Chapter 1

Generally covariant dynamics

(Paper 55)
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Abstract

Generally covariant translational and rotational dynamics are developed on the basis of Einstein Cartan Evans (ECE) field theory. Translational or central dynamics are defined as the limit of vanishing Cartan torsion where the Einstein Hilbert (EH) theory of gravitation is recovered. Rotational dynamics are defined in the limit where the translational Riemann curvature form vanishes and where the rotational Riemann form is dual to the Cartan torsion form. The mutual influence of translation and rotation is defined by the two Cartan structure equations and the two Bianchi identities of differential geometry. The equations of generally covariant rotational and translational dynamics are developed in the same form as the equations of generally covariant electrodynamics.

Keywords: Einstein Cartan Evans (ECE) field theory, generally covariant dynamics, generally covariant electrodynamics.

1.1 Introduction

Classical dynamics has been developed continuously for more than four hundred years. Major advances occurred in the sixteenth and seventeenth centuries [1], notably by Galileo, Brahe, Kepler and Newton, who synthesized the laws of classical translational dynamics. Later, rotational dynamics were developed by Euler and Coriolis, who inferred accelerations not present in Newtonian dynamics. Notable contributions came from Lagrange, Laplace and Hamilton using variational calculus. Following upon the results of the Michelson Morley experiment, length contraction was suggested by Fitzgerald, and developed into the theory of special relativity with notable contributions from Lorentz, Poincaré, Einstein and several others. Einstein inferred the theory of translational special relativity in 1905 and developed it into the theory of translational general relativity. The Einstein Hilbert (EH) field equation of translational dynamics was inferred independently by Einstein and Hilbert, and published in 1916. Later, Einstein and Cartan corresponded on the need for incorporating Cartan torsion into general relativity. In this paper the two Cartan structure equations and the two Bianchi identities of differential or Cartan geometry are inferred to be the equations of generally covariant dynamics, in which both translational and rotational motions are considered. These dynamics are valid in any frame of reference moving arbitrarily with respect to any other frame. The equations of these dynamics are therefore generally covariant as required by the theory of relativity and by objective natural philosophy.

In Section 1.2 the two Cartan structure equations and the two Bianchi identities are developed in a form which is identical to the equations of generally covariant electrodynamics [2]– [15] within a scalar factor $A^{(0)}$, essentially a primordial voltage. EH translational dynamics are defined as the limit where the Cartan torsion form vanishes and rotational dynamics are defined as the limit where the translational Riemann form vanishes. For rotational dynamics, the rotational Riemann form is dual to the Cartan torsion form, and the spin connection is dual to the tetrad. In Section 1.3 the Newtonian equations and principle of equivalence are inferred from the second Bianchi identity with zero Cartan torsion and the Euler equation is inferred from the first Cartan structure equation. In Section 1.4 the equations of generally covariant translational and rotational dynamics are developed in vector notation in the same form as the equations of generally covariant electrodynamics. Generally covariant dynamics provides several new inferences and suggests several phenomena not present in the EH limit. These may be tested with respect to cosmological anomalies where EH theory is not sufficient. There appear resonance solutions in both generally covariant dynamics and electrodynamics, and the ECE theory also gives the equations needed to describe the interaction of gravitation and electrodynamics.

1.2 The equations of generally covariant dynamics

The equations of classical dynamics are given by ECE theory in any frame of reference moving arbitrarily with respect to any other frame. In a condensed notation with all indices suppressed for clarity [2]– [15] the equations of motion are given by Cartan geometry:

$$T = D \wedge q \tag{1.1}$$

$$R = D \wedge \omega \tag{1.2}$$

$$D \wedge T = R \wedge q \tag{1.3}$$

$$D \wedge q = 0, \tag{1.4}$$

$$D\wedge = d \wedge + \omega \wedge . \tag{1.5}$$

Here T is the torsion form, q is the tetrad form, ω is the spin connection, R is the Riemann form and D denotes the covariant exterior derivative of Cartan [2]–[15]. This notation is fully explained and developed elsewhere [2]– [15] in form, tensor and vector notation. The condensed indexless notation of Eq.(1.1) to (1.4) gives the basic structure most clearly. These equations of geometry are transformed into equations of classical dynamics using the Einstein Ansatz:

$$R = -kT \tag{1.6}$$

where in Eq.(1.6), R denotes scalar curvature, k is the Einstein constant and T is the index contracted canonical energy momentum tensor [16]). In general T contains contributions from all four fundamental fields (gravitational, electromagnetic, weak and strong). In the EH theory only the gravitational contribution is considered.

