Chapter 9

Resonant Counter Gravitation

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Abstract

Generally covariant unified field theory has been used to show that the equations of classical electrodynamics are unified with those of gravitation using standard Cartan geometry (Einstein Cartan Evans (ECE) field theory). By expressing the ECE field equations in terms of the potential field, linear inhomogeneous field equations are obtained for each of the fundamental laws of electrodynamics unified with gravitation. These equations have resonant solutions, and in this paper the possibility of resonant counter gravitation is demonstrated by showing that the Riemann curvature can be affected by the electromagnetic field. Examples are the Coulomb law and Ampère law respectively of electro-statics and magneto-statics. At resonance the effect is greatly amplified (as for any resonant phenomenon), so in theory, circuits can be built for effective resonant counter gravitation and used in the aerospace industry.

Keywords: Resonant counter gravitation; Einstein Cartan Evans (ECE) field theory; generally covariant unified field theory; linear inhomogeneous differential equations; resonance.

9.1 Introduction

The principle of general relativity is the fundamental hallmark of objective physics, a natural philosophy that is independent of the observer, independent of subjective input. The principle means that every equation of physics has to be generally covariant, meaning that it must retain its form under any type of coordinate transformation. The principle must evidently be applied to all equations of physics, including electrodynamics. Only in this way can an objective unified theory of physics emerge - a generally covariant unified field theory [1]. It is well established [2] that the principle of general relativity as applied to gravitational theory by Einstein and Hilbert [3] is very accurate when compared with experimental data, but the principle of general relativity is not applied to electrodynamics in the standard model. In the latter [4] electrodynamics is a theory of special relativity in which the field is thought of as an entity independent of the frame. The space-time of electrodynamics in the standard model is the Minkowski ("flat") space-time. As a result standard model electrodynamics is not generally covariant, it is Lorentz covariant, and as such cannot be unified with generally covariant gravitational theory in the standard model. It is well known that Riemann geometry with the Christoffel connection is the geometrical basis of gravitational general relativity. However in this type of geometry the torsion tensor is zero [5]. It was first suggested by Cartan [6] that the electromagnetic field be the torsion form of Cartan geometry. In 2003 [7]– [40] a generally covariant unified field theory was developed using this suggestion and using standard Cartan geometry. It has since been developed in many directions [1, 7]– [40].

In Section 9.2 the field equations of ECE theory are expressed as linear inhomogeneous equations with resonant solutions. The Riemann term is isolated and it is shown that the electromagnetic part of the unified field can change the Riemann curvature, i.e. change the gravitational field. At resonance this effect is greatly amplified. In Section 9.3 this general conclusion is exemplified using the Coulomb and Ampère laws unified with gravitation. This means that a static electric or static magnetic configuration can change the gravitational field. In order to maximize the effect numerical methods of solution are needed to model a circuit which optimizes resonant counter gravitation. An assembly of such circuits can be placed aboard a device such as an aircraft or spacecraft, and is expected to be particularly effective in regions of near zero gravitation in outer space. Under the usual laboratory conditions it is well known that the electromagnetic and gravitational fields are essentially independent and have no influence on each other. This is observed experimentally in the Coulomb and Newton inverse square laws for example. If two charged masses are considered, then changing the charge on one of them has no effect on the Newton inverse square law. Similarly changing the mass of one of them has no effect on the Coulomb inverse square law. However, it is known through the Eddington effect that gravitation and electromagnetism interact and ECE theory was the first to give a classical explanation of the Eddington effect [7]– [40]. Einstein's famous prediction was based on photon mass and a semi-classical treatment. The Eddington effect is however tiny in magnitude, the enormous mass of the sun bends grazing light by a few seconds of arc only. Therefore resonant counter gravitation is the only practical method of counter gravitation. All claims to have observed an effect of electromagnetism on gravitation without resonance are almost certainly artifactual. Recently however the Mexican group of AIAS have observed resonantly enhanced electric power from ECE spacetime, the output power from a circuit was observed reproducibly [41] to exceed input power by a factor of one hundred thousand. This has been explained using ECE theory by the use of linear inhomogeneous differential equations of the same type as used in this paper for counter gravitation. The two phenomena are explained by a generally covariant unified field theory based on Cartan geometry.

9.2 The Resonance Equations Of ECE Field Theory

The overall aim of this section is to develop the ECE field equations to define the effect of electromagnetism on gravitation. In order to do this the Riemann term is isolated on the right hand side of the following field equations:

$$d \wedge F + \omega \wedge F = A^{(0)} R \wedge q, \tag{9.1}$$

$$d \wedge \widetilde{F} + \omega \wedge \widetilde{F} = A^{(0)} \widetilde{R} \wedge q, \tag{9.2}$$

$$F = d \wedge A + \omega \wedge A. \tag{9.3}$$

In these equations we have used a notation [1] which suppresses the various indices on the quantities on the left and right hand sides. This concise notation is used to reveal the basic structure of the equations. Later they will be developed into differential form, tensor and vector notation. Here F denotes the electromagnetic field, ω the spin connection, R the Riemann curvature and q the tetrad. The symbol $\wedge$ denotes Cartan's wedge product. The tilde denotes the Hodge dual [1] and A the potential field. Finally $A^{(0)}$ is the proportionality constant between F and the Cartan torsion:

$$F = A^{(0)} T \tag{9.4}$$

which is the ECE ansatz [1]. So Eqs. 9.1 and 9.2 balance electromagnetic terms on the left hand side and on the right hand side a gravitational term $R \wedge q$ multiplied by $A^{(0)}$. Using Eq. 9.3 in Eq. 9.1 gives a linear inhomogeneous equation:

$$d \wedge (d \wedge A + \omega \wedge A) + \omega \wedge (d \wedge A + \omega \wedge A) = A^{(0)} R \wedge q, \tag{9.5}$$

with resonance solutions [42]. Therefore resonance amplification of the effect of electromagnetism on $R \wedge q$ is possible in general relativity. In the standard model gravitation is described by:

$$R \wedge q = 0 \tag{9.6}$$

which is the Ricci cyclic equation [1] of Einstein Hilbert field theory. In tensorial notation the Ricci cyclic equation is:

$$R_{\sigma\mu\nu\rho} + R_{\sigma\rho\mu\nu} + R_{\sigma\nu\rho\mu} = 0 \tag{9.7}$$

where $R_{\sigma\mu\nu\rho}$ is the index lowered Riemann curvature tensor. Eq. 9.6 or equivalently Eq. 9.7 are true if and only if the Christoffel connection is assumed:

$$\Gamma^{\kappa}{}_{\mu\nu} = \Gamma^{\kappa}{}_{\nu\mu}. \tag{9.8}$$

The assumption 9.8 implies that the torsion tensor is zero:

$$T^{\kappa}{}_{\mu\nu} = \Gamma^{\kappa}{}_{\mu\nu} - \Gamma^{\kappa}{}_{\nu\mu} = 0. \tag{9.9}$$

