TENSOR TAYLOR SERIES METHOD FOR VACUUM EFFECTS.

by

M. W. Evans and H. Eckardt,

Civil List and AIAS / UPITEC,

(www.archive.com, www.webarchive.org.uk, www.aias.us, www.upitec.org, www.et3m.net)

ABSTRACT

The tensorial Taylor series is defined in detail, and applied to compute the effect of vacuum fluctuations on various laws of physics. The tensor Taylor series applies to scalars, and in consequence a scalar potential of any kind can be used directly. The effect of the vacuum on a vector field of force is computed through the scalar components. The Lamb shift is an example of the method when the scalar potential is the Coulomb potential, and where the vacuum fluctuations take effect at second order in the Taylor series. The vacuum affects the Newtonian inverse square law at fourth and sixth orders of the Taylor series. The whole of physics can be developed for vacuum effects using this method.

Keywords: ECE2 theory, tensor Taylor series, vacuum effects in physics.

UFT 397

1. INTRODUCTION

In recent papers of this series {1 - 41} the effect of vacuum fluctuations on material matter has been investigated using a development of the well known theory of the Lamb shift, in which isotropically averaged vacuum fluctuations are considered. The method is applied in Section 2 to the Coulomb potential, the Newtonian inverse square law and the dipole magnetic flux density. A new and generally applicable law of physics is inferred. Physics as it is currently understood is a theory that is developed in the hypothetical absence of the vacuum. However, what is actually observed experimentally in any part of physics always includes the influence of the vacuum, which must therefore always be taken into account. For example, the Schroedinger or Dirac theories of atomic hydrogen are theories developed in the absence of any consideration of the vacuum, and result in the energy degeneracy of the 2S and 2P states. The vacuum lifts this energy degeneracy giving the well known Lamb shift. The fully developed theory must include the effect of the vacuum.

Similarly, Newtonian universal gravitation is a theory developed in the absence of any consideration of the vacuum. In this theory the inverse square law of gravitational attraction of m and M results in a conic section orbit of m about M, notably the elliptical orbit. However, what is actually observed is a precessing ellipse, and the precession must be due to the effect of the vacuum (also known as the aether or spacetime). In section 2 it is shown that after isotropic averaging the vacuum affects the inverse square law at fourth, sixth and higher orders in the tensor Taylor series. Therefore the vacuum must produce orbital precession, because the latter is what is observed experimentally as is well known. Finally, the effect of the vacuum on the magnetic dipole potential and field is considered. The theory in the absence of the vacuum gives a well known fine and hyperfine spectral structure, which must therefore be changed by the vacuum in an experimentally measurable way.

In all these theories the vector spin connection of ECE2 can be computed.

This paper (UFT397) is a short synopsis of detailed calculations in the notes accompanying UFT397 on www.aias.us and www.upitec.org. The notes must be read with the paper. Note 397(1) (labelled 396(1) for historical reasons, but attached to UFT397) gives a detailed description of the meaning of the tensorial Taylor series, a very powerful and fundamental method. Note 397(2) applies the tensor Taylor series to the Coulomb law to give the Lamb shift. This note is a baseline note that defines the way in which the theory must be developed in other areas of physics. Note 397(3) applies the theory to Newtonian universal gravitation, Note 397(4) applies it to the magnetic dipole potential and field, and Note 397(5) gives numerical calculations.

Section 3 is a description of the computational methods used, notably the methods used in isotropic averaging. The tensor Taylor expansion rapidly becomes intricates ins structure, so the computer is needed at an early stage.

2. TENSOR TAYLOR SERIES AND APPLICATIONS

Consider the effect of a vacuum fluctuation $\delta \underline{r}$ on any scalar function f. The tensor Taylor series in components format is:

$$\Delta f = f(r+\delta r) - f(r) = \frac{\partial f}{\partial r^j}(\delta r)^j + \frac{1}{2!}\frac{\partial^2 f}{\partial r^j \partial r^k}(\delta r)^j(\delta r)^k + \frac{1}{3!}\frac{\partial^3 f}{\partial r^j \partial r^k \partial r^l}(\delta r)^j(\delta r)^k(\delta r)^l + \frac{1}{4!}\frac{\partial^4 f}{\partial r^j \partial r^k \partial r^l \partial r^m}(\delta r)^j(\delta r)^k(\delta r)^l(\delta r)^m + \cdots \quad -(1)$$

in which there is summation over repeated indices . Therefore the first term is

$$\Delta f^{(1)} = \frac{\partial f}{\partial r^1}(\delta r)^1 + \frac{\partial f}{\partial r^2}(\delta r)^2 + \frac{\partial f}{\partial r^3}(\delta r)^3$$

$$= \delta X\frac{\partial f}{\partial X} + \delta Y\frac{\partial f}{\partial Y} + \delta Z\frac{\partial f}{\partial Z} = \delta \underline{r}\cdot\underline{\nabla} f \quad -(2)$$

where:

$$\delta \underline{r} = \delta X \underline{i} + \delta Y \underline{j} + \delta Z \underline{k} \quad -(3)$$

and

$$\underline{\nabla} f = \frac{\partial f}{\partial X} \underline{i} + \frac{\partial f}{\partial Y} \underline{j} + \frac{\partial f}{\partial Z} \underline{k}. \quad -(4)$$

The second term is:

$$\Delta f^{(2)} = \frac{1}{2!}\left[\frac{\partial^2 f}{\partial r^1 \partial r^k}(\delta r)^1(\delta r)^k + \frac{\partial^2 f}{\partial r^2 \partial r^k}(\delta r)^2(\delta r)^k + \frac{\partial^2 f}{\partial r^3 \partial r^k}(\delta r)^3(\delta r)^k\right]. \quad -(5)$$

Now sum over the k index to give:

