Chapter 10

Proof Of The Evans Lemma From The Tetrad Postulate

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Abstract

Two rigorous proofs of the Evans Lemma are developed from the fundamental tetrad postulate of differential geometry. Both proofs show that the Lemma is the subsidiary geometrical proposition upon which is based the Evans wave equation. The latter is the source of all wave equations in physics and generally covariant quantum mechanics.

Key words: Evans lemma; tetrad postulate; unified field theory; Evans wave equation.

10.1 Introduction

Recently a generally covariant unified field theory has been developed [1]– [12], a theory which is based rigorously on standard differential geometry. The basic theorems of standard differential geometry [13]– [15] include the Cartan structure relations, the Bianchi identities, and the tetrad postulate. Recently it has been proven [16] that the first Cartan structure equation is an equality of two tetrad postulates. Cartan geometry seems to be entirely sufficient for a unified field theory based on Einstein's idea that physics is geometry. This is the fundamental idea of relativity theory - that all physics must be both objective to all observers and rigorously causal. This includes quantum mechanics, and it has recently been shown experimentally in many ways, summarized in ref [10], that the Heisenberg uncertainty principle is incorrect qualitatively. The generally covariant unified field theory [1]– [12] suggests a causal quantum mechanics based on differential geometry, and suggests a development [10] of the Heisenberg uncertainty principle to make it compatible with experimental data.

Section 10.2 gives an outline of the advantages of Cartan geometry over Riemann geometry in the development of a unified field theory. Section 10.3 gives the first proof of the Lemma, and this is followed in Section 10.4 by a second proof which reveals the existence of a subsidiary condition.

10.2 Advantages Of Cartan Geometry

Without Cartan geometry it is much more difficult, if not impossible, to develop an objective unified field theory. The reason is that the fundamental structure equations of Cartan, and the fundamental Bianchi identities, are easily recognized as having the structure of generally covariant electromagnetic theory. This is by no means clear in Riemann geometry. To illustrate the advantage of Cartan geometry we discuss the four fundamental equations below, first in Cartan geometry and then in Riemann geometry.

The first Cartan structure equation gives the relation between the electromagnetic field and the electromagnetic potential [1]– [12]. In Cartan geometry it is:

$$T^a = D \wedge q^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{10.1}$$

and in Riemann geometry it is:

$$T^\kappa{}_{\mu\nu} = \Gamma^\kappa{}_{\mu\nu} - \Gamma^\kappa{}_{\nu\mu}. \tag{10.2}$$

Here T^a is the torsion form, q^a is the tetrad form, $\omega^a{}_b$ is the spin connection, $T^\kappa{}_{\mu\nu}$ is the torsion tensor and $\Gamma^\kappa{}_{\mu\nu}$ is the gamma connection of Riemann geometry. Eq.(10.1) in generally covariant electromagnetic theory [1]– [12] becomes:

$$F^a = D \wedge A^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{10.3}$$

where F^a is the electromagnetic field two-form and A^a is the electromagnetic potential. In the special relativistic limit Eq.(10.3) reduces to the familiar relation between field and potential in Maxwell Heaviside field theory [17]:

$$F = d \wedge A. \tag{10.4}$$

It is seen by comparison of Eq.(10.1) and (10.3) that Cartan geometry leads almost directly to the correctly and generally covariant theory of electrodynamics, Eq.(10.3). However the equivalent of Eq.(10.1) in Riemann geometry, Eq.(10.2), leads to no such inference.

The field equations of electrodynamics in the Evans unified field theory are based on the first Bianchi identity of differential geometry, which in its most condensed form may be written as:

$$D \wedge T^a = R^a{}_b \wedge q^b. \tag{10.5}$$

Eq.(10.5) is equivalent to:

$$d \wedge T^a = -\left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b\right). \tag{10.6}$$

Here $R^a{}_b$ is the Riemann form [1]– [15]. Using the Evans ansatz

$$A^a = A^{(0)} q^a \tag{10.7}$$

Eq.(10.6) becomes the homogeneous field equation of generally covariant electrodynamics:

$$d \wedge F^a = -A^{(0)} \left(q^b \wedge R^a{}_b + \omega^a{}_b \wedge T^b\right) = \mu_0 j^a \tag{10.8}$$

where j^a is the homogeneous current. Eq.(10.8) leads to the Faraday law of induction and the Gauss law of magnetism when j^a is very small:

$$j^a \sim 0. \tag{10.9}$$

Eq.(10.9) in turn leads to the free space condition:

$$R^a{}_b \wedge q^b = \omega^a{}_b \wedge T^b \tag{10.10}$$

one possible solution of which is circular polarization [1]– [12].

