\mu \otimes \partial_\nu \\ &= (\partial_\mu X^\nu + \Gamma^\nu{}_{\mu\lambda}X^\lambda)dx^\mu \otimes \partial_\nu.\end{aligned} \qquad (8.25)$$

In the mixed basis:

$$\begin{aligned}(DX)_2 &= (D_\mu X^a)\, dx^\mu \otimes \hat{e}_{(a)} \\ &= \left(\partial_\mu X^a + \omega^a{}_{\mu b}X^b\right) dx^\mu \otimes \hat{e}_{(a)} \\ &= q^\sigma{}_a \left(q^a{}_\nu \partial_\mu X^\nu + X^\nu \partial_\mu q^a{}_\nu \right. \\ &\quad \left. + \omega^a{}_{\mu b} q^b{}_\lambda X^\lambda\right) dx^\mu \otimes \partial\sigma\end{aligned} \qquad (8.26)$$

where we have used the commutation rule for tensors. Now switch dummy indices σ to μ and use:

$$q^\nu{}_a q^a{}_\nu = 1 \qquad (8.27)$$

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to obtain:

$$(DX)_1 = \left(\partial_\mu X^\nu + q^\nu{}_a \partial_\mu q^a{}_\lambda X^\lambda + q^\nu{}_a q^b{}_\lambda \omega^a{}_{\mu b} X^\lambda \right) dx^\mu \otimes \partial_\nu \qquad (8.28)$$

Now compare Eq.(8.25) and Eq.(8.28) to give:

$$\Gamma^\nu{}_{\mu\lambda} = q^\nu{}_a \partial_\mu q^a{}_\lambda + q^\nu{}_a q^b{}_\lambda \omega^a{}_{\mu b} \qquad (8.29)$$

Multiply both sides of Eq.(8.29) by $q^a{}_\nu$:

$$q^a{}_\nu \Gamma^\nu{}_{\mu\lambda} = \partial_\mu q^a{}_\lambda + q^b{}_\lambda \omega^a{}_{\mu b} \qquad (8.30)$$

to obtain the tetrad postulate, Q. E. D.:

$$D_\mu q^a{}_\lambda = \partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu = 0. \qquad (8.31)$$

3. Proof from Basic Definition.

For any vector V^a [3]:

$$V^a = q^a{}_\nu V^\nu \qquad (8.32)$$

and using the Leibnitz Theorem:

$$D_\mu V^a = q^a{}_\nu D_\mu V^\nu + V^\nu D_\mu q^a{}_\nu. \qquad (8.33)$$

Using the result:

$$D_\mu q^a{}_\nu = 0 \qquad (8.34)$$

obtained in proofs (1) and (2), it is proven here that Eqs.(8.32) and (8.34) imply:

$$D_\mu q^a{}_\lambda = \partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu \qquad (8.35)$$

From Eqs.(8.33) and (8.34):

$$\partial_\mu V^a + \omega^a{}_{\mu b} V^b = q^a{}_\nu \left(\partial_\mu V^\nu + \Gamma^\nu{}_{\mu\lambda} V^\lambda \right). \qquad (8.36)$$

From Eq.(8.32):

$$\partial_\mu V^a = V^\nu \partial_\mu q^a{}_\nu + q^a{}_\nu \partial_\mu V^\nu \qquad (8.37)$$

and

$$\omega^a{}_{\mu b} V^b = \omega^a{}_{\mu b} q^b{}_\nu V^\nu. \qquad (8.38)$$

Add Eqs.(8.37) and (8.38):

$$\partial_\mu V^a + \omega^a{}_{\mu b} V^b = q^a{}_\nu \partial_\mu V^\nu + V^\nu \partial_\mu q^a{}_\nu + \omega^a{}_{\mu b} q^b{}_\nu V^\nu \qquad (8.39)$$

Comparing Eqs.(8.36) and (8.39):

$$q^a{}_\nu \Gamma^\nu{}_{\mu\lambda} V^\lambda = V^\nu \left(\partial_\mu q^a{}_\nu + \omega^a{}_{\mu b} q^b{}_\nu \right) \qquad (8.40)$$

and switching dummy indices $\nu \to \lambda$, we obtain:

$$\partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - q^a{}_\nu \Gamma^\nu{}_{\mu\lambda} = 0. \qquad (8.41)$$

This equation has been obtained from the assumption (8.34), so it follows that:

$$D_\mu q^a{}_\nu = \partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - q^a{}_\nu \Gamma^\nu{}_{\mu\lambda} = 0 \qquad (8.42)$$

Q.E.D.

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4. Proof from the First Cartan Structure Equation [9].