In the standard model the spin connection is missing because the Minkowski frame is not spinning, and so in the standard model:

$$d \wedge F = 0, \tag{9.10}$$

$$d \wedge \widetilde{F} = \mu_0 J, \tag{9.11}$$

$$F = d \wedge A. \tag{9.12}$$

Using Eq. 9.12 in Eq. 9.10 gives the Poincaré Lemma:

$$d \wedge (d \wedge A) = 0 \tag{9.13}$$

which does not have resonance solutions. In order to understand the influence of electromagnetism on gravitation numerical solutions of Eq. 9.5 are needed. At resonance the effect is greatly amplified. In the standard model J is not recognized as originating in elements of the Riemann tensor. The interaction between electromagnetism and gravitation is defined by:

$$R \wedge q \neq 0 \tag{9.14}$$

and by a non-zero and asymmetric spin connection. Both conditions are needed. It is important to note that for rotational motion, as for example in a free space electromagnetic field in ECE theory [1], the spin connection is dual to the tetrad:

$$\omega^a{}_b = -\frac{\kappa}{2}\epsilon^a{}_{bc} q^c \tag{9.15}$$

where κ is a wave-number and $\epsilon^a{}_{bc}$ is the index raised Levi-Civita tensor in the tangent space-time. Eq. 9.15 implies that the Cartan torsion tensor is dual to the Riemann tensor:

$$R^a{}_b = -\frac{\kappa}{2}\epsilon^a{}_{bc} T^c. \tag{9.16}$$

So it must be clearly understood that there is a Riemann spin tensor for free electromagnetism in ECE field theory. There is also a Riemann tensor for gravitation, the well known curvature Riemann tensor. When electromagnetism and gravitation are mutually influential $R \wedge q$ is not zero, and Eqs. 9.15 and 9.16 no longer apply. This is the condition needed for resonant counter gravitation. If $R \wedge q$ is zero then electromagnetism does not influence gravitation. Similarly if the spin connection is dual to the tetrad there is no mutual influence, and when the Cartan torsion is dual to the Riemann spin tensor, there is no mutual influence. These conclusions follow directly from Cartan geometry. For the free electromagnetic field, the homogeneous field equation [1] reduces to:

$$d \wedge F^a = 0 \tag{9.17}$$

which for each polarization index a, and using vector notation, gives the Gauss law applied to magnetism:

$$\nabla \cdot \mathbf{B}^a = 0 \tag{9.18}$$

and the Faraday law of induction:

$$\boldsymbol{\nabla} \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mathbf{0} \tag{9.19}$$

The inhomogeneous field equation of ECE theory [1] is:

$$d \wedge \widetilde{F} = A^{(0)} \left(\widetilde{R} \wedge q - \omega \wedge \widetilde{T} \right). \tag{9.20}$$

When the electromagnetic field is free of gravitation:

$$\left(\widetilde{R} \wedge q\right)_{e/m} = \left(\omega \wedge \widetilde{T}\right)_{e/m} \tag{9.21}$$

and the inhomogeneous field equation 9.20 reduces to:

$$d \wedge \widetilde{F} = A^{(0)} \left(\widetilde{R} \wedge q\right)_{grav} := \mu_0 J. \tag{9.22}$$

This means that the inhomogeneous current is derived from the mass of an electron in the Einstein Hilbert limit, i.e purely from the curving of space-time as described by the Schwarzschild metric [1]. When the electromagnetic and gravitational fields are mutually independent, there is no interaction between the spinning and curving of space-time. In this limit Eq. 9.22 gives for each index a the Coulomb Law:

$$\boldsymbol{\nabla} \cdot \mathbf{D}^a = \rho^a \tag{9.23}$$

and the Ampère Maxwell law:

$$\boldsymbol{\nabla} \cdot \mathbf{D}^a - \frac{\partial \mathbf{D}^a}{\partial t} = \mathbf{J}^a. \tag{9.24}$$

In the weak field limit of gravitation uninfluenced by electromagnetism the Newton inverse square law is also recovered.

In standard differential form notation Eq. 9.1 is:

$$d \wedge F^a + \omega^a{}_b \wedge F^b = A^{(0)} R^a{}_b \wedge q^b \tag{9.25}$$

and this in tensor notation is:

$$\begin{aligned} &\partial_\mu F^a{}_{\nu\rho} + \partial_\nu F^a{}_{\rho\mu} + \partial_\rho F^a{}_{\mu\nu} + \omega^a{}_{\mu b} F^b{}_{\nu\rho} + \omega^a{}_{\nu b} F^b{}_{\rho\mu} + \omega^a{}_{\rho b} F^b{}_{\mu\nu} \\ &\quad = A^{(0)} \left(R^a{}_{b\nu\mu} q^b{}_\rho + R^a{}_{b\rho\nu} q^b{}_\mu + R^a{}_{b\mu\rho} q^b{}_\nu\right). \end{aligned} \tag{9.26}$$

Now use:

$$R \wedge q = -q \wedge R \tag{9.27}$$

and the right hand side of Eq. 9.26 becomes:

$$-A^{(0)} \left(q^b{}_\mu R^a{}_{b\nu\rho} + q^b{}_\nu R^a{}_{b\rho\mu} + q^b{}_\rho R^a{}_{b\mu\nu}\right). \tag{9.28}$$

Eq. 9.27 is the same as:

$$\partial_\mu \widetilde{F}^{a\mu\nu} + \omega^a{}_{\mu b} \widetilde{F}^{a\mu\nu} = -A^{(0)} q^b{}_\mu \widetilde{R}^a{}_b{}^{\mu\nu}. \tag{9.29}$$

Similarly Eq. 9.2 becomes:

$$\partial_\mu F^{a\mu\nu} + \omega^a{}_{\mu b} F^{a\mu\nu} = -A^{(0)} q^b{}_\mu R^a{}_b{}^{\mu\nu}. \tag{9.30}$$

In the standard model Eq. 9.29 is:

$$\partial_\mu \widetilde{F}^{\mu\nu} = 0 \tag{9.31}$$

and Eq. 9.30 is:

$$\partial_\mu F^{\mu\nu} = \mu_0 J^\nu. \tag{9.32}$$

From Eq. 9.31 we obtain the Gauss law and Faraday law of induction of the standard model, and from Eq. 9.32 we obtain the standard model's Coulomb Law and Ampère Maxwell Law. In ECE theory these laws must be obtained from Eqs. 9.29 and 9.30. The details are given in Appendix K of ref. [1].

The Coulomb Law in vector notation in ECE theory will be derived and explained in detail later in this section. The result is:

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a + {\boldsymbol{\omega}^a}_b{}' \cdot \mathbf{E}^b = -c\mathbf{A}^{b'} \cdot {\mathbf{R}^a}_b. \tag{9.33}$$

Similarly the Ampère Maxwell Law in vector notation in ECE theory is:

$$\boldsymbol{\nabla} \times \mathbf{B}^a + {\boldsymbol{\omega}'^a}_b \times \mathbf{B}^b - \frac{1}{c^2}\left(\frac{\partial \mathbf{E}^a}{\partial t} + {\omega'^a}_{ob}\mathbf{E}^b\right) = \mu_0 \mathbf{J}^{a'}. \tag{9.34}$$

These laws are required to understand the effect of electromagnetism on gravitation, and to design devices for resonant counter gravitation. Gravitation is represented by the Riemann terms on the right hand sides of Eqs. 9.33 and 9.34, and electromagnetism by the terms on the left hand sides. The equations therefore show that elements of the Riemann tensor can be affected by electric and magnetic fields. The engineering challenge is to maximize the effect with resonance amplification. The latter possibility is seen by writing out Eq. 9.3 in vector notation [7]– [40]:

$$\mathbf{E}^a = -\frac{\partial \mathbf{A}^a}{\partial t} - c\boldsymbol{\nabla} A^{0a} - c{\omega^{0a}}_b\mathbf{A}^b + c{\boldsymbol{\omega}^a}_b A^{0b} \tag{9.35}$$

and

$$\mathbf{B}^a = \boldsymbol{\nabla} \times \mathbf{A}^a - {\boldsymbol{\omega}^a}_b \times \mathbf{A}^b. \tag{9.36}$$