None of these inferences are clear, however, from the equivalent of Eq.(10.5) in Riemann geometry [1]– [15]:

$$\begin{aligned} R^\lambda{}_{\rho\mu\nu} = \partial_\mu \Gamma^\lambda{}_{\nu\rho} - \partial_\nu \Gamma^\lambda{}_{\mu\rho} + \Gamma^\lambda{}_{\mu\sigma}\Gamma^\sigma{}_{\nu\sigma} - \Gamma^\lambda{}_{\nu\sigma}\Gamma^\sigma{}_{\mu\rho} \\ et \quad cyclicum \end{aligned} \tag{10.11}$$

and so it would be difficult if not impossible to construct a unified field theory from Riemann geometry.

Cartan geometry also helps to clarify and simplify the structure of Einstein's original gravitational theory. This is accomplished using the second Cartan structure equation and the second Bianchi identity of Cartan geometry. The former is:

$$R^a{}_b = D \wedge \omega^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{10.12}$$

and the latter is:

$$D \wedge R^a{}_b = d \wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b + \omega^c{}_b \wedge R^a{}_c \quad = 0. \tag{10.13}$$

Eq.(10.12) bears an obvious similarity to Eq.(10.1) and again this is indicative of the fact that electromagnetism and gravitation are parts of a unified field based on Cartan geometry. The equivalent of Eq.(10.12) in Riemann geometry is, however:

$$R^\sigma{}_{\lambda\nu\mu} = \partial_\nu \Gamma^\sigma{}_{\mu\lambda} - \partial_\mu \Gamma^\sigma{}_{\nu\lambda} + \Gamma^\sigma{}_{\nu\rho}\Gamma^\rho{}_{\mu\lambda} - \Gamma^\sigma{}_{\nu\rho}\Gamma^\rho{}_{\nu\lambda} \tag{10.14}$$

and there is no resemblance to Eq.(10.2), the first Cartan structure equation written in terms of Riemann geometry. The same is true of the second Bianchi identity (10.13) of Cartan geometry, which is closely similar to the first Bianchi identity (10.5). However, the equivalent of Eq.(10.13) in Riemann geometry is:

$$D_\rho R^\kappa{}_{\sigma\mu\nu} + D_\mu R^\kappa{}_{\sigma\nu\rho} + D_\nu R^\kappa{}_{\sigma\rho\mu} = 0 \tag{10.15}$$

and is entirely different in structure form Eq.(10.11), the first Bianchi identity in Riemann geometry.

10.3 First Proof Of The Evans Lemma From The Tetrad Postulate

This first proof of the Evans Lemma [1]– [12] is based on the tetrad postulate of Cartan geometry [13]– [16], which is proven in eight ways in refs. [12] and [16]. The proof in ref (10.16) is a particularly clear demonstration of the fundamental nature of the tetrad postulate, because it is basically a restatement of the first Cartan structure equation without which there would be no Cartan geometry. The tetrad postulate is:

$$D_\mu q^a{}_\nu = 0 \tag{10.16}$$

where D_μ denotes covariant derivative [13] and may be thought of as the equivalent in Cartan geometry of the metric compatibility condition of Riemann geometry [1]– [16]. The metric compatibility condition has been tested experimentally to one part in one hundred thousand [16] by the NASA Cassini experiments designed to test the original 1915 theory of general relativity. The latter is based on the metric compatibility condition and the concomitant relation between the symmetric Christoffel symbol and the symmetric metric. However, the tetrad postulate is more generally applicable and is valid for all types of connection [1]– [16].

The Evans Lemma [1]– [12] is obtained from the identity:

$$D^\mu \left(D_\mu q^a{}_\nu\right) := 0 \tag{10.17}$$

i.e. from:

$$D^\mu (0) := 0. \tag{10.18}$$

Eq (10.18) implies [1]– [13] that:

$$\partial^\mu (0) := 0 \tag{10.19}$$

and so we obtain:

$$\partial^\mu \left(D_\mu q^a{}_\nu\right) := 0 \tag{10.20}$$

or

$$\partial^\mu \left(\partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu\right) := 0 \tag{10.21}$$

where $\omega^a{}_{\mu b}$ is the spin connection and $\Gamma^\nu{}_{\mu\lambda}$ is the general gamma connection [1]– [16]. The specialized Christoffel connection of the 1915 theory is obtained when the gamma connection is symmetric in its lower two indices; in order to develop a unified field theory we need the most general gamma connection, which is asymmetric in its lower two indices. Thus, for a given upper index, the most general gamma connection is a sum of a gamma connection symmetric in its lower two indices and a gamma connection antisymmetric in its lower two indices. The former is the Christoffel connection of gravitation and the latter is the gamma connection for electromagnetism [1]– [16].