$$\begin{aligned}
\Delta f^{(2)} = \frac{1}{2!}\Big[& \frac{\partial^2 f}{\partial r^1 \partial r^1}(\delta r)^1(\delta r)^1 + \frac{\partial^2 f}{\partial r^1 \partial r^2}(\delta r)^1(\delta r)^2 + \frac{\partial^2 f}{\partial r^1 \partial r^3}(\delta r)^1(\delta r)^3 \\
& + \frac{\partial^2 f}{\partial r^2 \partial r^1}(\delta r)^2(\delta r)^1 + \frac{\partial^2 f}{\partial r^2 \partial r^2}(\delta r)^2(\delta r)^2 + \frac{\partial^2 f}{\partial r^2 \partial r^3}(\delta r)^2(\delta r)^3 \\
& + \frac{\partial^2 f}{\partial r^3 \partial r^1}(\delta r)^3(\delta r)^1 + \frac{\partial^2 f}{\partial r^3 \partial r^2}(\delta r)^3(\delta r)^2 + \frac{\partial^2 f}{\partial r^3 \partial r^3}(\delta r)^3(\delta r)^3\Big] \\
= \frac{1}{2!}\Big[& (\delta X)^2 \frac{\partial^2 f}{\partial X^2} + (\delta X)(\delta Y)\frac{\partial^2 f}{\partial X \partial Y} + (\delta X)(\delta Z)\frac{\partial^2 f}{\partial X \partial Z} \\
& + (\delta Y)(\delta X)\frac{\partial^2 f}{\partial Y \partial X} + (\delta Y)^2 \frac{\partial^2 f}{\partial Y^2} + (\delta Y)(\delta Z)\frac{\partial^2 f}{\partial Y \partial Z} \\
& + (\delta Z)(\delta X)\frac{\partial^2 f}{\partial Z \partial X} + (\delta Z)(\delta Y)\frac{\partial^2 f}{\partial Z \partial Y} + (\delta Z)^2 \frac{\partial^2 f}{\partial Z^2}\Big] \\
= & \left(\delta X \frac{\partial}{\partial X} + \delta Y \frac{\partial}{\partial Y} + \delta Z \frac{\partial}{\partial Z}\right)\left(\delta X \frac{\partial f}{\partial X} + \delta Y \frac{\partial f}{\partial Y} + \delta Z \frac{\partial f}{\partial Z}\right) \\
= & \left(\delta \underline{r} \cdot \underline{\nabla}\right)\left(\delta \underline{r} \cdot \underline{\nabla} f\right) \\
= & \left(\delta \underline{r} \cdot \underline{\nabla}\right)\left(\left(\delta \underline{r} \cdot \underline{\nabla}\right) f\right) := \left(\delta \underline{r} \cdot \underline{\nabla}\right)^2 f
\end{aligned}$$

$$-(6)$$

The notation $(\delta \underline{r} \cdot \underline{\nabla})^2$ means "$\delta \underline{r} \cdot \underline{\nabla}$ operating on $\delta \underline{r} \cdot \underline{\nabla}$ operating on f". Having checked that the tensorial Taylor series (1) gives the vector Taylor expansion:

$$f(\underline{r} + \delta \underline{r}) - f(\underline{r}) = (\delta \underline{r} \cdot \underline{\nabla}) f(\underline{r}) + \frac{1}{2!} (\delta \underline{r} \cdot \underline{\nabla})^2 f(\underline{r}) + \frac{1}{3!} (\delta \underline{r} \cdot \underline{\nabla})^3 f(\underline{r}) + \frac{1}{4!} (\delta \underline{r} \cdot \underline{\nabla})^4 f(\underline{r}) + \ldots \quad -(7)$$

it is possible to proceed to the evaluation of higher order terms with the help of computer algebra, and then to apply isotropic averaging.

The result is a general and powerful method of calculating the effect of the vacuum on any scalar function f.

The clearest way to apply isotropic averaging is to use the Cartesian component results:

$$\langle \delta x \rangle = \langle \delta y \rangle = \langle \delta z \rangle = 0, \quad -(8)$$

$$\langle \delta x \, \delta y \rangle = \langle \delta x \, \delta z \rangle = \langle \delta y \, \delta z \rangle = 0, \quad -(9)$$

$$\langle \delta x^2 \rangle = \langle \delta y^2 \rangle = \langle \delta z^2 \rangle = \frac{1}{3} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle. \quad -(10)$$

It follows that:

$$\langle \Delta f^{(1)} \rangle = \langle \delta x \rangle \frac{\partial f}{\partial x} + \langle \delta y \rangle \frac{\partial f}{\partial y} + \langle \delta z \rangle \frac{\partial f}{\partial z} = 0 \quad -(11)$$

and:

$$\langle \Delta f^{(2)} \rangle = \frac{1}{2!} \left(\langle \delta x^2 \rangle \frac{\partial^2 f}{\partial x^2} + \langle \delta y^2 \rangle \frac{\partial^2 f}{\partial y^2} + \langle \delta z^2 \rangle \frac{\partial^2 f}{\partial z^2} \right)$$

$$= \frac{1}{6} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \nabla^2 f. \quad -(12)$$

This is a very useful and important result because the effect of the vacuum on any scalar function f can be calculated by finding $\langle \delta \underline{r} \cdot \delta \underline{r} \rangle$ for the vacuum. If f is the Coulomb potential, Eq. (12) gives a precise explanation of the Lamb shift as is well known, so there can be great confidence in applying this method to the rest of physics.

The meaning of the condensed notation (7) is by no means clear, and tensor notation as in Eq. (1) is not used by anyone but a minority of physicists. The clearest notation is therefore component notation. For example in Cartesian components:

$$
\begin{aligned}
\Delta f = {} & \delta x \frac{\partial f}{\partial x} + \delta y \frac{\partial f}{\partial y} + \delta z \frac{\partial f}{\partial z} \\
& + \frac{1}{2!}\left(\delta x \frac{\partial}{\partial x} + \delta y \frac{\partial}{\partial y} + \delta z \frac{\partial}{\partial z}\right)\left(\delta x \frac{\partial f}{\partial x} + \delta y \frac{\partial f}{\partial y} + \delta z \frac{\partial f}{\partial z}\right) \\
& + \frac{1}{3!}\left(\delta x \frac{\partial}{\partial x} + \delta y \frac{\partial}{\partial y} + \delta z \frac{\partial}{\partial z}\right)\left[\left(\delta x \frac{\partial}{\partial x} + \delta y \frac{\partial}{\partial y} + \delta z \frac{\partial}{\partial z}\right)\left(\delta x \frac{\partial f}{\partial x} + \delta y \frac{\partial f}{\partial y} + \delta z \frac{\partial f}{\partial z}\right)\right] \\
& + \frac{1}{4!}\left(\delta x \frac{\partial}{\partial x} + \delta y \frac{\partial}{\partial y} + \delta z \frac{\partial}{\partial z}\right)\left[\left(\delta x \frac{\partial}{\partial x} + \delta y \frac{\partial}{\partial y} + \delta z \frac{\partial}{\partial z}\right)\right. \\
& \left.\left[\left(\delta x \frac{\partial}{\partial x} + \delta y \frac{\partial}{\partial y} + \delta z \frac{\partial}{\partial z}\right)\left(\delta x \frac{\partial f}{\partial x} + \delta y \frac{\partial f}{\partial y} + \delta z \frac{\partial f}{\partial z}\right)\right]\right] \\
& + \cdots
\end{aligned}
\qquad -(13)
$$

and this can be developed with computer algebra to eliminate human error. This is carried out in Section 3.