By substituting Eqs. 9.35 and 9.36 into 9.33 and 9.34 linear inhomogeneous differential equations are obtained. The final step is to solve these numerically to design circuits that give resonance amplification of the effect of electromagnetism on gravitation. If the Riemann tensor is decreased, gravity is lessened, and conversely. This is a highly non-trivial problem in general and shows why previous attempts to understand this problem are naive. In the standard model these linear inhomogeneous differential equations are replaced by the d'Alembert wave equation [43] using the Lorentz gauge condition. The solutions are the Liennard- Wiechert potentials, and these are electromagnetic waves without resonance and without the information given by the interaction for gravitation and electromagnetism of ECE field theory.

The Ampère law of magneto-statics is obtained when there is no electric field present, only a magnetic field, so eq. 9.34 reduces to:

$$\boldsymbol{\nabla} \times \mathbf{B}^a + {\boldsymbol{\omega}'^a}_b \times \mathbf{B}^b = \mu_0 \mathbf{J}^{a'}. \tag{9.37}$$

As discussed in the introduction the standard model's Coulomb, Ampère and Newton inverse square laws hold to high precision [43]. Therefore no influence of electromagnetism on gravitation has hitherto been detected in the laboratory. The reason is that resonance amplification has not been used, and resonance amplification occurs only in general relativity, not in special relativity. However an influence of gravitation on electromagnetism has been detected in the Eddington effect. ECE field theory is the first self-consistent explanation of the

Eddington effect both on the classical and quantum levels. Einstein's famous prediction which led to the Eddington experiment was semi-classical and was based on Einstein Hilbert field theory, so used only gravitation and not the required unified field theory. Light in Einstein's prediction was a photon with mass, the concomitant electric and magnetic fields were not considered. The semi-classical theory happens to be very accurate for the solar system, [1], but in general effects are expected due to the interaction of gravitation and electromagnetism. These may occur not only in a cosmological context but also in an atom or a circuit on the microscopic scale. In the vicinity of an electron space-time curvature is large because of the small electron radius, and electric fields are intense, giving plenty of scope for the interaction of space-time spin and curvature.

The origin of Coulomb's law in ECE field theory is the inhomogeneous field equation [1]:

$$\partial_\mu F^{a\mu\nu} = \mu_0 J^{a\nu} = A^{(0)} \left(R^{a}{}_{b}{}^{\mu\nu} q^{b}{}_{\mu} - \omega^{a}{}_{\mu b} T^{b\mu\nu} \right) \tag{9.38}$$

and the law is obtained by using

$$\nu = 0 \tag{9.39}$$

in Eq. 9.38 to give:

$$\begin{aligned} &\partial_1 F^{a10} + \partial_2 F^{a20} + \partial_3 F^{a30} + \omega^{a}{}_{1b} T^{b10} + \omega^{a}{}_{2b} T^{b20} + \omega^{a}{}_{3b} T^{b30} \\ &\qquad = A^{(0)} \left(R^{a}{}_{b}{}^{10} q^{b}{}_{1} + R^{a}{}_{b}{}^{20} q^{b}{}_{2} + R^{a}{}_{b}{}^{30} q^{b}{}_{3} \right) \end{aligned} \tag{9.40}$$

which translates into the vector notation of Eq. 9.33. The primed quantities arise because the metric $g_{\mu\nu}$ must be used to raise and lower indices, so:

$$A^{a}{}_{\mu}{}' := g_{\mu\nu} A^{a\nu}, \tag{9.41}$$

$$\omega^{a}{}_{\mu b}{}' := g_{\mu\nu} \omega^{\nu a}{}_{b}, \tag{9.42}$$

where the unprimed quantities are defined by convention as metric free. Other conventions may be adopted, but in ECE theory the metric is not the Minkowski metric in general, so contra-variant and covariant quantities must be defined carefully. It is no longer sufficient just to switch the sign from positive (contravariant space part of a four-vector) to negative (covariant space part of a four-vector). These details must be programmed carefully in numerical applications.

The resonant version of Eq. 9.33 may now be developed from Eq. 9.35 substituted into Eq. 9.33 to give:

$$\begin{aligned} & c\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} A^{0a} + \boldsymbol{\nabla} \cdot \frac{\partial \mathbf{A}^a}{\partial t} + c\boldsymbol{\nabla} \cdot \left(\omega^{0a}{}_{b} \mathbf{A}^b - \boldsymbol{\omega}^{a}{}_{b} A^{0b} \right) \\ & + \boldsymbol{\omega}^{a}{}_{b}{}' \cdot \left(\frac{\partial \mathbf{A}^b}{\partial t} + c\boldsymbol{\nabla} A^{0b} + c\omega^{0b}{}_{c} \mathbf{A}^c - c\boldsymbol{\omega}^{b}{}_{c} A^{0c} \right) \\ & = -c\mathbf{A}^{b'} \cdot \mathbf{R}^{a}{}_{b}. \end{aligned} \tag{9.43}$$

If we restrict consideration to a static electric field configuration Eq. 9.43 simplifies as follows. In order to guide this simplification exercise consider first the standard model's static electric field:

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \rho / \epsilon_0, \tag{9.44}$$

$$\boldsymbol{\nabla} \times \mathbf{E} = 0, \tag{9.45}$$

$$\nabla^2 \Phi = -\rho/\epsilon_0, \tag{9.46}$$

where Φ is the scalar potential of the Poisson equation [43]. Eq. 9.45 implies that:

$$\mathbf{E} = -\boldsymbol{\nabla}\Phi. \tag{9.47}$$

For a time-dependent electric field:

$$\mathbf{E} = -\boldsymbol{\nabla}\Phi - \frac{\partial \mathbf{A}}{\partial t}, \tag{9.48}$$

so a static electric field means:

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{0}. \tag{9.49}$$

If we further assume for the sake of approximation that:

$$\boldsymbol{\nabla} \cdot \mathbf{A}^b = 0 \tag{9.50}$$

Eq. 9.43 simplifies to:

$$\begin{aligned} &\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} A^{0a} - \boldsymbol{\nabla} \cdot \left(\boldsymbol{\omega}^a{}_b A^{0b}\right) + \boldsymbol{\omega}^{a'}{}_b \cdot \boldsymbol{\nabla} A^{0b} \\ &+ \boldsymbol{\omega}^a{}_b{}' \cdot \boldsymbol{\omega}^{0b}{}_c \mathbf{A}^c - \boldsymbol{\omega}^a{}_b{}' \cdot \boldsymbol{\omega}^b{}_c A^{0c} = -\mathbf{A}^{b'} \cdot \mathbf{R}^a{}_b \\ &:= c\mu_0 J^{0a} \end{aligned} \tag{9.51}$$

This is still a complicated equation so to simplify further we consider the limit of weak interaction between the electromagnetic and gravitational fields [1]:

$$d \wedge \widetilde{F} = \mu_0 J \longrightarrow A^{(0)} \left(\widetilde{R} \wedge q\right)_{grav}. \tag{9.52}$$