To obtain the Evans Lemma rewrite Eq.(10.21) as follows:

$$\Box q^a{}_\mu = \partial^\mu \left(\Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda \right) := R q^a{}_\mu \tag{10.22}$$

where

$$R = q^\lambda{}_a \partial^\mu \left(\Gamma^\nu{}_{\mu\lambda} q^a{}_\mu - \omega^a{}_{\mu b} q^b{}_\lambda \right) \tag{10.23}$$

is scalar curvature, with units of inverse square metres. The Evans Lemma is therefore the prototypical wave equation of Cartan geometry:

$$\Box q^a{}_\mu = R q^a{}_\mu \tag{10.24}$$

Q.E.D.

Now consider the Einstein field equation [13]:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k T_{\nu\mu} \tag{10.25}$$

where $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the symmetric metric tensor, $T_{\mu\nu}$ is the symmetric canonical energy momentum tensor, and k is the Einstein constant. Multiply both sides of Eq. (10.25) by $g^{\mu\nu}$ and define the following scalars by index contraction:

$$R = g^{\mu\nu} R_{\mu\nu} \tag{10.26}$$

$$T = g^{\mu\nu} T_{\mu\nu} \tag{10.27}$$

Use:

$$g^{\mu\nu} g_{\mu\nu} = 4 \tag{10.28}$$

to obtain:

$$R = -kT. \tag{10.29}$$

From Eq.(10.24) and (10.29) we obtain the Evans wave equation [1]– [12]:

$$\left(\Box + kT \right) q^a{}_\mu = 0 \tag{10.30}$$

where:

$$R = -kT = g^{\mu\nu} R_{\mu\nu} = q^\lambda{}_a \partial^\mu \left(\Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda \right) \tag{10.31}$$

From the correspondence principle the Dirac equation is obtained in the limit:

$$kT \longrightarrow \left(\frac{mc}{\hbar} \right)^2 \tag{10.32}$$

with

$$T = \frac{m}{V_0} \tag{10.33}$$

giving the volume of the elementary particle in its rest frame:

$$V_0 = \frac{\hbar^2 k}{mc^2} = \frac{\hbar^2 k}{En_0} \tag{10.34}$$

This procedure also shows that the Dirac spinor is derived from the tetrad, which is the fundamental field [1]– [16] of the Palatini variation of general relativity [13]– [15]. In the unified field theory [1]– [12, 16] the tetrad is the fundamental field for all radiated and matter fields, (gravitation, electromagnetism, weak and strong fields and particle fields).

The Lemma is therefore a subsidiary condition based on Cartan geometry, a condition that leads via Eq.(10.29) to the Evans wave equation of physics. The various fields of physics are all defined by various tetrads [1]– [12], and all sectors of the unified field theory are generally covariant and rigorously objective. The need for general covariance in electrodynamics for example introduces the spin connection into electrodynamics, and with it the Evans spin field observed in the inverse Faraday effect [1]– [12]. Thereby electrodynamics is recognized as spinning spacetime, gravitation as curving spacetime. The two fields interact when the homogeneous current j^a is non zero [1]– [12]. A non-zero j^a however implies the experimental violation of the Faraday Law of induction and Gauss law of magnetism, and within contemporary experimental precision, this has not been observed in the laboratory.