If f is the Coulomb potential:

$$
f = -\frac{e^2}{4\pi \epsilon_0 r} \qquad -(14)
$$

between the electron and proton in the H atom, the Lamb shift may be calculated with great accuracy, by using the Dirac delta function $\delta_D(r)$ as follows:

$$
\nabla^2\left(\frac{1}{r}\right) = -4\pi\, \delta_D(r). \qquad -(15)
$$

It follows from Eq. (15) that:

$$\langle \Delta\phi^{(2)} \rangle = \frac{1}{6} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \left\langle \nabla^2 \left(-\frac{e^2}{4\pi\epsilon_0 r} \right) \right\rangle \quad -(16)$$

in which the expectation value is:

$$\left\langle \nabla^2 \left(-\frac{e^2}{4\pi\epsilon_0 r} \right) \right\rangle = -\frac{e^2}{4\pi\epsilon_0} \int \psi^*(r) \nabla^2 \left(\frac{1}{r} \right) \psi(r) \, dr = \frac{e^2}{\epsilon_0} |\psi(0)|^2 \quad -(17)$$

where ψ is the relevant wave function of atomic H. The wave functions of the Schroedinger theory are used as discussed further in Note 397(5). So the well known Schroedinger H atom is the theory in the absence of the vacuum.

In this theory, isotropic averaging of the vacuum fluctuations is used. The result is

$$\langle \Delta f^{(2)} \rangle = \frac{1}{2!} \left\langle \left(\delta X \frac{\partial}{\partial X} + \delta Y \frac{\partial}{\partial Y} + \delta Z \frac{\partial}{\partial Z} \right) \left(\delta X \frac{\partial f}{\partial X} + \delta Y \frac{\partial f}{\partial Y} + \delta Z \frac{\partial f}{\partial Z} \right) \right\rangle$$

$$= \frac{1}{6} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle \nabla^2 f \quad -(18)$$

to second order in the tensor Taylor series, in which we have used:

$$\langle \delta X^2 \rangle = \langle \delta Y^2 \rangle = \langle \delta Z^2 \rangle = \frac{1}{3} \langle \delta \underline{r} \cdot \delta \underline{r} \rangle. \quad -(19)$$

Third order terms of the tensor Taylor series vanish upon isotropic averaging, but at fourth order:

$$\langle \Delta f^{(4)} \rangle = \frac{1}{4!} \left\langle \left(\delta X \frac{\partial}{\partial X} + \delta Y \frac{\partial}{\partial Y} + \delta Z \frac{\partial}{\partial Z} \right) \left[\left(\delta X \frac{\partial}{\partial X} + \delta Y \frac{\partial}{\partial Y} + \delta Z \frac{\partial}{\partial Z} \right) \right. \right.$$

$$\left. \left. \left[\left(\delta X \frac{\partial}{\partial X} + \delta Y \frac{\partial}{\partial Y} + \delta Z \frac{\partial}{\partial Z} \right) \left(\delta X \frac{\partial f}{\partial X} + \delta Y \frac{\partial f}{\partial Y} + \delta Z \frac{\partial f}{\partial Z} \right) \right] \right] \right\rangle \quad -(20)$$

So the complete correction is:

$$\Delta f = \frac{1}{2!} \langle \Delta f^{(2)} \rangle + \frac{1}{4!} \langle \Delta f^{(4)} \rangle + \frac{1}{6!} \langle \Delta f^{(6)} \rangle + \cdots \quad -(21)$$

The usual theory of the Lamb shift uses:

$$\Delta f = \frac{1}{2!} \langle \Delta f^{(2)} \rangle \quad -(22)$$

and mode theory to give the constant result in which α is the fine structure constant

$$\langle \delta\underline{r} \cdot \delta\underline{r} \rangle = \frac{1}{2\epsilon_0 \pi^2} \frac{e^2}{\hbar c} \left(\frac{\hbar}{mc} \right)^2 \log_e \frac{4\epsilon_0 \hbar c}{e^2} = 8\pi\alpha \lambda_c^2 \log_e \frac{1}{\pi\alpha} \quad -(23)$$

and

$$\lambda_c = \frac{\hbar}{mc} \quad -(24)$$

is the Compton wavelength of the electron. In the H atom:

$$\psi_{2S}(0) = \frac{1}{(8\pi a_0^3)^{1/2}} \quad -(25)$$

and

$$\psi_{2P}(0) = 0 \quad -(26)$$

where a_0 is the Bohr radius:

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{me^2} = 5.29177 \times 10^{-11} \text{ m} \quad -(27)$$

So the degeneracy of 2S and 2P is removed by the vacuum, giving a result that can be measured experimentally with great precision. Note 397(5) puts numbers on these formulae, to show that the Lamb shift is a small but measurable effect:

$$\frac{\langle \delta\underline{r} \cdot \delta\underline{r} \rangle^{1/2}}{\langle r(2S) \rangle} = 1.611 \times 10^{-4}; \quad \frac{\langle \Delta U \rangle (2S)}{E(2S)} = 6.23 \times 10^{-7}. \quad -(28)$$

The above is a famous radiative correction for the effect of the vacuum. The Schroedinger or Dirac H atom are theories worked out in the hypothetical absence of the vacuum. The vacuum however, is ubiquitous and never absent. The subject of physics in its present state of development is worked out in the hypothetical absence of the vacuum.

Consider for example the Hooke / Newton inverse square law of gravitational attraction between masses m and M:

$$\underline{F} = -m\underline{M}G\frac{\underline{r}}{r^3} \qquad -(29)$$

Here G is Newton's constant. The law seems to have been first inferred by Hooke and thereafter greatly developed by Newton. Eq. (29) takes no account of the effect of the vacuum, and in Cartesian component form is:

$$F_X = -mMG\frac{X}{(X^2+Y^2+Z^2)^{3/2}} \qquad -(30)$$

$$F_Y = -mMG\frac{Y}{(X^2+Y^2+Z^2)^{3/2}} \qquad -(31)$$

$$F_Z = -mMG\frac{Z}{(X^2+Y^2+Z^2)^{3/2}} \qquad -(32)$$

The isotropically averaged change in these components due to vacuum fluctuations

$$\underline{r} \rightarrow \underline{r} + \delta\underline{r} \qquad -(33)$$

are, from Eq. (13)

$$\langle \Delta F_X \rangle = \Big\langle \frac{1}{2!}\left(\delta X\frac{\partial}{\partial X} + \delta Y\frac{\partial}{\partial Y} + \delta Z\frac{\partial}{\partial Z}\right)\left(\delta X\frac{\partial F_X}{\partial X} + \delta Y\frac{\partial F_X}{\partial Y} + \delta Z\frac{\partial F_X}{\partial Z}\right) + \frac{1}{4!}\left(\delta X\frac{\partial}{\partial X} + \delta Y\frac{\partial}{\partial Y} + \delta Z\frac{\partial}{\partial Z}\right)\left[\left(\delta X\frac{\partial}{\partial X} + \delta Y\frac{\partial}{\partial Z} + \delta Z\frac{\partial}{\partial Z}\right)\left[\left(\delta X\frac{\partial}{\partial X} + \delta Y\frac{\partial}{\partial Y} + \delta Z\frac{\partial}{\partial Z}\right)\left(\delta X\frac{\partial F_X}{\partial X} + \delta Y\frac{\partial F_X}{\partial Y} + \delta Z\frac{\partial F_X}{\partial Z}\right)\right]\right]\Big\rangle \qquad -(34)$$

$$+ \cdots$$

with similar expressions for $\langle \Delta F_Y \rangle$ and $\langle \Delta F_Z \rangle$.