   This proof has been given in all detail in ref. [9] and is summarized here for convenience. Similarly for Proofs (5) to (7). The first Cartan structure equation [3]– [8] is a fundamental equation of differential geometry first derived by Cartan. It defines the torsion form as the covariant exterior derivative of the tetrad form:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{8.43}$$

   i.e.

$$T^a{}_{\mu\nu} = \partial_\mu q^a{}_\nu - \partial_\nu q^a{}_\mu + \omega^a{}_{\mu b} q^b{}_\nu - \omega^a{}_{\nu b} q^b{}_\mu. \tag{8.44}$$

   Here $T^a{}_{\mu\nu}$ is the torsion two-form, $q^a{}_\mu$ is the tetrad one-form and $\omega^a{}_{\mu b}$ is the spin connection. The torsion tensor of Riemann geometry is defined [3]– [5] as:

$$T^\lambda{}_{\mu\nu} = q^\lambda{}_a T^a{}_{\mu\nu}. \tag{8.45}$$

   Using the tetrad postulate (8.31) in the form:

$$\Gamma^\lambda{}_{\mu\nu} = q^\lambda{}_a \partial_\mu q^a{}_\nu + q^\lambda{}_a q^b{}_\nu \omega^a{}_{\mu b} \tag{8.46}$$

   it is seen from Eqs.(8.44) to (8.46) that:

$$\begin{aligned}
T^\lambda{}_{\mu\nu} &= q^\lambda{}_a \left(\partial_\mu q^a{}_\nu + \omega^a{}_{\mu b} q^b{}_\nu \right) \\
&\quad - q^\lambda{}_a \left(\partial_\nu q^a{}_\mu + \omega^a{}_{\nu b} q^b{}_\mu \right) \\
&= \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}.
\end{aligned} \tag{8.47}$$

   Eq.(8.47) is the torsion tensor of Riemann geometry Q.E.D. Given the Cartan structure equation (8.43), therefore, the tetrad postulate is needed to derive the torsion tensor of Riemann geometry. The converse is also true.

5. Proof from the Second Cartan Structure Equation [3].

   Similarly this proof has been given in complete detail elsewhere [9]– [15] and is an elegant illustration of the tetrad postulate being used as the link between differential and Riemann geometry. The second Cartan structure equation defines the Riemann form as the covariant exterior derivative of the spin connection:

$$R^a{}_b = D \wedge \omega^a{}_b \tag{8.48}$$

   or

$$R^a{}_{b\nu\mu} = \partial_\nu \omega^a{}_{\mu b} - \partial_\mu \omega^a{}_{\nu b} + \omega^a{}_{\nu c} \omega^c{}_{\mu b} - \omega^a{}_{\mu c} \omega^c{}_{\nu b}. \tag{8.49}$$

   To establish this link the tetrad postulate is used in the form:

$$\omega^a{}_{\mu b} = q^a{}_\nu q^\lambda{}_b \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_b \partial_\mu q^a{}_\lambda \tag{8.50}$$

to write the spin connection in terms of the gamma connection. The Riemann tensor is defined as [3]– [5]:

$$R^{\sigma}{}_{\lambda\nu\mu} = q^{\sigma}{}_{a} q^{b}{}_{\lambda} R^{a}{}_{b\nu\mu} \tag{8.51}$$

and using the invertibility property of the tetrad tensor [3]:

$$q^{\lambda}{}_{c} q^{c}{}_{\lambda} = 1 \tag{8.52}$$

the Riemann tensor is correctly obtained [9]– [15] as:

$$R^{\sigma}{}_{\lambda\nu\mu} = \partial_{\nu}\Gamma^{\sigma}{}_{\mu\lambda} - \partial_{\mu}\Gamma^{\sigma}{}_{\nu\lambda} + \Gamma^{\sigma}{}_{\nu\rho}\Gamma^{\rho}{}_{\mu\lambda} - \Gamma^{\sigma}{}_{\mu\rho}\Gamma^{\rho}{}_{\nu\lambda} \tag{8.53}$$

Q. E. D. Therefore it has been shown that the Riemann form and the Riemann tensor are linked by the tetrad postulate. The Riemann form is defined by the second Cartan structure equation (8.48). The first and second Cartan structure equations are also known as the first and second Maurer - Cartan structure equations [3]. They are true for any type of spin connection.

6. Proof from the First Bianchi Identity.

The first Bianchi identity of differential geometry [3] is:

$$D \wedge T^{a} = R^{a}{}_{b} \wedge q^{b}. \tag{8.54}$$

This condensed notation denotes [9]– [15]:

$$(d \wedge T)^{a}{}_{\mu\nu\rho} = \partial_{\mu}T^{a}{}_{\nu\rho} + \partial_{\nu}T^{a}{}_{\rho\mu} + \partial_{\rho}T^{a}{}_{\mu\nu} \tag{8.55}$$

$$(\omega \wedge T)^{a}{}_{\mu\nu\rho} = \omega^{a}{}_{\mu b}T^{b}{}_{\nu\rho} + \omega^{a}{}_{\nu b}T^{b}{}_{\rho\mu} + \omega^{a}{}_{\rho b}T^{b}{}_{\mu\nu}. \tag{8.56}$$

The torsion form is defined as:

$$T^{a}{}_{\mu\nu} = \left(\Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu}\right) q^{a}{}_{\lambda}. \tag{8.57}$$