In the notation of Eqs.(1.1) to (1.5) the EH field theory of 1916 is:

$$T = 0 \tag{1.7}$$

$$R \wedge q = 0 \tag{1.8}$$

$$D \wedge R = 0 \tag{1.9}$$

In this limit therefore the torsion form vanishes. Eq.(1.8) is the Ricci cyclic equation of EH theory. In tensor notation Eq.(1.8) is the familiar cyclic combination of Riemann tensors:

$$R_{\sigma\mu\nu\rho} + R_{\sigma\rho\mu\nu} + R_{\sigma\nu\rho\mu} = 0 \tag{1.10}$$

Eq.(1.9) is the second Bianchi identity. In tensor notation it becomes:

$$D^{\mu} G_{\mu\nu} = 0 \tag{1.11}$$

where $G_{\mu\nu}$ is the Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{1.12}$$

In Eq.(1.12) $R_{\mu\nu}$ is the Ricci tensor and $g_{\mu\nu}$ is the symmetric metric tensor. The EH field equation is obtained from the second Bianchi identity (1.11) and the Noether Theorem:

$$D^{\mu} T_{\mu\nu} = 0 \tag{1.13}$$

From Eqs.(1.11) and (1.13) we obtain the EH field equation:

$$D \wedge R = 0 \tag{1.14}$$

Here $T_{\mu\nu}$ is the canonical energy momentum tensor, whose index contracted form in EH theory is:

$$T = g^{\mu\nu} T_{\mu\nu} \tag{1.15}$$

It can be seen that the EH theory is limited by the omission of the Cartan torsion form and is therefore confined to the pure translational part of the general equations of dynamics, Eqs.(1.1) to (1.5). Pure rotational dynamics are defined as [2]– [15]:

$$T = D \wedge q \tag{1.16}$$

$$d \wedge T = 0 \tag{1.17}$$

and it is seen that the translational part of the Riemann or curvature form is zero. Pure rotation is defined by:

$$R \wedge q = \omega \wedge T \tag{1.18}$$

which means that the Cartan torsion is dual to the rotational part of the Riemann form. In this limit the Cartan torsion is the vector valued two-form dual to the rotational Riemann form, a tensor valued two-form. This duality is defined in the tangent (Minkowski) spacetime [2]– [15] of differential geometry and is analogous to the type of duality between for example an anti-symmetric tensor in three dimensions and an axial vector in three dimensions. The rotational Riemann form is the dual of the Cartan torsion form. The translational Riemann form on the other hand is not dual to the Cartan torsion form. The translational Riemann form may therefore be zero when the Cartan form is non-zero and vice-versa. In EH theory only the translational Riemann form is considered, the rotational Riemann form and the Cartan torsion form are not considered in EH theory. In the latter the connection is the symmetric Christoffel connection and EH theory does not consider the interaction between rotation and translation. The equations needed to describe this interaction are Eqs.(1.1) to (1.5).

In the notation [15] of standard differential geometry Eqs.(1.1) to (1.4) become:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{1.19}$$

$$R^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{1.20}$$

$$d \wedge T^a + \omega^a{}_b \wedge T^b = R^a{}_b \wedge q^b \tag{1.21}$$

$$d \wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b - R^a{}_c \wedge \omega^c{}_b = 0 \tag{1.22}$$

in which the indices are those of the tangent (Minkowski) spacetime at point P to the base manifold. The indices of the base manifold are Greek indices which are always the same on both sides of any equation of Cartan geometry. So in the standard notation [15] of Cartan geometry the Greek indices are not written out. In refs. [2]– [14] however the equations of Cartan geometry are written out in full in form, tensor and vector notation.