The change in $\underline{F}$ due to the vacuum is therefore:

$$\underline{\Delta F} = \underline{F}(\underline{r}+\delta \underline{r}) - \underline{F}(\underline{r}) = \Delta F_X \, \underline{i} + \Delta F_Y \, \underline{j} + \Delta F_Z \, \underline{k} \quad -(35)$$

so:

$$\underline{F}(\underline{r}+\delta\underline{r}) = -mMG\frac{\underline{r}}{r^3} + \underline{\Delta F} \quad -(36)$$

is the force of attraction with vacuum effects considered, the force between m and M with vacuum effects considered. As shown in Section 3 using computer algebra, there are no second order effects of the vacuum on the inverse square law (29), but terms to fourth and sixth order and higher order correct the inverse square law for the effect of the vacuum. So the elliptical orbit of Newton is changed by the vacuum. It is well known that the observed orbit is a precessing ellipse, so the vacuum must produce a precessing ellipse. The vacuum fluctuations after isotropic averaging must be such as to produce the precessing ellipse for any object in the universe. Other precessions must also be due to the vacuum, producing a completely new cosmology.

In rigorous theory the force law (36) must produce the precessing orbit by modifying the well known Newtonian method. In order to see how precession can emerge, the results of Note 377(4) can be used, in which the ECE2 lagrangian:

$$\mathcal{L} = -mc^2\left(1 - \frac{\dot{\underline{r}}\cdot\dot{\underline{r}}}{c^2}\right)^{1/2} + \frac{mMG}{|\underline{r}|} \quad -(37)$$

when used with the Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial \underline{r}} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}} \quad -(38)$$

gives the force law:

$$\underline{F} = m\ddot{\underline{r}} = -mMG\frac{\underline{r}}{r^3}\left(1 - \frac{v_0^2}{c^2}\right)^{3/2} \quad -(39)$$

in which the relativistic momentum is

$$\underline{p} = \frac{\partial \mathcal{L}}{\partial \dot{\underline{r}}} = \gamma m \underline{v}_0 \quad -(40)$$

and in which $\underline{v}_0$ is the Newtonian velocity. In UFT377 it was shown that Eq. (39) gives a precessing orbit.

In the solar system:

$$v_0 \ll c \quad -(41)$$

so:

$$\underline{F} = -\frac{mMG\underline{r}}{r^3}\left(1 - \frac{v_0^2}{c^2}\right)^{3/2} \sim -\frac{mMG\underline{r}}{r^3}\left(1 - \frac{3}{2}\frac{v_0^2}{c^2}\right) \quad -(42)$$

Comparing Eqs. (36) and (42):

$$\Delta \underline{F} = \frac{3}{2}\frac{v_0^2}{c^2}\frac{mMG\underline{r}}{r^3} \quad -(43)$$

will give a precessing orbit, Q. E. D. In the first instance the isotropically averaged fluctuations at fourth and sixth orders in the tensor Taylor expansion can be adjusted to give the experimentally observed precession. The theory can be refined by using Monte Carlo or molecular dynamics type computer simulations of the vacuum to compute the fluctuations, assuming that the vacuum is made up of vacuum particles as in previous UFT papers.

Finally consider the effect of the vacuum on the well known magnetic vector potential:

$$\underline{A}_0 = \frac{\mu_0}{4\pi}\frac{\underline{m} \times \underline{r}}{r^3} \quad -(44)$$

where μ_0 is the vacuum permeability and $\underline{m}$ is the magnetic dipole moment. In this case the tensor Taylor series is applied to the three scalar components of $\underline{A}_0$, for example the Cartesian components. As shown in Section 3, the vacuum begins to affect the magnetic vector potential at order four in the Taylor expansion. There are no second order effects but there are higher order effects which can be observed experimentally with fine and hyperfine spectral structure.

In the standard model of physics the magnetic flux density $\underline{B}_0$ is defined by:

$$\underline{B}_0 = \underline{\nabla} \times \underline{A}_0 \quad -(45)$$

but in ECE2 theory:

$$\underline{B} = \underline{\nabla} \times \underline{A}_0 - \underline{\omega} \times \underline{A}_0 \quad -(46)$$

where $\underline{\omega}$ is the vector spin connection which defines the vacuum flux density:

$$\underline{B}(\text{vac}) = -\underline{\omega} \times \underline{A}_0 . \quad -(47)$$

So ECE2 theory automatically considers the vacuum correction:

$$\Delta \underline{B}_0 = \underline{B}(\text{vac}) = \underline{B} - \underline{B}_0 . \quad -(48)$$

From vector analysis the complete dipole magnetic flux density is:

$$\underline{B}_0 = \frac{\mu_0}{4\pi r^3}\left(3\,\underline{m}\cdot\frac{\underline{r}\,\underline{r}}{r^2} - \underline{m}\right) \quad -(49)$$

which in the Cartesian coordinate system is:

$$\underline{B}_0 = \frac{\mu_0}{4\pi}\left[\frac{3(m_X X + m_Y Y + m_Z Z)}{(X^2+Y^2+Z^2)^{5/2}}\left(X\underline{i} + Y\underline{j} + Z\underline{k}\right) - \frac{\left(m_X\underline{i} + m_Y\underline{j} + m_Z\underline{k}\right)}{(X^2+Y^2+Z^2)^{3/2}}\right] \quad -(50)$$

For the X component for example the correction due to the vacuum is:

$$\langle \Delta B_{0X} \rangle = \Big\langle \frac{1}{2!}\left(\delta X \frac{\partial}{\partial X} + \delta Y \frac{\partial}{\partial Y} + \delta Z \frac{\partial}{\partial Z}\right)\left(\delta X \frac{\partial B_{0X}}{\partial X} + \delta Y \frac{\partial B_{0Y}}{\partial Y} + \delta Z \frac{\partial B_{0Z}}{\partial Z}\right)$$

$$+ \frac{1}{4!}\left(\delta X \frac{\partial}{\partial X} + \delta Y \frac{\partial}{\partial Y} + \delta Z \frac{\partial}{\partial Z}\right)\left[\left(\delta X \frac{\partial}{\partial X} + \delta Y \frac{\partial}{\partial Y} + \delta Z \frac{\partial}{\partial Z}\right)\right.$$

$$\left.\left[\left(\delta X \frac{\partial}{\partial X} + \delta Y \frac{\partial}{\partial Y} + \delta Z \frac{\partial}{\partial Z}\right)\left(\delta X \frac{\partial B_{0X}}{\partial X} + \delta Y \frac{\partial B_{0X}}{\partial Y} + \delta Z \frac{\partial B_{0X}}{\partial Z}\right)\right]\right] \Big\rangle$$

$$+ \cdots \qquad -(51)$$

and the complete correction in three dimensions is:

$$\langle \Delta \underline{B}_0 \rangle = \langle \Delta B_{0X} \rangle \underline{i} + \langle \Delta B_{0Y} \rangle \underline{j} + \langle \Delta B_{0Z} \rangle \underline{k}. \qquad -(52)$$

However, from Eq. (48):

$$\langle \Delta \underline{B}_0 \rangle = \langle \underline{B}(vac) \rangle = -\underline{\omega} \times \underline{A}_0 \qquad -(53)$$

so the isotropically averaged vacuum magnetic flux density is:

$$\langle \underline{B}(vac) \rangle = -\underline{\omega} \times \underline{A}_0 \qquad -(54)$$

and the spin connection vector can be found.