In this limit:

$$\left(\widetilde{R} \wedge q\right)_{e/m} \sim \left(\omega \wedge \widetilde{T}\right)_{e/m}, \quad \widetilde{T}_{grav} \sim 0. \tag{9.53}$$

The structure of Eq. 9.33 simplifies to:

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a \sim -c\mathbf{A}^{b'} \cdot \mathbf{R}^a{}_b \tag{9.54}$$

and the spin connection in Eq. 9.35 can be considered to be approximately dual to the tetrad. So Eq. 9.43 simplifies further to

$$\boldsymbol{\nabla} \cdot \left(-\frac{\partial \mathbf{A}^a}{\partial t} - c\boldsymbol{\nabla} A^{0a} - c\omega^{0a}{}_b \mathbf{A}^b + c\boldsymbol{\omega}^a{}_b A^{0b}\right) \sim -c\mathbf{A}^{b'} \cdot \mathbf{R}^a{}_b \tag{9.55}$$

with

$$\omega^a{}_b \sim -\frac{\kappa}{2} q^c \epsilon^a{}_{bc}. \tag{9.56}$$

If we use a static electric field and asume Eq. 9.50, Eq. 9.55 simplifies to:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} A^{0a} - \boldsymbol{\nabla} \cdot \left(\boldsymbol{\omega}^a{}_b A^{0b}\right) \sim -\mathbf{A}^{b'} \cdot \mathbf{R}^a{}_b. \tag{9.57}$$

If we do not assume Eq. 9.50 we obtain:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} A^{0a} + \boldsymbol{\nabla} \cdot \left(\omega^{0a}{}_b \mathbf{A}^b\right) - \boldsymbol{\nabla} \cdot \left(\boldsymbol{\omega}^a{}_b A^{0b}\right) \sim -\mathbf{A}^{b'} \cdot \mathbf{R}^a{}_b. \tag{9.58}$$

Eq. 9.57 is a Hooke's Law type of resonance equation with a driving term on the right hand side, Eq. 9.58 has an additional damping term. In both cases gravitation is resonantly affected by a static electric field. In order for this to occur the spin connection must be identically non-zero, meaning that κ in Eq. 9.56 must be identically non-zero. In the limit of an identically static electric field, κ is identically zero, and we recover the standard model's Coulomb law. In so doing we lose the possibility of influencing gravitation with a static electric field.

The Ampère Maxwell law can be expressed [1] in ECE theory as:

$$\boldsymbol{\nabla} \times \mathbf{B}^a - \frac{1}{c^2}\frac{\partial \mathbf{E}^a}{\partial t} = \mu_0 \mathbf{J}^a \tag{9.59}$$

where

$$\mathbf{J}^a = J^a{}_x \mathbf{i} + J^a{}_y \mathbf{j} + J^a{}_z \mathbf{k}. \tag{9.60}$$

To isolate the Riemann term Eq. 9.61 is developed as:

$$\boldsymbol{\nabla} \times \mathbf{B}^a + \boldsymbol{\omega}'^a{}_b \times \mathbf{B}^b - \frac{1}{c^2}\left(\frac{\partial \mathbf{E}^a}{\partial t} + \omega'^a{}_{0b}\mathbf{E}^b\right) = \mu_0 \mathbf{J}^{a'} \tag{9.61}$$

where [1]:

$$J^a{}_x{}' = -\frac{A^{(0)}}{\mu_0}\left(R^a{}_0{}^{10} + R^a{}_2{}^{12} + R^a{}_3{}^{13}\right), \tag{9.62}$$

$$J^a{}_y{}' = -\frac{A^{(0)}}{\mu_0}\left(R^a{}_0{}^{20} + R^a{}_1{}^{21} + R^a{}_3{}^{23}\right), \tag{9.63}$$

$$J^a{}_z{}' = -\frac{A^{(0)}}{\mu_0}\left(R^a{}_0{}^{30} + R^a{}_1{}^{31} + R^a{}_2{}^{32}\right), \tag{9.64}$$

We first check Eq. 9.61 for units. We obtain in S.I.:

$$A^{(0)} = JsC^{-1}m^{-1} = voltsm^{-1}, \tag{9.65}$$

$$R = m^{-2}, \tag{9.66}$$

$$\mu_0 = Js^2C^{-2}m^{-1}, \quad J = Am^{-2} = Cs^{-1}m^{-2}. \tag{9.67}$$

When electromagnetism and gravitation are independent of each other the elements of the Riemann tensor appearing in Eq. 9.61 are precisely those of the Schwarzschild metric [1], elements which represent the curvature of space-time due to the mass of an electron or ensemble of electrons. However when electromagnetism and gravitation influence each other the elements of the Riemann tensor are changed, and this gives rise to the possibility of resonant counter gravitation.

The magneto-static Ampère law can be developed into a linear inhomogeneous differential equation by using Eq. 9.36 in Eq. 9.37 to give:

$$\begin{aligned} &\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{A}^a) - \boldsymbol{\nabla} \times \left(\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b\right) \\ &+\boldsymbol{\omega}'^a{}_b \times (\boldsymbol{\nabla} \times \mathbf{A}^a) - \boldsymbol{\omega}'^a{}_b \times \left(\boldsymbol{\omega}^b{}_c \times A^c\right) \\ &= \mu_0 \mathbf{J}^{a'}. \end{aligned} \tag{9.68}$$

This equation must be solved in general on a computer, but some simplifying assumptions may be made as for the Coulomb Law. The gravitational term on the right hand side of Eq. 9.68 is balanced by the magnetic terms on the left hand side. Eq. 9.68 reduces to the Ampère law of the standard model for each index a when the spin connection vanishes.

In tensor notation the Coulomb and Ampère Maxwell laws are given in resonant form by substituting

$$F^{a\mu\nu} = \partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + \omega^{a\mu}{}_b A^{b\nu} - \omega^{a\nu}{}_b A^{b\mu} \tag{9.69}$$

into Eq. 9.30 to give the linear inhomogeneous tensorial equation:

$$\begin{aligned}
&\Box A^{a\nu} - \partial^\nu \left(\partial_\mu A^{a\mu}\right) + \omega^{a\mu}{}_b \partial_\mu A^{b\nu} - \omega^{a\nu}{}_b \partial_\mu A^{b\mu} \\
&+ \omega^a{}_{\mu b} \left(\partial^\mu A^{b\mu} - \partial^\nu A^{b\mu}\right) + \left(\partial_\mu \omega^{a\mu}{}_b\right) A^{b\nu} - \left(\partial_\mu \omega^{a\nu}{}_b\right) A^{b\mu} \\
&+ \omega^a{}_{\mu b} \omega^{b\mu}{}_c A^{c\nu} - \omega^a{}_{\mu b} \omega^{b\nu}{}_c A^{c\mu} \\
&= -A^{(0)} R^a{}_b{}^{\mu\nu} q^b{}_\mu = -A^{(0)} R^{a\mu\nu}{}_\mu
\end{aligned} \tag{9.70}$$

in which the gravitational term on the right hand side is balanced by the electromagnetic terms on the left hand side. Eq. 9.70 is therefore the tensorial equivalent of Eq. 9.5.