The equations of Cartan geometry may be written as follows in the same overall format as the ECE equations of electrodynamics [2]– [14]. The first and second Cartan structure equations can be written as differential field equations in which the left hand side involves only the Cartan exterior derivative, and the right hand side is a combination of terms defining currents or source terms. Thus Eqs.(1.21) and (1.22) can be written as:

$$d \wedge T^a = j^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \tag{1.23}$$

$$d \wedge R^a{}_b = j^a{}_b = R^a{}_c \wedge \omega^c{}_b - \omega^a{}_c \wedge R^c{}_b \tag{1.24}$$

Here j^a is the homogeneous current of ECE electrodynamics within the factor $A^{(0)}$. Therefore dynamics and electrodynamics both originate in Cartan geometry, and are unified. The Hodge duals [2]– [14] of Eqs.(1.23) and (1.24) are:

$$d \wedge \widetilde{T}^a = J^a \tag{1.25}$$

$$d \wedge \widetilde{R}^a{}_b = J^a{}_b \tag{1.26}$$

and J^a is the inhomogeneous current of ECE electrodynamics within the factor $A^{(0)}$. Similarly the two structure equations (1.19) and (1.20) may be written as follows:

$$d \wedge q^a = j_1^a = T^a - \omega^a{}_b \wedge q^b \tag{1.27}$$

$$d \wedge \omega^a{}_b = j_{1b}^a = R^a{}_b - \omega^a{}_c \wedge \omega^c{}_b \tag{1.28}$$

whose Hodge duals are:

$$d \wedge \widetilde{q}^a = J_1^a \tag{1.29}$$

$$d \wedge \widetilde{\omega}^a{}_b = J_{1b}^a \tag{1.30}$$

Eqs.(1.23),(1.24), (1.25) and (1.26) may be written in tensor and vector notation, producing much novel information on dynamics and cosmology. Later in this paper it will be shown that Newtonian dynamics is a limit of the vector equation:

$$\boldsymbol{\nabla} \cdot \mathbf{R}^a{}_b \,(\text{orbital}) = j^{a(0)}{}_b \tag{1.31}$$

an equation which is one out of two vector equations given by the single form equation (1.26).

1.3 Newton and Euler equations

In Eq.(1.31), $R^a{}_b$ is the orbital part of the translational Riemann form, a quantity which is defined fully in Section 1.4. In this section the Newton inverse square law is derived straightforwardly from Eq.(1.31). First recognize that the Newtonian force $\mathbf{F}$ is derived from the force form $F^a{}_b$ defined by:

$$\mathbf{F}^a{}_b = -m_1 m_2 G R^a{}_b \mathbf{F} \tag{1.32}$$

Here R is the scalar curvature of Cartan geometry [2]– [14] and c is the speed of light in vacuo, the universal constant of relativity theory. The force form is:

$$F^a{}_b = m_1 g^a{}_b \tag{1.33}$$

where m_1 is a scalar quantity with the units of kilograms. Thus m_1 is recognized as mass and $g^a{}_b$ is a tensor valued two-form with the units of acceleration. Eq.(1.31) is mathematically equivalent to:

$$\mathbf{F}^a{}_b = -G\frac{m_1 m_2}{r^2}\mathbf{k}^a{}_b = m_1 \mathbf{g}^a{}_b \tag{1.34}$$

where G is the Newton gravitational constant defined by:

$$k = \frac{8\pi G}{c^2} \tag{1.35}$$

and where r is the distance between two masses m_1 and m_2. Restoring the indices of the base manifold to equation (1.33) gives:

$$F^a{}_{b\mu\nu} = m_1 g^a{}_{b\mu\nu} \tag{1.36}$$

an equation which shows that the force ${F^a}_{b\mu\nu}$ and the acceleration ${g^a}_{b\mu\nu}$ are both proportional to the orbital part of the translational Riemann form. In the Newtonian limit the base manifold approaches a Minkowski spacetime, so:

$$\mathbf{F}^a{}_b = \mathbf{F}, \quad \mathbf{g}^a{}_b = \mathbf{g} \tag{1.37}$$

and Eq.(1.34 becomes the Newton inverse square law:

$$\mathbf{F} = -G\frac{m_1 m_2}{r^2}\mathbf{k} \tag{1.38}$$

and Eq.(1.33) becomes the Newton force law:

$$\mathbf{F} = m_1 \mathbf{g} \tag{1.39}$$

Both laws are derived from the orbital part of the translational Riemann form of Cartan geometry, and this is the principle of equivalence of inertia and acceleration. The principle of equivalence states that must exist a scalar quantity m_1. It may be seen from Eq.(1.32) that mass does not enter into the relation between force and curvature, and this is the result of the well known Galileo experiment where two different masses dropped from the same height in the Earth's gravitational field hit the ground at the same time from the apocryphal Tower of Pisa. (In fact [1] Galileo proved this result using inclined planes.)

This simple derivation of Newtonian dynamics is confirmed as follows with reference to the well known Schwarzschild metric (SM), which by Birkhoff's Theorem is the unique spherically symmetric vacuum solution of the EH field equation (1.14). The SM is thus a solution of pure translational dynamics and takes no account of torsion in cosmology.