The vacuum effects show up as small shifts in spin spin fine and hyperfine structure, for example in ESR and NMR, so can be measured experimentally.

3. NUMERICAL AND GRAPHICAL ANALYSIS

Section by co author Horst Eckardt

Tensor Taylor series method for vacuum effects

M. W. Evans[*], H. Eckardt[†]
Civil List, A.I.A.S. and UPITEC

(www.webarchive.org.uk, www.aias.us,
www.atomicprecision.com, www.upitec.org)

3 Numerical and graphical analysis

3.1 General formulas

The detailed Taylor series of a function of vector arguments was given in Eq. (13). The isotropic averaging of $\Delta f(\mathbf{r})$ can be performed on this abstract level, giving the 2nd order Taylor terms

$$\langle \Delta f \rangle^{(2)} = \frac{\langle \delta\mathbf{r} \cdot \delta\mathbf{r} \rangle}{6} \left(\frac{d^2}{dX^2} f + \frac{d^2}{dY^2} f + \frac{d^2}{dZ^2} f \right), \tag{55}$$

the 4th order terms

$$\langle \Delta f \rangle^{(4)} = \frac{\left\langle (\delta\mathbf{r} \cdot \delta\mathbf{r})^2 \right\rangle}{216} \left(\frac{d^4}{dX^4} f + \frac{d^4}{dY^4} f + \frac{d^4}{dZ^4} f + 6 \left(\frac{d^4}{dY^2 dZ^2} f + \frac{d^4}{dX^2 dZ^2} f + \frac{d^4}{dX^2 dY^2} f \right) \right) \tag{56}$$

and the 6th order terms

$$\begin{aligned} \langle \Delta f \rangle^{(6)} = \frac{\left\langle (\delta\mathbf{r} \cdot \delta\mathbf{r})^3 \right\rangle}{19440} & \left(\frac{d^6}{dX^6} f + \frac{d^6}{dY^6} f + \frac{d^6}{dZ^6} f \right. \\ & + 15 \left(\frac{d^6}{dY^4 dZ^2} f + \frac{d^6}{dY^2 dZ^4} f + \frac{d^6}{dX^4 dZ^2} f + \frac{d^6}{dX^4 dY^2} f + \frac{d^6}{dX^2 dZ^4} f + \frac{d^6}{dX^2 dY^4} f \right) \\ & \left. +90 \frac{d^6}{dX^2 dY^2 dZ^2} f \right). \end{aligned} \tag{57}$$

In the following we consider some examples.

3.2 Vector potential of a magnetic dipole

The vector potential of a magnetic dipole has been given by Eq. (44). The vector components of $\mathbf{A}_0$ experience vacuum corrections of fourth order onward. The

[*]email: emyrone@aol.com
[†]email: mail@horst-eckardt.de

X component in the first two non-vanishing orders is

$$\langle \Delta A_X \rangle^{(4)} = \left\langle (\delta\mathbf{r}\cdot\delta\mathbf{r})^2 \right\rangle \frac{35Y\left(3Z^4 - 5Y^2 Z^2 - 3X^2 Z^2 + Y^4 - 5X^2 Y^2 + 3X^4\right)}{18(X^2+Y^2+Z^2)^{\frac{11}{2}}}, \tag{58}$$

$$\langle \Delta A_X \rangle^{(6)} = \frac{\left\langle (\delta\mathbf{r}\cdot\delta\mathbf{r})^3 \right\rangle}{9(X^2+Y^2+Z^2)^{\frac{15}{2}}} \cdot \tag{59}$$
$$7X\left(8Z^6 - 75Y^2 Z^4 - 15X^2 Z^4 - 75Y^4 Z^2 + 300X^2 Y^2 Z^2 \right.$$
$$\left. -21X^4 Z^2 + 8Y^6 - 15X^2 Y^4 - 21X^4 Y^2 + 2X^6\right).$$

The undistorted vector potential $\mathbf{A}_0$ has been graphed in Fig. 1 for constant values of $\delta\mathbf{r}$. The averaged vacuum terms $\langle \Delta A_X \rangle^{(4)}$ and $\langle \Delta A_X \rangle^{(6)}$ are plotted in Figs. 2 and 3. According to their degree of approxmiation, they have a high symmetry. There is a behaviour like a source field potential at the centre. When both contributions are added to the total potential of Fig. 1, the result of Fig. 4 emerges. In the central part the corrections due to vacuum fluctuations dominate the structure, representing a multipole expansion of the vacuum terms.

3.3 Coulomb potential

The isotropically averaged, non-vanishing fluctuations of the normalized Coulomb potential

$$U = -\frac{1}{(X^2+Y^2+Z^2)^{1/2}} \tag{60}$$

are

$$\langle \Delta U \rangle^{(4)} = \left\langle (\delta\mathbf{r}\cdot\delta\mathbf{r})^2 \right\rangle \frac{7\left(Z^4 - 3Y^2 Z^2 - 3X^2 Z^2 + Y^4 - 3X^2 Y^2 + X^4\right)}{18(Z^2+Y^2+X^2)^{\frac{9}{2}}} \tag{61}$$

and

$$\langle U \rangle^{(6)} = \frac{\left\langle (\delta\mathbf{r}\cdot\delta\mathbf{r})^3 \right\rangle}{9(X^2+Y^2+Z^2)^{\frac{15}{2}}} \cdot \tag{62}$$
$$7X\left(2Z^6 - 15Y^2 Z^4 - 15X^2 Z^4 - 15Y^4 Z^2 + 180X^2 Y^2 Z^2 \right.$$
$$\left. -15X^4 Z^2 + 2Y^6 - 15X^2 Y^4 - 15X^4 Y^2 + 2X^6\right).$$

These are formally similar to those of the vector potential components. Both contributions are graphed in Fig. 5 together with the Coulomb potential (in X direction). The fourth-order fluctuation is repulsive, while the sixth-order fluctuation is attractive like the original potential. All components together (Fig. 6) give a correction to the potential which leads to a steeper descent near to the central charge.