10.4 Second Proof Of The Evans Lemma From The Tetrad Postulate

Rewrite Eq.(10.17) as:

$$D^\mu \left(\partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu \right) = 0 \tag{10.35}$$

and use the Leibnitz Theorem for covariant derivatives [13] to obtain:

$$\begin{aligned} (D^\mu \partial_\mu)\, q^a{}_\lambda + \partial_\mu \left(D^\mu q^a{}_\lambda\right) + \left(D^\mu \omega^a{}_{\mu b}\right) + \omega^a{}_{\mu b} \left(D^\mu q^b{}_\lambda\right) \\ - \left(D^\mu \Gamma^\nu{}_{\mu\lambda}\right) q^a{}_\nu - \Gamma^\nu{}_{\mu\lambda} \left(D^\mu q^a{}_\nu\right) = 0. \end{aligned} \tag{10.36}$$

Using Eq.(10.16) in Eq. (10.36) we obtain:

$$(D^\mu \partial_\mu)\, q^a{}_\lambda + \left(D^\mu \omega^a{}_{\mu b}\right) q^b{}_\lambda - \left(D^\mu \Gamma^\nu{}_{\mu\lambda}\right) q^a{}_\nu = 0. \tag{10.37}$$

Now use:

$$D^\mu \partial_\mu = \partial^\mu \partial_\mu + \ldots \tag{10.38}$$

$$D^\mu \omega^a{}_{\mu b} = \partial^\mu \omega^a{}_{\mu b} + \ldots \tag{10.39}$$

$$D^\mu \Gamma^\nu{}_{\mu\lambda} = \partial^\mu \Gamma^\nu{}_{\mu\lambda} + \ldots \tag{10.40}$$

to rewrite Eq.(10.37) as

$$\partial^\mu \left(\partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu \right) + \ldots = 0 \tag{10.41}$$

i.e.

$$\partial^{\mu}\left(D_{\mu} q^{a}{}_{\lambda}\right)+\ldots=0 \tag{10.42}$$

By comparison of Eq.(10.20) and (10.42) it is seen that there must be a subsidiary condition which ensures that the remainder term on the left hand side of Eq.(10.42) be zero. This condition provides a useful analytical constraint on the geometry of the Lemma.

In order to proceed we define the covariant derivatives:

$$D_{\mu} V^{\nu}=\partial_{\mu} V^{\nu}+\Gamma^{\nu}{}_{\mu \lambda} V^{\lambda} \tag{10.43}$$

$$D_{\mu} V_{\nu}=\partial_{\mu} V_{\nu}-\Gamma_{\nu \mu}{}^{\lambda} V_{\lambda} \tag{10.44}$$

Therefore if

$$V^{\nu}=0 \tag{10.45}$$

then

$$V^{0}=V^{1}=V^{2}=V^{3}=0 \tag{10.46}$$

and in Eq.(10.46):

$$\begin{gathered}
D_{\mu} V^{0}=\partial_{\mu} V^{0}+\Gamma^{\nu}{}_{\mu \lambda} V^{\lambda} \\
\vdots \\
D_{\mu} V^{3}=\partial_{\mu} V^{3}+\Gamma^{\nu}{}_{\mu \lambda} V^{\lambda} .
\end{gathered} \tag{10.47}$$

However:

$$V^{\lambda}=0, \lambda=0,1,2,3 \tag{10.48}$$

which implies

$$\begin{gathered}
D_{\mu} V^{0}=\partial_{\mu} V^{0}=0 \\
\vdots \\
D_{\mu} V^{3}=\partial_{\mu} V^{3}=0
\end{gathered} \tag{10.49}$$

and implies the subsidiary condition:

$$\Gamma^{\nu}{}_{\mu \lambda} V^{\lambda}=0 . \tag{10.50}$$

The same reasoning applies to the subsidiary condition in Eq.(10.42). Therefore when all components of a vector or tensor are zero:

$$D^{\mu} D_{\mu}=\partial^{\mu} D_{\mu} . \tag{10.51}$$

The components of a vector or tensor are not scalars, so in general D^{μ} acting on a vector component or tensor component is not the same as ∂^{μ} acting on the same component. This is clear from Eqs.(10.43) and (10.44).

The covariant divergence is now defined from the expression for the covariant divergence of a vector:

$$D_{\mu} V^{\mu}=\partial_{\mu} V^{\mu}+\Gamma^{\mu}{}_{\mu \lambda} V^{\lambda} \tag{10.52}$$

i.e.:

$$D_\mu \partial^\mu = \partial_\mu \partial^\mu + \Gamma^\mu{}_{\mu\lambda} \partial^\lambda. \tag{10.53}$$

Rewriting dummy indices inside the connection:

$$D_\mu \partial^\mu = \Box + \Gamma^\nu{}_{\nu\mu} \partial^\mu. \tag{10.54}$$

The wave equation (10.37) is therefore:

$$\left(\Box + \Gamma^\nu{}_{\nu\mu} \partial^\mu \right) q^a{}_\lambda - R_1 q^a{}_\lambda = 0 \tag{10.55}$$

where:

$$-R_1 q^a{}_\lambda := \left(D^\mu \omega^a{}_{\mu b} \right) q^b{}_\lambda - \left(D^\mu \Gamma^\nu{}_{\mu\lambda} \right) q^a{}_\nu. \tag{10.56}$$