The SM in in spherical polar coordinates [2]– [15] is:

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right)c^2dt^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 \tag{1.40}$$

where M is a parameter with units of kilograms identified with mass. If this parameter is identically zero Eq.(1.40) becomes the Minkowski metric in spherical polar coordinates:

$$ds^2 = c^2dt^2 - dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 \tag{1.41}$$

The six non-zero elements of the Riemann tensor in the SM are as follows [2]–[15]:

$$R^0{}_{101} = e^{2(\beta-\alpha)}\left(\partial_0^2\beta + (\partial_0\beta)^2 - \partial_0\alpha\partial_0\beta\right) + \partial_1\alpha\partial_1\beta - \partial_1^2\alpha - (\partial_1\alpha)^2 \tag{1.42}$$

$$R^0{}_{202} = -re^{-2\beta}\frac{\partial_1\alpha}{r^2} \tag{1.43}$$

$$R^0{}_{303} = \sin^2\theta R^0{}_{202} \tag{1.44}$$

$$R^1{}_{212} = re^{-2\beta}\frac{\partial_1\beta}{r^2} \tag{1.45}$$

$$R^1{}_{323} = R^2{}_{323} = \left(1 - e^{-2\beta}\right)\frac{\sin^2\theta}{r^2} \tag{1.46}$$

where the parameters are defined by:

$$e^{2\alpha} = e^{-2\beta} = 1 - \frac{2GM}{rc^2} \tag{1.47}$$

The three orbital components of the SM are ${R^0}_{101}$, ${R^0}_{202}$, and ${R^0}_{303}$ and the three spin components are ${R^1}_{212}$, ${R^1}_{313}$, and ${R^2}_{323}$. It is seen that:

$$ {R^0}_{101} = e^{-4\alpha}\left(-\left(\partial_1\alpha\right)^2 - \partial_1^2\alpha - \left(\partial_1\alpha\right)^2\right) \tag{1.48}$$

and that:

$$ {R^0}_{202} + {R^0}_{303} = -\frac{GM}{c^2r^3}\left(1+\sin^2\theta\right) \tag{1.49}$$

The Newtonian limit is recovered from:

$$\nabla\cdot {\mathbf{g}^a}_b = -c^2\left({R^0}_{202} + {R^0}_{303}\right) = \frac{2GM}{r^3} \tag{1.50}$$

This result allows us to infer:

$$ {\mathbf{g}^a}_b = \frac{1}{m}{\mathbf{F}^a}_b = -\frac{GM}{r^2}{\mathbf{k}^a}_b \tag{1.51}$$

and to recover Eq.(1.32) in the form:

$$ {\mathbf{F}^a}_b = -\frac{GmM}{r^2}{\mathbf{k}^a}_b \tag{1.52}$$

Thus the scalar curvature R of Eq.(1.32) may be identified as:

$$ {\mathbf{R}^a}_b = -\frac{{\mathbf{k}^a}_b}{r^2} \tag{1.53}$$

More generally the SM produces well known departures from Newtonian dynamics, giving for example the perihelion advance of planets and the result of the Eddington experiment [2]– [15], now verified to one part in one hundred thousand accuracy. The above derivation of the Newton inverse square law from the orbital components of the Riemann tensor of the SM assumes that the ${R^0}_{101}$ element goes to zero faster than the sum of the ${R^0}_{202}$ and ${R^0}_{303}$ elements as M goes to zero and uses the geometry $\theta = 0$ in spherical polar coordinates.

The Riemann form in the SM is therefore the antisymmetric tensor:

$$ {R^a}_{b\mu\nu} = \begin{bmatrix} 0 & {R^0}_{101} & {R^0}_{202} & {R^0}_{303} \\ {R^0}_{110} & 0 & {R^1}_{212} & {R^1}_{313} \\ {R^0}_{220} & {R^1}_{221} & 0 & {R^2}_{323} \\ {R^0}_{330} & {R^1}_{331} & {R^2}_{332} & 0 \end{bmatrix} \tag{1.54}$$

and consists of orbital and spin components. The six non vanishing elements of the SM are precisely the three orbital elements and the three spin elements. The Riemann tensor is obtained from the Riemann form as follows [2]– [15]:

$$ {R^\rho}_{\sigma\mu\nu} = {q^b}_\sigma {q^\rho}_a {R^a}_{b\mu\nu} \tag{1.55}$$