3.4 Gravitational force

The normalized gravitational force (identical to Coulomb force)

$$\mathbf{F} = -\frac{\mathbf{r}}{(X^2+Y^2+Z^2)^{3/2}} \tag{63}$$

leads to the non-vanishing Taylor series fluctuations (X component)

$$\langle \Delta F_X \rangle^{(4)} = \left\langle (\delta\mathbf{r}\cdot\delta\mathbf{r})^2 \right\rangle \frac{35X\left(3Z^4 - 3Y^2\,Z^2 - 5X^2\,Z^2 + 3Y^4 - 5X^2\,Y^2 + X^4\right)}{18(Z^2+Y^2+X^2)^{\frac{11}{2}}} \tag{64}$$

and

$$\begin{aligned}\langle \Delta F_X \rangle^{(6)} = & \frac{\left\langle (\delta\mathbf{r}\cdot\delta\mathbf{r})^3 \right\rangle}{9(X^2+Y^2+Z^2)^{\frac{15}{2}}}\cdot \\ & 7X\,\left(8Z^6 - 75Y^2\,Z^4 - 15X^2\,Z^4 - 75Y^4\,Z^2 + 300X^2\,Y^2\,Z^2\right. \\ & \left.-21X^4\,Z^2 + 8Y^6 - 15X^2\,Y^4 - 21X^4\,Y^2 + 2X^6\right).\end{aligned} \tag{65}$$

The fluctuations (Fig. 7) are similar in sign to those of the potential, but the force field (Fig. 8) has a pronounced saddle. Also in this case the vacuum fluctuations effect a broadening of the central region, like an effective central mass with extended radius, not a point mass.

As stated in section 2, relativistic corrections to Newton's force law (42) can be expanded for small velocities v_0 to a second-order correction in v_0/c:

$$\Delta\mathbf{F} \approx \frac{3}{2}\frac{v_0^2}{c^2}\frac{mMG}{r^3}\mathbf{r}. \tag{66}$$

This term has to be equal to the Taylor expansion of $\Delta\mathbf{F}$ whose first two non-vanishing terms are given by Eqs. (64, 65). Correctly, one has to use

$$-\frac{mMG}{r^3}\mathbf{r}\left(1-\frac{v_0^2}{c^2}\right)^{3/2} = -\frac{mMG}{r^3}\mathbf{r} + \langle \Delta\mathbf{F} \rangle^{(4)} + \langle \Delta\mathbf{F} \rangle^{(6)} + ... \tag{67}$$

The exponential term at the left hand side can be expressed by Newton's generalized binomial theorem:

$$\left(1-\frac{v_0^2}{c^2}\right)^{3/2} = 1 + \frac{7{v_0}^{12}}{1024c^{12}} + \frac{3{v_0}^{10}}{256c^{10}} + \frac{3{v_0}^{8}}{128c^{8}} + \frac{{v_0}^{6}}{16c^{6}} + \frac{3{v_0}^{4}}{8c^{4}} - \frac{3{v_0}^{2}}{2c^{2}} + \ldots. \tag{68}$$

Using the terms of lowest order at the left hand side, we have

$$-\frac{mMG}{r^3}\mathbf{r}\left(1 - \frac{3}{2}\frac{v_0^2}{c^2} + \frac{3}{8}\frac{v_0^4}{c^4} + \frac{1}{16}\frac{v_0^6}{c^6} + ...\right) = \langle \Delta\mathbf{F} \rangle^{(4)} + \langle \Delta\mathbf{F} \rangle^{(6)} + ... \tag{69}$$

The right hand side gives a dependence on $\delta\mathbf{r}\cdot\delta\mathbf{r}$ so that in principle it is possible to compute the size of fluctuations δr from the orbit. Please note that the result depends on the orbital velocity v_0 which is given by the orbital relation $v_0(\mathbf{r})$. This relation is known if the orbital dynamics is known, for example from a Lagrange solution of (42). Resolving Eq. (69) for δr gives a polynomial of degree 6 in the case above. There is a real-valued solution for $(\delta r)^2$ but the terms are such complicated that this method of determining δr cannot be handled even by computer algebra with reasonable effort.

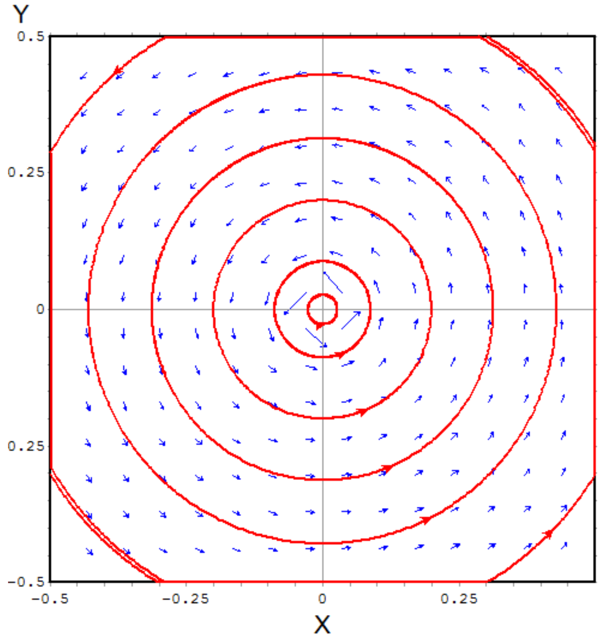

Figure 1: Undistorted vector potential $\mathbf{A}_0$ of a magnetic dipole field.

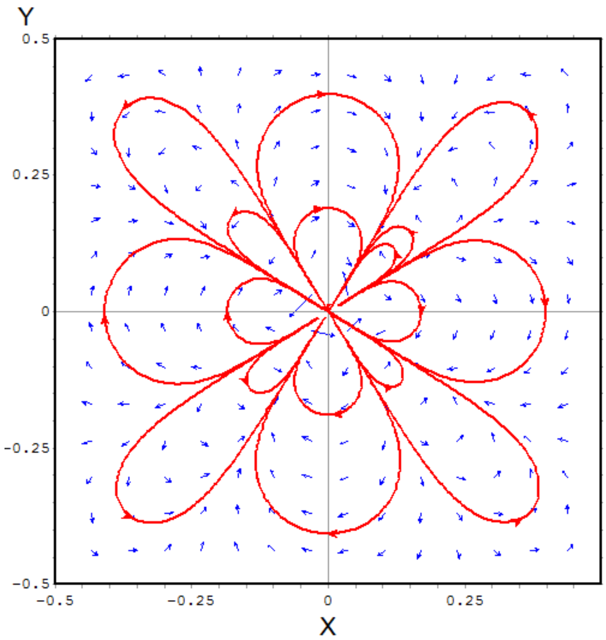

Figure 2: fluctuations of 4th order of the magnetic vector potential.