9.3 Basic Definitions And Conventions For Numerical Solutions

The electric and magnetic fields in ECE theory are defined from Cartan geometry by Eqs. 9.35 and 9.36. In these equations the tetrad is defined by:

$$q^a{}_\mu{}' = g_{\mu\nu} q^{\nu a}. \tag{9.71}$$

For each index a the contravariant tetrad is defined as the four-vector:

$$\begin{aligned}
q^{\nu a} &= \left(q^{0a}, \mathbf{q}^a\right) \\
&= \left(q^{0a}, q^{1a}, q^{2a}, q^{3a}\right) \\
&= \left(q^{0a}, q^a{}_x, q^a{}_y, q^a{}_z\right).
\end{aligned} \tag{9.72}$$

Similarly the spin connection is defined by:

$$\omega^a{}_{\mu b}{}' = g_{\mu\nu} \omega^{\nu a}{}_b \tag{9.73}$$

adopting the convention:

$$\begin{aligned}
\omega^{\nu a}{}_b &= \left(\omega^{0a}{}_b, \boldsymbol{\omega}^a{}_b\right) \\
&= \left(\omega^{0a}{}_b, \omega^{1a}{}_b, \omega^{2a}{}_b, \omega^{3a}{}_b\right) \\
&= \left(\omega^{0a}{}_b, \omega^a{}_{xb}, \omega^a{}_{yb}, \omega^a{}_{zb}\right).
\end{aligned} \tag{9.74}$$

The role of the spin connection in ECE theory can be illustrated with reference to the fundamentally important Evans spin field $B^{(3)}$ [1] observed in the inverse Faraday effect. The spin connection means that electromagnetism is Cartan torsion, so the frame is spinning. Similarly gravitation is Riemann or Cartan curvature, so the frame is curving. A spinning or curving frame means that there must be a connection present. If there is no connection the space-time is Minkowski space-time, called flat" space-time, the space-time of special relativity. The spin field is defined in the complex circular basis [7]– [40] with polarization indices denoted by:

$$a = (1), (2), (3) \tag{9.75}$$

so:

$$\mathbf{q}^{(1)} \times \mathbf{q}^{(2)} = i\mathbf{q}^{(3)*}, \tag{9.76}$$

$$\mathbf{q}^{(2)} \times \mathbf{q}^{(3)} = i\mathbf{q}^{(1)*}, \tag{9.77}$$

$$\mathbf{q}^{(3)} \times \mathbf{q}^{(1)} = i\mathbf{q}^{(2)*}. \tag{9.78}$$

The spin field is a special case of:

$$\mathbf{B}^a{}_{spin} = -A^{(0)}\boldsymbol{\omega}^a{}_b \times \mathbf{q}^b \tag{9.79}$$

and exists only in general relativity. Its existence therefore shows that classical electrodynamics is a theory of general relativity, and not of special relativity. This is a fundamentally important finding, because the spin field is an experimental observable of the inverse Faraday effect, which therefore shows experimentally that classical electrodynamics is a theory of general relativity. After realizing this, the unification of electromagnetism with gravitation follows self-consistently from the rules of Cartan geometry, so the spin field is fundamentally important for the development of a generally covariant unified field theory and for the study of resonant counter gravitation. This point is emphasized here by some technical details as follows.

If we consider a circularly polarized electromagnetic field independent of gravitation, and use for example:

$$a = 3 \tag{9.80}$$

the spin field is:

$$\mathbf{B}^3 = -A^{(0)}\left(\boldsymbol{\omega}^3{}_1 \times \mathbf{q}^1 + \boldsymbol{\omega}^3{}_2 \times \mathbf{q}^2\right) \tag{9.81}$$

where summation over repeated covariant contravariant indices has been used, together with:

$$\boldsymbol{\omega}^3{}_3 = \mathbf{0}. \tag{9.82}$$

Eq. 9.82 follows because for circular polarization [1]:

$$\omega^a{}_{\mu b} = -\frac{\kappa}{2}\epsilon^a{}_{bc}q^c{}_\mu \tag{9.83}$$

where:

$$\epsilon^a{}_{bc} = \eta^{ad}\epsilon_{dbc} \tag{9.84}$$

and where η^{ad} is the Minkowski metric of the tangent space-time of Cartan geometry. Thus

$$\eta^{ad} = \eta_{ad} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \mathrm{diag}\,(-1, 1, 1, 1) \tag{9.85}$$

and

$$\begin{aligned} \epsilon_{dbc} &= 1, \text{even permutation} \\ &= -1, \text{odd permutation} \end{aligned} \tag{9.86}$$

i.e.

$$\epsilon_{123} = -\epsilon_{132} = 1 \quad \text{etc.}, \quad \epsilon^1{}_{23} = \epsilon 123 = 1 \quad \text{etc.} \tag{9.87}$$

Therefore the spin connection elements are:

$$\omega^1{}_2 = -\frac{\kappa}{2}\epsilon^1{}_{23}q^3 = -\frac{\kappa}{2}q^3, \tag{9.88}$$

$$\omega^3{}_1 = -\frac{\kappa}{2}\epsilon^3{}_{12}q^2 = -\frac{\kappa}{2}q^2, \tag{9.89}$$

$$\omega^3{}_2 = -\frac{\kappa}{2}\epsilon^3{}_{21}q^1 = -\frac{\kappa}{2}q^1, \tag{9.90}$$

and the spin field is [1]:

$$\mathbf{B}^3 = A^{(0)}\frac{\kappa}{2}\left(\mathbf{q}^2 \times \mathbf{q}^1 - \mathbf{q}^1 \times \mathbf{q}^2\right) = -A^{(0)}\kappa\mathbf{q}^1 \times \mathbf{q}^2. \tag{9.91}$$

Finally switch to the complex circular basis and use:

$$A^1 = A^{(0)}q^1, \quad A^2 = A^{(0)}q^2 \tag{9.92}$$

to find the original Evans spin field [1]:

$$\mathbf{B}^{(3)} = \mathbf{B}^{(3)*} = -i\frac{\kappa}{A^{(0)}}\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} = -ig\mathbf{A}^{(1)} \times \mathbf{A}^{(2)}. \tag{9.93}$$

Historically the spin field was proposed in 1992 [1] from the experimental existence of the conjugate product $\mathbf{A}^{(1)} \times \mathbf{A}^{(2)}$ in the inverse Faraday effect and developed in several ways [7]– [40] using gauge theory. The spin field was incorporated into a generally covariant unified field theory from 2003 onwards [1]. The gauge theoretical methods were replaced completely by the fully self consistent methods of Cartan geometry. In gauge theory the indices a are abstract entities, in the final generally covariant unified field theory they are indices of the tangent spacetime as in standard Cartan geometry and as such have a clearly defined geometrical role which is rigorously self-consistent and consistent with the principle of general relativity. Gauge theory on the other hand superimposes an abstract a index on a flat Minkowski space-time, and so gauge theory cannot lead to a generally covariant unified field theory. Gauge theory cannot be used to investigate the mutual interaction of gravitation and electromagnetism, and neither can string theory. Only a geometrically based theory can do this, and for self-consistently the theory must be one that is in accordance with the fundamental principle of general relativity.