Similarly:

$$R^{a}{}_{b} \wedge q^{b} = \left(R^{\sigma}{}_{\mu\nu\rho} + R^{\sigma}{}_{\nu\rho\mu} + R^{\sigma}{}_{\rho\mu\nu}\right) q^{a}{}_{\sigma}. \tag{8.58}$$

Use of the Leibnitz Theorem and the tetrad postulate in the form:

$$\partial_{\mu}q^{a}{}_{\sigma} + \omega^{a}{}_{\mu b}q^{b}{}_{\sigma} = \Gamma^{\lambda}{}_{\mu\sigma}q^{a}{}_{\lambda} \tag{8.59}$$

leads correctly [9]– [15] to:

$$\begin{aligned}
&\partial_{\mu}\Gamma^{\lambda}{}_{\nu\rho} - \partial_{\nu}\Gamma^{\lambda}{}_{\mu\rho} + \Gamma^{\lambda}{}_{\mu\sigma}\Gamma^{\sigma}{}_{\nu\rho} - \Gamma^{\lambda}{}_{\nu\sigma}\Gamma^{\sigma}{}_{\mu\rho} \\
&+\partial_{\nu}\Gamma^{\lambda}{}_{\rho\mu} - \partial_{\rho}\Gamma^{\lambda}{}_{\nu\mu} + \Gamma^{\lambda}{}_{\nu\sigma}\Gamma^{\sigma}{}_{\rho\mu} - \Gamma^{\lambda}{}_{\rho\sigma}\Gamma^{\sigma}{}_{\nu\mu} \\
&+\partial_{\rho}\Gamma^{\lambda}{}_{\nu\nu} - \partial_{\mu}\Gamma^{\lambda}{}_{\rho\nu} + \Gamma^{\lambda}{}_{\rho\sigma}\Gamma^{\sigma}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\sigma}\Gamma^{\sigma}{}_{\rho\nu} \\
&\quad = R^{\lambda}{}_{\rho\mu\nu} + R^{\lambda}{}_{\mu\nu\rho} + R^{\lambda}{}_{\nu\rho\mu}
\end{aligned} \tag{8.60}$$

allowing the identification of the Riemann tensor for any gamma connection:

$$R^\lambda{}_{\rho\mu\nu} = \partial_\mu\Gamma^\lambda{}_{\nu\rho} - \partial_\nu\Gamma^\lambda{}_{\mu\rho} + \Gamma^\lambda{}_{\mu\sigma}\Gamma^\sigma{}_{\nu\rho} - \Gamma^\lambda{}_{\nu\sigma}\Gamma^\sigma{}_{\mu\rho} \qquad (8.61)$$

Q.E.D. Therefore it has been shown that the tetrad postulate is the necessary and sufficient condition to link the first Bianchi identity (8.54) and the equivalent in Riemann geometry, Eq.(8.60).

7. Proof from the Second Bianchi Identity

The second Bianchi identity of differential geometry is [3, 9]– [15]:

$$D \wedge R^a{}_b = d \wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b + \omega^c{}_b \wedge R^a{}_c \\ = 0. \qquad (8.62)$$

Using the results of Proof (7), and using by implication the tetrad postulate again, we correctly obtain [9]– [15] the second Bianchi identity of Riemann geometry:

$$D_\rho R^\kappa{}_{\sigma\mu\nu} + D_\mu R^\kappa{}_{\sigma\nu\rho} + D_\nu R^\kappa{}_{\sigma\rho\mu} = 0 \qquad (8.63)$$

Q. E.D. Therefore it has been shown that the tetrad postulate is the necessary and sufficient link between the second Bianchi identity of differential geometry [3] and the second Bianchi identity of Riemann geometry.

8.3   Physical Meaning Of The Metric Compatibility Condition And The Tetrad Postulate

The metric compatibility condition of Riemann geometry means that the metric tensor is covariantly constant [3, 9]– [15]: the covariant derivative of the metric tensor vanishes. If the metric is not covariantly constant then the metric is not compatible. The Einstein Hilbert variation of general relativity (the original 1915 theory) is based on metric compatibility [3,9]– [15]. The theory is accurate for central gravitation of the sun to one part in one hundred thousand [2]. Metric compatibility is used and also the assumption that the torsion tensor vanishes. These assumptions lead to the definition of the Christoffel symbol used by Einstein in his original theory of general relativity. Metric compatibility can also be assumed without the assumption of zero torsion. In this case we obtain the Palatini variation of general relativity in which metric compatibility becomes the tetrad postulate as described in Sections 8.1 and 8.2. The advantages of the Palatini variation are well known and the tetrad postulate has recently been shown to be the geometrical origin of all the wave equations of physics [9]– [15]. In a unified field theory a non-zero torsion form and torsion tensor are always needed to describe the electromagnetic sector. Only when the gravitational and electromagnetic sectors become independent can we use the original Einstein Hilbert variation of gravitational general relativity, with its vanishing torsion tensor and symmetric or Christoffel connection.

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