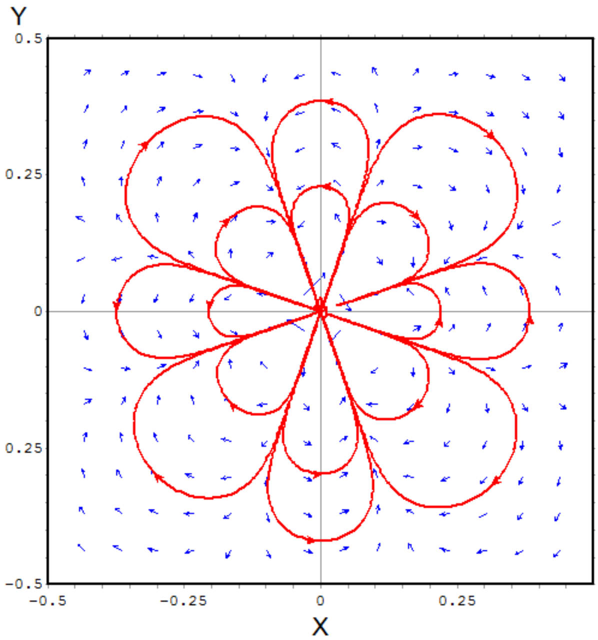

Figure 3: fluctuations of 6th order of the magnetic vector potential.

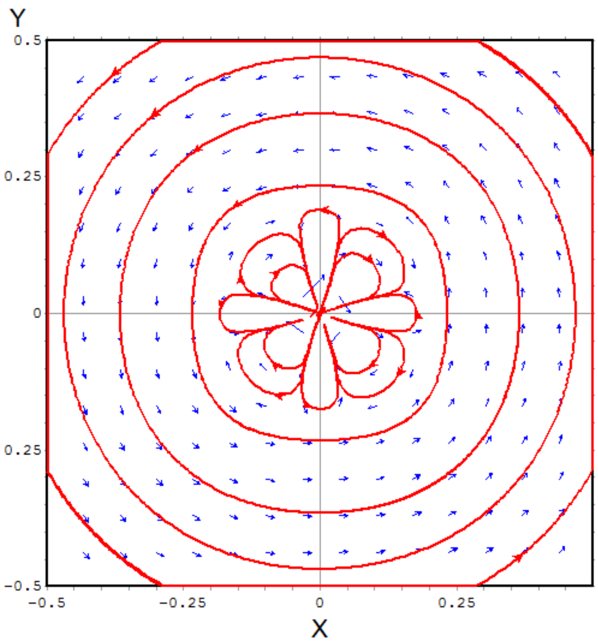

Figure 4: Total magnetic vector potential with fluctuation terms.

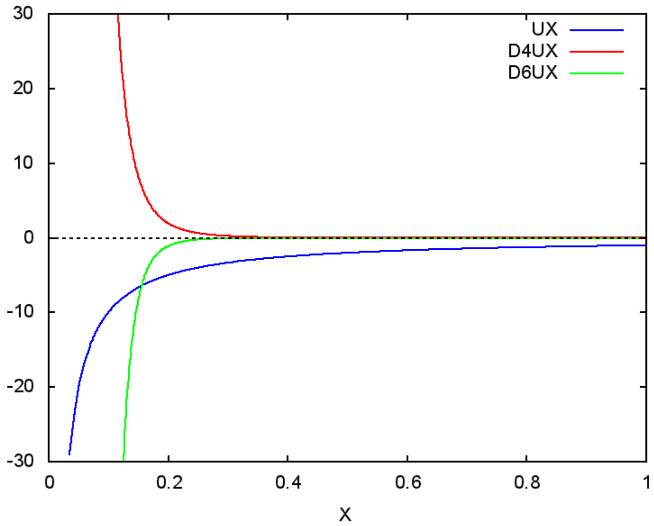

Figure 5: Coulomb potential and 4th order and 6th order fluctuations.

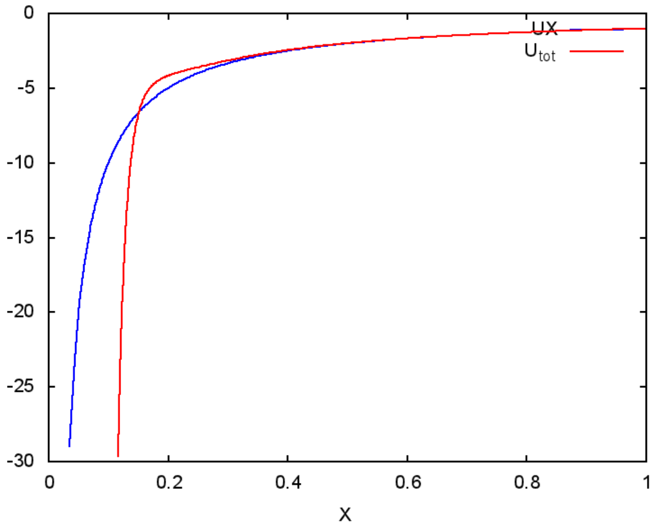

Figure 6: Total Coulomb potential with fluctuations.

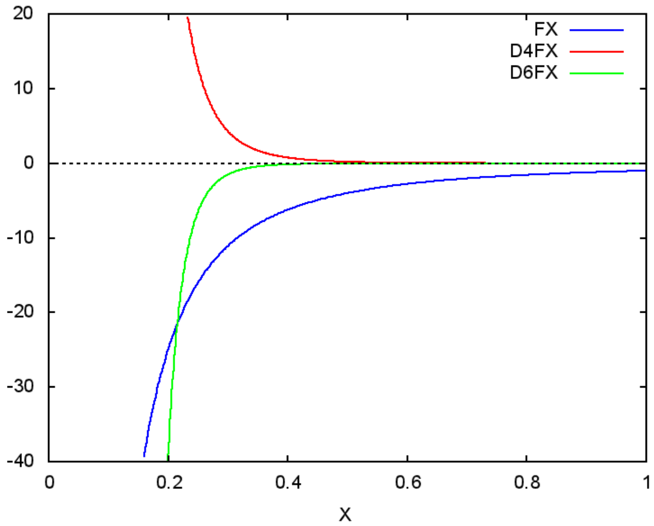

Figure 7: Gravitational force and 4th order and 6th order fluctuations.

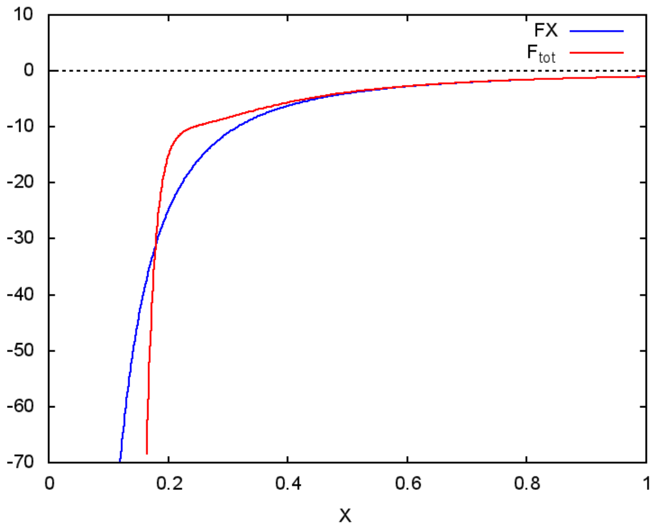

Figure 8: Total gravitational force with fluctuations.