The orbital Riemann vector is defined from Eq.(1.54) as:

$$\mathbf{R}^a{}_b\,(\text{orbital}) = R^0{}_{101}\mathbf{i}^a{}_b + R^0{}_{202}\mathbf{j}^a{}_b + R^0{}_{303}\mathbf{k}^a{}_b \tag{1.56}$$

and the spin Riemann vector as:

$$\mathbf{R}^a{}_b\,(\text{Spin}) = R^2{}_{323}\mathbf{i}^a{}_b + R^1{}_{331}\mathbf{j}^a{}_b + R^1{}_{212}\mathbf{k}^a{}_b \tag{1.57}$$

This derivation can be developed and summarized as follows. Starting with the second Bianchi identity in form notation:

$$D \wedge R^a{}_b = 0 \tag{1.58}$$

i.e.

$$d \wedge R^a{}_b = j^a{}_b = R^a{}_c \wedge \omega^c{}_b - \omega^a{}_c \wedge R^c{}_b \tag{1.59}$$

the homogeneous field equation of translational dynamics is obtained in the tensor notation:

$$\partial_\mu \widetilde{R}^a{}_b{}^{\mu\nu} = \widetilde{j}^a{}_b{}^\nu \tag{1.60}$$

The Hodge dual gives the inhomogeneous field equation of translational dynamics:

$$\partial_\mu R^a{}_b{}^{\mu\nu} = J^a{}_b{}^\nu \tag{1.61}$$

The operators ∂_μ are the Minkowski differential operators:

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t},\quad \frac{\partial}{\partial X},\quad \frac{\partial}{\partial Y},\quad \frac{\partial}{\partial Z}\right) \tag{1.62}$$

and their space part refers therefore to a Cartesian frame of reference with Cartesian unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$. Thus in vector notation in the SM:

$$\mathbf{R}^a{}_b\,(\text{orbital}) = R^a{}_{b01}\mathbf{i} + R^a{}_{b02}\mathbf{j} + R^a{}_{b03}\mathbf{k} \tag{1.63}$$

and

$$\mathbf{R}^a{}_b\,(\text{Spin}) = R^2{}_{323}\mathbf{i} + R^1{}_{331}\mathbf{j} + R^1{}_{212}\mathbf{k} \tag{1.64}$$

It is also possible to define:

$$\mathbf{R}^a{}_b\,(\text{orbital}) = R^a{}_b{}^{01}\mathbf{i} + R^a{}_b{}^{02}\mathbf{j} + R^a{}_b{}^{03}\mathbf{k} \tag{1.65}$$

and

$$\mathbf{R}^a{}_b\,(\text{Spin}) = R^2{}_3{}^{23}\mathbf{i} + R^1{}_3{}^{31}\mathbf{j} + R^1{}_2{}^{12}\mathbf{k} \tag{1.66}$$

In the SM:

$$\mathbf{R}^0{}_1 = R^0{}_1{}^{01}\mathbf{i}, \quad \mathbf{R}^0{}_2 = R^0{}_2{}^{02}\mathbf{j}, \quad \mathbf{R}^0{}_3 = R^0{}_3{}^{03}\mathbf{k} \tag{1.67}$$

Now define:

$$\mathbf{R} = R^0{}_1{}^{01}\mathbf{i} + R^0{}_2{}^{02}\mathbf{j} + R^0{}_3{}^{03}\mathbf{k} \tag{1.68}$$

to obtain:

$$\mathbf{R}\,(\text{orbital}) = \mathbf{R} \tag{1.69}$$

and similarly:

$$\mathbf{R}\,(\text{Spin}) = R^2{}_3{}^{23}\mathbf{i} + R^1{}_3{}^{31}\mathbf{j} + R^1{}_2{}^{12}\mathbf{k} \tag{1.70}$$

The translational equations of motion are therefore:

$$\partial_\mu \widetilde{R}^{\mu\nu} = \widetilde{j}^\nu, \quad \partial_\mu R^{\mu\nu} = J^\mu \tag{1.71}$$

where the translational field tensor can be defined as:

$$R^{\mu\nu} = \begin{bmatrix} 0 & R^0{}_1{}^{01} & R^0{}_2{}^{02} & R^0{}_3{}^{03} \\ R^0{}_1{}^{10} & 0 & R^1{}_2{}^{12} & R^1{}_3{}^{13} \\ R^0{}_2{}^{20} & R^1{}_2{}^{21} & 0 & R^2{}_3{}^{23} \\ R^0{}_3{}^{30} & R^1{}_3{}^{31} & R^2{}_3{}^{32} & 0 \end{bmatrix} \tag{1.72}$$