Using the tetrad postulate:

$$\partial_\mu q^a{}_\lambda = -\omega^a{}_{\mu b} q^b{}_\lambda + \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu \tag{10.57}$$

so

$$\partial^\mu q^a{}_\lambda = -\omega^{\mu a}{}_b q^b{}_\lambda + \Gamma^{\mu\nu}{}_\lambda q^a{}_\nu. \tag{10.58}$$

Therefore in Eq.(10.55):

$$\Box q^a{}_\lambda - \Gamma^\nu{}_{\nu\mu} \omega^{\mu a}{}_b q^b{}_\lambda + \Gamma^\nu{}_{\nu\mu} \Gamma^{\mu\nu}{}_\lambda q^a{}_\nu - R_1 q^a{}_\lambda. \tag{10.59}$$

Finally define:

$$-R_2 q^a{}_\lambda := -\Gamma^\nu{}_{\nu\mu} \omega^{\mu a}{}_b q^b{}_\lambda + \Gamma^\nu{}_{\nu\mu} \Gamma^{\mu\nu}{}_\lambda q^a{}_\nu \tag{10.60}$$

to obtain the Evans Lemma:

$$\begin{aligned} \Box q^a{}_\lambda &= R q^a{}_\lambda, \\ R &= R_1 + R_2, \end{aligned} \tag{10.61}$$

Q.E.D.

10.5 Discussion

The Evans lemma is an eigenequation for R, and is accompanied by Eq.(10.23), which is a subsidiary condition for R. These geometrical equations provide a source for all the wave equations of physics in general relativity. Such a concept does not exist in the standard model, where quantum mechanics and general relativity are mutually incompatible. Develop Eq.(10.23) using [13]:

$$\partial^\mu = g^{\mu\sigma} \partial_\sigma \tag{10.62}$$

to give:

$$R = g^{\mu\sigma} q^\lambda{}_a \left(\Gamma^\nu{}_{\mu\lambda} \partial_\sigma q^a{}_\nu + q^a{}_\nu \partial_\sigma \Gamma^\nu{}_{\mu\lambda} - \omega^a{}_{\mu b} \partial_\sigma q^b{}_\lambda - q^b{}_\lambda \partial_\sigma \omega^a{}_{\mu b} \right). \tag{10.63}$$

Now use the tetrad postulates:

$$\partial_\sigma q^a{}_\nu + \omega^a{}_{\sigma b} q^b{}_\nu - \Gamma^\rho{}_{\sigma\nu} q^a{}_\rho = 0, \tag{10.64}$$

$$\partial_\sigma q^b{}_\lambda + \omega^b{}_{\sigma c} q^c{}_\lambda - \Gamma^\nu{}_{\sigma\lambda} q^a{}_\nu = 0, \tag{10.65}$$

to find:

$$\begin{aligned} R = g^{\mu\sigma} \big(& q^\lambda{}_a q^a{}_\rho \Gamma^\nu{}_{\mu\lambda} \Gamma^\rho{}_{\sigma\nu} - q^\lambda{}_a q^b{}_\nu \Gamma^\nu{}_{\mu\lambda} \omega^a{}_{\sigma b} \\ & + q^\lambda{}_a q^a{}_\nu \partial_\sigma \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_a q^b{}_\nu \omega^a{}_{\mu b} \Gamma^\nu{}_{\sigma\lambda} \\ & + q^\lambda{}_a q^c{}_\lambda \omega^a{}_{\mu b} \omega^b{}_{\sigma c} - q^\lambda{}_a q^b{}_\lambda \partial_\sigma \omega^a{}_{\mu b} \big) . \end{aligned} \tag{10.66}$$