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and the staff of AIAS and others for many interesting discussions. Dave Burleigh, CEO of Annexa Inc., is thanked for voluntary posting, site maintenance and feedback maintenance. Alex Hill is thanked for many translations, and Robert Cheshire nd Michael Jackson for broadcasting and video preparation.

REFERENCES

{1} M. W. Evans, H. Eckardt, D. W. Lindstrom, D. J. Crothers and U. E. Bruchholtz, "Principles of ECE Theory, Volume Two" (ePubli, Berlin 2017).

{2} M. W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "Principles of ECE Theory, Volume One" (New Generation, London 2016, ePubli Berlin 2017).

{3} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (UFT301 on www.aias.us and Cambridge International 2010).

{4} M. W. Evans, H. Eckardt and D. W. Lindstrom "Generally Covariant Unified Field Theory" (Abramis 2005 - 2011, in seven volumes softback, open access in various UFT papers, combined sites www.aias.us and www.upitec.org).

{5} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis 2007, open access as UFT302, Spanish translation by Alex Hill).

{6} H. Eckardt, "The ECE Engineering Model" (Open access as UFT203, collected equations).

{7} M. W. Evans, "Collected Scientometrics" (open access as UFT307, New Generation, London, 2015).

{8} M .W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific 2001, open access in the Omnia Opera section of www.aias.us).

{9} M . W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley Interscience, New York, 1992, 1993, 1997 and 2001) in two editions and six volumes, hardback, softback and e book.

{10} M. W. Evans and J. - P. Vigier, "The Enigmatic Photon" (Kluwer, Dordrecht, 1994 to 1999) in five volumes hardback and five volumes softback, open source in the Omnia Opera Section of www.aias.us).

{11} M. W. Evans, Ed. "Definitive Refutations of the Einsteinian General Relativity" (Cambridge International Science Publishing, 2012, open access on combined sites).

{12} M. W. Evans, Ed., J. Foundations of Physics and Chemistry (Cambridge International Science Publishing).

{13} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory (World Scientific 1974).

{14} G. W. Robinson, S. Singh, S. B. Zhu and M. W. Evans, "Water in Biology, Chemistry and Physics" (World Scientific 1996).

{15} W. T. Coffey, M. W. Evans, and P. Grigolini, "Molecular Diffusion and Spectra" (Wiley Interscience 1984).

{16} M. W. Evans, G. J. Evans, W. T. Coffey and P. Grigolini", "Molecular Dynamics and the Theory of Broad Band Spectroscopy (Wiley Interscience 1982).

{17} M. W. Evans, "The Elementary Static Magnetic Field of the Photon", Physica B, 182(3), 227-236 (1992).

{18} M. W. Evans, "The Photon's Magnetic Field: Optical NMR Spectroscopy" (World Scientific 1993).

{19} M. W. Evans, "On the Experimental Measurement of the Photon's Fundamental Static Magnetic Field Operator, B(3): the Optical Zeeman Effect in Atoms", Physica B, 182(3), 237 - 143 (1982).

{20} M. W. Evans, "Molecular Dynamics Simulation of Induced Anisotropy: I Equilibrium Properties" , J. Chem. Phys., 76, 5473 - 5479 (1982).

{21} M. W. Evans, "A Generally Covariant Wave Equation for Grand Unified Theory" Found. Phys. Lett., 16, 513 - 547 (2003).

{22} M. W. Evans, P. Grigolini and P. Pastori-Parravicini, Eds., "Memory Function Approaches to Stochastic Problems in Condensed Matter" (Wiley Interscience, reprinted 2009).

{23} M. W. Evans, "New Phenomenon of the Molecular Liquid State: Interaction of Rotation and Translation", Phys. Rev. Lett., 50, 371, (1983).

{24} M .W. Evans, "Optical Phase Conjugation in Nuclear Magnetic Resonance: Laser NMR Spectroscopy", J. Phys. Chem., 95, 2256-2260 (1991).

{25} M. W. Evans, "New Field induced Axial and Circular Birefringence Effects" Phys. Rev. Lett., 64, 2909 (1990).

{26} M. W. Evans, J. - P. Vigier, S. Roy and S. Jeffers, "Non Abelian Electrodynamics", "Enigmatic Photon V olume 5" (Kluwer, 1999)

{27} M. W. Evans, reply to L. D. Barron "Charge Conjugation and the Non Existence of the Photon's Static Magnetic Field" , Physica B, 190, 310-313 (1993).

{28} M. W. Evans, "A Generally Covariant Field Equation for Gravitation and Electromagnetism" Found. Phys. Lett., 16, 369 - 378 (2003).

{29} M. W. Evans and D. M. Heyes, "Combined Shear and Elongational Flow by Non Equilibrium Electrodynamics", Mol. Phys., 69, 241 - 263 (1988).

{30} Ref. (22), 1985 printing.

{31} M. W. Evans and D. M. Heyes, "Correlation Functions in Couette Flow from Group Theory and Molecular Dynamics", Mol. Phys., 65, 1441,- 1453 (1988).

{32} M. W. Evans, M. Davies and I. Larkin, Molecular Motion and Molecular Interaction in the Nematic and Isotropic Phases of a Liquid Crystal Compound", J. Chem. Soc. Faraday II, 69, 1011-1022 (1973).

{33} M. W. Evans and H. Eckardt, "Spin Connection Resonance in Magnetic Motors", Physica B., 400, 175 - 179 (2007).

{34} M. W. Evans, "Three Principles of Group Theoretical Statistical Mechanics", Phys. Lett. A, 134, 409 - 412 (1989).

{35} M. W. Evans, "On the Symmetry and Molecular Dynamical Origin of Magneto Chiral Dichroism: "Spin Chiral Dichroism in Absolute Asymmetric Synthesis" Chem. Phys. Lett., 152, 33 - 38 (1988).

{36} M. W. Evans, "Spin Connection Resonance in Gravitational General Relativity", Acta Physica Polonica, 38, 2211 (2007).

{37} M. W. Evans, "Computer Simulation of Liquid Anisotropy, III. Dispersion of the Induced Birefringence with a Strong Alternating Field", J. Chem. Phys., 77, 4632-4635 (1982).

{38} M. W. Evans, "The Objective Laws of Classical Electrodynamics, the Effect of Gravitation on Electromagnetism" J. New Energy Special Issue (2006).

{39} M. W. Evans, G. C. Lie and E. Clementi, "Molecular Dynamics Simulation of Water from 10 K to 1273 K", J. Chem. Phys., 88, 5157 (1988).

{40} M. W. Evans, "The Interaction of Three Fields in ECE Theory: the Inverse Faraday Effect" Physica B, 403, 517 (2008).

{41} M. W. Evans, "Principles of Group Theoretical Statistical Mechanics", Phys. Rev., 39, 6041 (1989).