In these equations the covariant derivative [1] is defined as usual by applying a correction to the flat four-derivative:

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \boldsymbol{\nabla}\right). \tag{9.94}$$

The contravariant flat space derivative is:

$$\partial^\mu = \eta^{\mu\nu}\partial_\nu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\boldsymbol{\nabla}\right) \tag{9.95}$$

and the flat space d'Alembertian operator is:

$$\Box = \partial^\mu \partial_\mu. \tag{9.96}$$

It follows that Eq. 9.95 must also be used to define ∂^μ in ECE space-time, defined as the four-dimensional space-time with curvature and torsion both present in general. The contravariant derivative in ECE space-time is:

$$D^\mu = g^{\mu\nu} D_\nu \tag{9.97}$$

but:

$$D^\mu = \partial^\mu + \cdots \tag{9.98}$$

therefore the d'Alembertian operator remains the same in ECE space-time, and is defined by:

$$D^\mu D_\mu = \Box + \cdots \tag{9.99}$$

Therefore $\Box$ is the "flat part" of $D^\mu D_\mu$. Similarly:

$$g^{\mu\nu} = \eta^{\mu\nu} + \cdots \tag{9.100}$$

and $\nu^{\mu\nu}$ is the "flat part" of $g^{\mu\nu}$. As discussed by Carroll [7]– [40] in his chapter 3, the derivative operator ∂_μ in flat space-time is a map from (k, l) tensor fields to (k, l+1) tensor fields, the derivative operator acts linearly on its arguments and obeys the Leibnitz Theorem for tensor products. The derivative operator D_μ of ECE theory therefore performs the functions of ∂_μ in a way that is independent of coordinates. This property is fundamentally required by general relativity. Since D_μ obeys the Leibnitz Theorem it may always be written as the ∂_μ plus a linear transformation. In Riemann geometry and for a given vector V^ν the covariant derivative is therefore:

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu{}_{\mu\lambda} V^\lambda \tag{9.101}$$

where $\Gamma^\nu{}_{\mu\lambda}$ is the connection. Thus ∂^μ in ECE theory is the same as Eq. 9.95, and ∂_μ in ECE theory is the same as Eq. 9.94. It follows that the d'Alembertian operator of ECE theory is defined by Eq. 9.96, i.e.:

$$\Box = \partial^\mu \partial_\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2. \tag{9.102}$$

In order to produce a numerical solution of the ECE field equation the differential operators must be defined as in flat space-time, i.e. as:

$$\begin{aligned} \partial_\mu &= \left(\frac{1}{c}\frac{\partial}{\partial t}, \boldsymbol{\nabla}\right) = (\partial_0, \partial_1, \partial_2, \partial_3) \\ &= \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \end{aligned} \tag{9.103}$$

and

$$\begin{aligned} \partial^\mu &= \left(\frac{1}{c}\frac{\partial}{\partial t}, -\boldsymbol{\nabla}\right) = \left(\partial^0, \partial^1, \partial^2, \partial^3\right) \\ &= \left(\frac{1}{c}\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right). \end{aligned} \tag{9.104}$$

With these definitions and conventions the ECE electromagnetic field tensor is:

$$F^{a\mu\nu} = -F^{a\nu\mu} = \partial^\mu A^{\nu a} - \partial^\nu A^{\nu a} + \omega^{\mu a}{}_b A^{b\nu} - \omega^{\nu a}{}_b A^{b\mu} \tag{9.105}$$

which compares with the standard model's:

$$F^{\mu\nu} = -F^{-\nu\mu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \tag{9.106}$$

The polarization index and spin connection are missing in the standard model. In S.I. units the following convention is adopted for the field tensor:

$$F^{a\mu\nu} = \begin{bmatrix} 0 & -E^{a1}/c & -E^{a2}/c & -E^{a3}/c \\ E^{a1}/c & 0 & -B^{a3} & B^{a2} \\ E^{a2}/c & B^{a3} & 0 & -B^{a1} \\ E^{a3}/c & -B^{a2} & B^{a1} & 0 \end{bmatrix}. \tag{9.107}$$

In this convention the units of the field tensor are those of magnetic flux density (tesla) or electric field strength (volt m^{-1}) divided by c. Other conventions for the field tensor may be used if preferred, provided that care is taken that all S.I. units are balanced on the right and left hand sides of any equation. In Eq. 9.107:

$$\begin{aligned} E^{a1} &= E^a{}_x, \quad E^{a2} = E^a{}_y, \quad E^{a3} = E^a{}_z, \\ B^{a1} &= B^a{}_x, \quad B^{a2} = B^a{}_y, \quad B^{a3} = B^a{}_z, \end{aligned} \tag{9.108}$$

thus:

$$F^{a01} = -E^{a1}/c = -F^{a10}, \tag{9.109}$$

$$F^{a02} = -E^{a2}/c = -F^{a20}, \tag{9.110}$$

$$F^{a03} = -E^{a3}/c = -F^{a30}, \tag{9.111}$$

and

$$F^{a12} = -F^{a21} = -B^{a3}, \tag{9.112}$$

$$F^{a13} = -F^{a31} = -B^{a2}, \tag{9.113}$$

$$F^{a23} = -F^{a32} = -B^{a1}. \tag{9.114}$$

Therefore:

$$\begin{aligned} F^{a01} &= -\frac{1}{c}E^{a1} = \partial^0 A^{a1} - \partial^1 A^{a0} + \omega^a{}_b{}^0 A^{b1} - \omega^a{}_b{}^1 A^{b0} \\ &= -\frac{1}{c}E^a{}_x = \frac{1}{c}\frac{\partial A^a{}_x}{\partial t} + \frac{\partial A^{a0}}{\partial x} + \omega^a{}_b{}^0 A^b{}_x - \omega^a{}_{xb}{}^1 A^{b0} \end{aligned} \tag{9.115}$$

from which we obtain Eq. 9.35. In the standard model (S. I. units):

$$\mathbf{E} = -\frac{\partial A}{\partial t} - c\boldsymbol{\nabla} A^0 := -\frac{\partial \mathbf{A}}{\partial t} - \boldsymbol{\nabla}\phi. \tag{9.116}$$

Similarly:

$$\begin{aligned} F^{a12} &= -B^{a3} = -B^a{}_z = \partial^1 A^{a2} - \partial^2 A^{a1} + \omega^a{}_b{}^1 A^{b2} - \omega^a{}_b{}^2 A^{b1} \\ &= -\frac{\partial A^a{}_y}{\partial x} + \frac{\partial A^a{}_x}{\partial y} + \omega^a{}_{xb} A^b{}_y - \omega^a{}_{yb} A^b{}_x. \end{aligned} \tag{9.117}$$

Now use the definition of the vector curl:

$$\boldsymbol{\nabla} \times \mathbf{A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{bmatrix} \tag{9.118}$$

and vector cross product:

$$\mathbf{A} \times \mathbf{B} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix} \tag{9.119}$$

to obtain Eq. 9.36.