Eliminate the tetrads using:

$$R^\sigma{}_{\lambda\nu\mu} = q^\sigma{}_a q^b{}_\lambda R^a{}_{b\nu\mu} \tag{10.67}$$

$$R^a{}_{b\nu\mu} = q^a{}_\sigma q^\lambda{}_b R^\sigma{}_{\lambda\nu\mu} \tag{10.68}$$

$$R^a{}_{b\nu\mu} = \partial_\nu \omega^a{}_{\mu b} - \partial_\mu \omega^a{}_{\nu b} + \omega^a{}_{\nu c} \omega^c{}_{\mu b} - \omega^a{}_{\mu c} \omega^c{}_{\nu b} \tag{10.69}$$

$$R^\sigma{}_{\lambda\nu\mu} = \partial_\nu \Gamma^\sigma{}_{\mu\lambda} - \partial_\mu \Gamma^\sigma{}_{\nu\lambda} + \Gamma^\sigma{}_{\nu\rho} \Gamma^\rho{}_{\mu\lambda} - \Gamma^\sigma{}_{\mu\rho} \Gamma^\rho{}_{\nu\lambda}. \tag{10.70}$$

The Riemann tensor $R^\sigma{}_{\lambda\nu\mu}$ and the Riemann form $R^a{}_{b\nu\mu}$ are antisymmetric respectively in σ and λ and in a and b. Using this antisymmetry Eq.(10.66) reduces to:

$$R = -g^{\mu\sigma} q^\lambda{}_a q^b{}_\nu \left(\Gamma^\nu{}_{\mu\lambda} \omega^a{}_{\sigma b} + \omega^a{}_{\mu b} \Gamma^\nu{}_{\sigma\lambda} \right) . \tag{10.71}$$

Now simplify and remove the tetrads using:

$$q^\lambda{}_a \Gamma^\nu{}_{\mu\lambda} = \Gamma^\nu{}_{\mu a} \tag{10.72}$$

$$q^b{}_\nu \omega^a{}_{\sigma b} = \omega^a{}_{\sigma\nu} \tag{10.73}$$

$$q^\lambda{}_a \omega^a{}_{\mu b} = \omega^\lambda{}_{\mu b} \tag{10.74}$$

$$q^b{}_\nu \Gamma^\nu{}_{\sigma\lambda} = \Gamma^b{}_{\sigma\lambda} \tag{10.75}$$

to give:

$$R = -g^{\mu\sigma} \left(\Gamma^\nu{}_{\mu a} \omega^a{}_{\sigma\nu} + \omega^\lambda{}_{\mu b} \Gamma^b{}_{\sigma\lambda} \right) . \tag{10.76}$$

Finally re-arrange dummy indices $b \to a, \lambda \to \nu$ to give:

$$R = -g^{\mu\sigma} \left(\Gamma^\nu{}_{\mu a} \omega^a{}_{\sigma\nu} + \omega^\nu{}_{\mu b} \Gamma^a{}_{\sigma\nu} \right) . \tag{10.77}$$

Eq.(10.77) is the required generalization to unified field theory of the equation for the scalar curvature R used in the original 1915 theory of gravitational general relativity, i.e. is the generalization of

$$R = g^{\mu\sigma} R_{\mu\sigma} \tag{10.78}$$

where $R_{\mu\sigma}$ is the Ricci tensor.

Comparing Eqs.(10.77) and (10.78):

$$R_{\mu\sigma} = -\left(\Gamma^\nu{}_{\mu a} \omega^a{}_{\sigma\nu} + \omega^\nu{}_{\mu a} \Gamma^a{}_{\sigma\nu} \right) \tag{10.79}$$

is the Ricci tensor in the unified field theory. Using Eq.(10.29) it is found that:

$$kT = g^{\mu\sigma} \left(\Gamma^\nu{}_{\mu a} \omega^a{}_{\sigma\nu} + \omega^\nu{}_{\mu a} \Gamma^a{}_{\sigma\nu} \right) \tag{10.80}$$

Eq.(10.80) is the subsidiary condition for the Evans wave equation (10.30), which in general relativity must be solved simultaneously with Eq.(10.80). In the limit of special relativity however, there is only one equation to solve - the Dirac equation - because:

$$g^{\mu\sigma}\left(\Gamma^{\nu}{}_{\mu a}\omega^{a}{}_{\sigma\nu}+\omega^{\nu}{}_{\mu a}\Gamma^{a}{}_{\sigma\nu}\right)=\left(\frac{mc}{\hbar}\right)^{2}. \tag{10.81}$$

In conclusion the Evans lemma and wave equation have been derived rigorously form Cartan geometry and the Einstein equation (10.29). As inferred by Einstein (10.17) the latter must be interpreted as being valid for all fields, not only the gravitational field. Eq.(10.29) is more fundamental than the more well known Einstein field equation (10.25) because various field equation can be constructed [1]– [12] from Eq. (10.29).

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