The following displays give a summary of the translation of notation. In ECE theory:

$$\boxed{F = d \wedge A + \omega \wedge A} \rightarrow \boxed{\begin{aligned} \mathbf{E}^a &= -\frac{\partial \mathbf{A}^a}{\partial t} - c\boldsymbol{\nabla} A^{a0} \\ &\quad - c\omega^a{}_b{}^0 \mathbf{A}^b + c\boldsymbol{\omega}^a{}_b A^{b0}, \\ \mathbf{B}^a &= \boldsymbol{\nabla} \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times A^b. \end{aligned}} \tag{9.120}$$

In the standard model (S.I. units):

$$\boxed{F = d \wedge A} \rightarrow \boxed{\begin{aligned} \mathbf{E} &= -\frac{\partial \mathbf{A}}{\partial t} - \boldsymbol{\nabla}\phi \\ \mathbf{B} &= \boldsymbol{\nabla} \times \mathbf{A} \end{aligned}}. \tag{9.121}$$

In order to develop the resonance formulation of the Faraday law of induction in ECE field theory it is convenient to use the ECE Faraday law of induction in its dielectric form [7]– [40]:

$$\boldsymbol{\nabla} \times (\epsilon_r \mathbf{E}^a) + \frac{\partial}{\partial t}\left(\frac{\mathbf{B}^a}{\mu_r}\right) = \mathbf{0} \tag{9.122}$$

where μ_r and ϵ_r are respectively the relative permeability and permittivity of ECE space-time considered as a dielectric. The homogeneous current of Eq. 9.1 is re-defined in the dielectric formulation as:

$$\widetilde{\mathbf{j}}^a := \frac{\partial \mathbf{M}^a}{\partial t} - c^2 \boldsymbol{\nabla} \times \mathbf{P}^a \tag{9.123}$$

where the magnetization is:

$$\mathbf{M}^a = \left(\frac{1}{\mu_0} - \frac{1}{\mu}\right)\mathbf{B}^a \tag{9.124}$$

and the polarization is:

$$\mathbf{P}^a = (\epsilon - \epsilon_0)\,\mathbf{E}^a. \tag{9.125}$$

In Eqs. 9.123–9.125 and ϵ_0 respectively are the vacuum permeability and permittivity, and μ and ϵ are the permeability and permittivity of ECE space-time regarded as a dielectric. Using Eqs. 9.35 and 9.36 in Eq. 9.122 gives a resonance equation in the dielectric formulation. The magnetization is:

$$\mathbf{M}^a = A^{(0)} \left(\frac{1}{\mu_0} - \frac{1}{\mu} \right) \left(\boldsymbol{\nabla} \times \mathbf{q}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{q}^b \right) \tag{9.126}$$

and the polarization is:

$$\mathbf{P}^a = A^{(0)} \left(\epsilon - \epsilon_0 \right) \left(-\frac{\partial \mathbf{q}^a}{\partial t} - \boldsymbol{\nabla} q^{0a} - c\boldsymbol{\omega}^{0a}{}_b \mathbf{q}^b + c\boldsymbol{\omega}^a{}_b q^{0b} \right). \tag{9.127}$$

The homogeneous current is:

$$\begin{aligned} \widetilde{\mathbf{j}}^a = A^{(0)} \Bigg(\frac{\partial}{\partial t} \left(\left(\frac{1}{\mu_0} - \frac{1}{\mu} \right) \left(\boldsymbol{\nabla} \times \mathbf{q}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{q}^b \right) \right) \\ -c^2 \boldsymbol{\nabla} \times \left(\left(\epsilon - \epsilon_0 \right) \left(-\frac{\partial \mathbf{q}^a}{\partial t} - c\boldsymbol{\nabla} q^{0a} - c\omega^{0a}{}_b \mathbf{q}^b + c\boldsymbol{\omega}^a{}_b q^{0b} \right) \right) \Bigg). \end{aligned} \tag{9.128}$$

The numerical task is to find resonance solutions of Eq. 9.128. In general μ and ϵ are functions of ct, X, Y and Z:

$$\epsilon = \epsilon \left(ct, X, Y, Z \right), \tag{9.129}$$

$$\mu = \mu \left(ct, X, Y, Z \right), \tag{9.130}$$

and in general both ϵ and μ are tensorial quantities (as for example in crystals). They are scalars only in an isotropic homogeneous dielectric. We may approximate Eq. 9.128 by considering ϵ and μ as scalars, so Eq. 9.128 simplifies using:

$$\frac{\partial}{\partial t} \left(\left(\frac{1}{\mu_0} - \frac{1}{\mu} \right) \boldsymbol{\nabla} \times \mathbf{A}^a \right) = \left(\frac{1}{\mu_0} - \frac{1}{\mu} \right) \boldsymbol{\nabla} \times \frac{\partial \mathbf{A}^a}{\partial t}, \tag{9.131}$$

$$c\boldsymbol{\nabla} \times \left(\left(\epsilon - \epsilon_0 \right) \boldsymbol{\nabla} A^{0a} \right) = 0, \tag{9.132}$$

and which is a linear inhomogeneous differential equation with mixed derivatives. Eq. 9.128 has only two input parameters ϵ and μ.

When the electromagnetic and gravitational fields are independent:

$$\mu = \mu_0, \quad \epsilon = \epsilon_0, \widetilde{\mathbf{j}} = \mathbf{0}, \tag{9.133}$$

and the relative permittivity and permeability become:

$$\epsilon_r = 1, \quad \mu_r = 1. \tag{9.134}$$

In this limit of independent fields we obtain self consistently the Faraday law of induction of ECE field theory with no homogeneous current:

$$\boldsymbol{\nabla} \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mathbf{0}. \tag{9.135}$$

In this limit the spin connection is dual to the tetrad as in Eq. 9.15 and the Riemann spin form is dual to the torsion form as in Eq. 9.16. In this limit the Evans spin field is obtained self consistently as in Eq. 9.93. Note carefully that Eq. 9.135 is a standard model Faraday law of induction for each index a [1, 7]– [40].

In the standard model the Evans spin field is missing, because electrodynamics in the standard model is a theory of special relativity (the Maxwell Heaviside field theory). Therefore in the standard model classical electrodynamics is incompatible with the principle of general relativity, and this is a major weak point of the standard model because the Evans spin field is an experimental observable in the inverse Faraday effect, indicating that classical electrodynamics must originate in the torsion of space-time, torsion indicating the presence of a spin connection that is, indeed, detected experimentally in the inverse Faraday effect. Classical electrodynamics is not an entity superimposed on flat space-time (the nineteenth century view) because in this view there is no spin connection and no inverse Faraday effect, contrary to reproducible data. The historical origin of this major weak point is well known but worth recounting briefly as follows. Classical electrodynamics in its modern vector formulation was developed in the late nineteenth century (from James Clerk Maxwell's original quaternion equations of the mid nineteenth century), by Oliver Heaviside, before special relativity was developed. Heaviside's vectorial equations for electrodynamics were put in tensorial form by Lorentz and Poincarè at the turn of the twentieth century and were assumed to be Lorentz covariant. Only later, in 1905, did Einstein develop special relativity for the whole of physics. In 1915 Einstein and Hilbert developed the generally covariant field equation of gravitation in general relativity, but electrodynamics remained a Lorentz covariant theory of special relativity. Therefore gravitation and electrodynamics could not be unified, being conceptually (i.e fundamentally) different. Attempts at unification have been made ever since and the first successful generally covariant unified field theory is now generally recognised [7]– [40] as being ECE theory. This did not begin to emerge until 2003. ECE theory now gives the basic understanding needed to evaluate resonant counter gravitation and many other phenomena new to physics [7]– [40].

If the Faraday law of induction is considered from Eq. 9.1 to be [7]– [40]:

$$\boldsymbol{\nabla} \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mu_0 \widetilde{\mathbf{j}}^a, \tag{9.136}$$

and if we restrict consideration to scalar, time-independent μ and ϵ, Eqs. 9.126 and 9.127 used in Eq. 9.123 give:

$$\begin{gathered}(\frac{1}{\mu_0} - \frac{1}{\mu} + c^2(\epsilon - \epsilon_0))\boldsymbol{\nabla} \times \frac{\partial \mathbf{A}^a}{\partial t} - (\frac{1}{\mu_0} - \frac{1}{\mu})\frac{\partial}{\partial t}(\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b) \\ c^3 \boldsymbol{\nabla} \times (\omega^{0a}{}_b \mathbf{A}^b - \boldsymbol{\omega}^a{}_b A^{0b}) = \widetilde{\mathbf{j}}^a\end{gathered} \tag{9.137}$$

Now use the approximation:

$$\omega^{\mu a}{}_b \longrightarrow -\frac{\kappa}{2} q^{\mu c} \epsilon^a{}_{bc} \tag{9.138}$$

which is equivalent to:

$$\widetilde{\mathbf{j}} \rightarrow \mathbf{0}. \tag{9.139}$$

For index $a = 1$:

$$\begin{gathered}\left(\frac{1}{\mu_0} - \frac{1}{\mu} + c^2(\epsilon - \epsilon_0)\right)\boldsymbol{\nabla} \times \frac{\partial \mathbf{A}^1}{\partial t} - \left(\frac{1}{\mu_0} - \frac{1}{\mu}\right)\frac{\partial}{\partial t}\left(\boldsymbol{\omega}^1{}_2 \times \mathbf{A}^2 + \boldsymbol{\omega}^1{}_3 \times \mathbf{A}^3\right) \\ -\frac{\kappa}{2} c^3 \boldsymbol{\nabla} \times \left(q^{03}\mathbf{A}^2 - q^{02}\mathbf{A}^3 - \mathbf{q}^3 A^{02} + \mathbf{q}^2 A^{03}\right) = \widetilde{\mathbf{j}}^1 \rightarrow \mathbf{0}\end{gathered} \tag{9.140}$$

i.e.:

$$\begin{aligned}&\left(\left(\frac{1}{\mu_0}-\frac{1}{\mu}\right)+c^2\left(\epsilon-\epsilon_0\right)\right)\boldsymbol{\nabla}\times\frac{\partial\mathbf{A}^1}{\partial t}+\kappa\left(\frac{1}{\mu_0}-\frac{1}{\mu}\right)\frac{\partial}{\partial t}\left(\mathbf{q}^2\times\mathbf{A}^3\right)\\&-\frac{\kappa}{2}c^3\boldsymbol{\nabla}\times\left(q^{03}\mathbf{A}^2-\mathbf{q}^{02}A^3-\mathbf{q}^3A^{02}+\mathbf{q}^2A^{03}\right)=\widetilde{\mathbf{j}}^1\rightarrow\mathbf{0}.\end{aligned}\tag{9.141}$$

Now use:

$$q^{03}=q^{02}=A^{03}=A^{02}\rightarrow 0\tag{9.142}$$

and switch to the complex circular basis to obtain:

$$\left(\frac{1}{\mu_0}-\frac{1}{\mu}+c^2\left(\epsilon-\epsilon_0\right)\right)\boldsymbol{\nabla}\times\frac{\partial\mathbf{A}^{(1)*}}{\partial t}+\kappa\left(\frac{1}{\mu_0}-\frac{1}{\mu}\right)\frac{\partial\mathbf{A}^{(1)*}}{\partial t}=\widetilde{\mathbf{j}}^{(1)*}\rightarrow\mathbf{0}.\tag{9.143}$$

The structure of this equation is:

$$x\boldsymbol{\nabla}\times\frac{\partial\mathbf{A}^{(2)}}{\partial t}+y\kappa\frac{\partial\mathbf{A}^{(2)}}{\partial t}=\widetilde{\mathbf{j}}^{(2)}\rightarrow\mathbf{0}\tag{9.144}$$

where the scalars x and y are:

$$x=\frac{1}{\mu_0}-\frac{1}{\mu}+c^2\left(\epsilon-\epsilon_0\right),\tag{9.145}$$

$$y=\frac{1}{\mu_0}-\frac{1}{\mu}.\tag{9.146}$$

Eq. 9.144 is again a linear inhomogeneous differential equation with resonant solutions. So in ECE theory the fundamental equations of classical electrodynamics all develop a resonant structure never considered previously in the history of physics and engineering because a generally covariant unified field theory was not available.

Finally in this section the standard model's Lorentz force law is developed into a generally covariant equation of unified field theory. This shows how gravitation is expected to affect the law. In the standard model the Lorentz force law originates in the Lorentz transformation of the field tensor [1]:

$$F'^{\mu\nu}=\frac{\partial x'^{\mu}}{\partial x^{\rho}}\frac{\partial x'^{\nu}}{\partial x^{\sigma}}F^{\rho\sigma}\tag{9.147}$$

where x^μ is the four-coordinate:

$$x^\mu=\left(ct,X,Y,Z\right).\tag{9.148}$$

In the standard model the Lorentz transformation is used from K to a frame K' translating uniformly at $\mathbf{v}$ with respect to K . Using the Lorentz transformation in Eq. 9.147 gives [7]– [40] in S.I. units:

$$\mathbf{E}'=\gamma\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)-\frac{\gamma^2}{\gamma+1}\frac{\mathbf{v}}{c}\left(\frac{\mathbf{v}}{c}\cdot\mathbf{E}\right),\tag{9.149}$$

$$\mathbf{B}'=\gamma\left(\mathbf{B}-\frac{\mathbf{v}}{c^2}\times\mathbf{E}\right)-\frac{\gamma^2}{\gamma+1}\frac{\mathbf{v}}{c}\left(\frac{\mathbf{v}}{c}\cdot\mathbf{B}\right),\tag{9.150}$$

where:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \tag{9.151}$$

The Lorentz force law as usually given in the textbooks as:

$$\mathbf{F} = e\mathbf{E}' = e\gamma\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \tag{9.152}$$

and is an approximation to Eq. 9.149 when:

$$v \ll c, \quad \gamma \neq 1. \tag{9.153}$$

The non-relativistic limit of the Lorentz force law is obtained from the approximation 9.153 in the limit:

$$\gamma \rightarrow 1 \tag{9.154}$$

and is the familiar:

$$\mathbf{F} = e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right). \tag{9.155}$$

In ECE theory [7]– [40] the Lorentz force law is obtained from the rules [7]– [40] of general coordinate transformation of the torsion tensor in Cartan geometry, i.e.:

$$T^{a'}{}_{\mu'\nu'} = \Lambda^{a'}{}_{a} \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} T^{a}{}_{\mu\nu} \tag{9.156}$$

where only $\Lambda^{a'}{}_{a}$ is a Lorentz transformation matrix and where $\partial x^{\mu}/\partial x^{\mu'}$ and $\partial x^{\nu}/\partial x^{\nu'}$ are general coordinate transformation matrices. The electromagnetic field tensor is [7]– [40]:

$$F^{a}{}_{\mu\nu} = A^{(0)} T^{a}{}_{\mu\nu} \tag{9.157}$$

so the Lorentz force in ECE field theory manifests itself in:

$$F^{a'}{}_{\mu'\nu'} = A^{(0)} T^{a'}{}_{\mu'\nu'} \tag{9.158}$$

multiplied by charge. Contained within the Cartan torsion is the spin connection, which is related to the Riemann curvature and to gravitation.

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