In vector notation Eq.(1.70) are:

$$\nabla \cdot \mathbf{R}\,(\text{Spin}) = \widetilde{j}^0 \tag{1.73}$$

$$\nabla \times \mathbf{R}\,(\text{orbital}) + \frac{1}{c}\frac{\partial \mathbf{R}}{\partial t}\,(\text{Spin}) = \widetilde{\mathbf{j}} \tag{1.74}$$

$$\nabla \cdot \mathbf{R}\,(\text{orbital}) = J^0 \tag{1.75}$$

$$\nabla \times \mathbf{R}\,(\text{Spin}) - \frac{1}{c}\frac{\partial \mathbf{R}}{\partial t}\,(\text{orbital}) = \mathbf{J} \tag{1.76}$$

Eq.(1.73) is analogous to the generally covariant Gauss law of magnetism [2]–[14], Eq.(1.74) to the generally covariant Faraday law of induction, Eq.(1.75) to the generally covariant Coulomb Law and Eq.(1.76) to the generally covariant Ampere Maxwell law.

The equations of pure rotational dynamics are Eqs.(1.16) and (1.17). To end this section Eq.(1.16) is developed by defining the angular momentum tetrad [2]–[14] as:

$$J^a{}_\mu = J^{(0)} q^a{}_\mu \tag{1.77}$$

The first Cartan structure equation (1.16) then defines the generally covariant torque equation:

$$N^a = c\left(d \wedge J^a + \omega^a{}_b \wedge J^b\right) \tag{1.78}$$

In the classical non-relativistic limit (1.17) the Euler equation of motion is:

$$\mathbf{N} = \left(\frac{d\mathbf{J}}{dt}\right)_{\text{lab}} = \left(\frac{d\mathbf{J}}{dt}\right)_{\text{moving}} + \boldsymbol{\omega} \times \mathbf{J} \tag{1.79}$$

where:

$$\mathbf{N} = \left(\frac{d\mathbf{J}}{dt}\right)_{\text{lab}} \tag{1.80}$$

is valid only in an inertial Cartesian frame. Eq.(1.78) on the other hand is generally covariant, it is valid in any frame of reference moving arbitrarily with respect to any other frame of reference. It defines the generally covariant torque N^a as the Cartan torsion T^a within a scalar factor $cJ^{(0)}$. Therefore the Cartan torsion always defines torque in general relativity. In Eulerian dynamics the torque is still constructed from classical Newtonian dynamics, (classical non-relativistic torque ($\mathbf{N}$) is the arm ($\mathbf{r}$) cross multiplied by Newtonian force ($\mathbf{F}$)).

It has been demonstrated already that source of Newtonian dynamics is the Riemann form, not the torsion form. Therefore ECE theory gives new insight to the nature of both rotational and translational dynamics in the required generally covariant formulation. Einstein Hilbert (EH) field theory deals only with translational dynamics and is restricted to a particular Riemann geometry where the Christoffel connection is symmetric [2]– [15].

Eq.(1.78) is closely analogous to the equation defining the electromagnetic field:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{1.81}$$

wherein the $\omega^a{}_b \wedge A^b$ term originates in a spinning frame of reference, i.e. spinning space-time itself. In general relativity (ECE theory) there is no Cartan torsion if space-time is not spinning, so the generally covariant torque vanishes in a static or purely curving but not spinning space-time:

$$N^a = 0 \tag{1.82}$$

This limit of zero torque is the generally covariant description of translational dynamics, which reduces to Newtonian dynamics as we have argued in this section. The classical non-relativistic torque [17] has therefore been inferred historically from:

$$\mathbf{N} = \dot{\mathbf{J}} = \mathbf{r} \times \mathbf{F} \tag{1.83}$$

The classical, non-relativistic angular momentum $\mathbf{J}$ is defined as in Eq.(1.80). The total time derivative of $\mathbf{J}$ is the classical non-relativistic torque $\mathbf{N}$. Therefore classical non-relativistic rotational dynamics, as inferred historically by Euler and Coriolis and others, is not a self-consistent theory of general relativity, because it attempts to describe torque without Cartan torsion.

The self-consistent and generally covariant description of rotational dynamics in objective physics must be Eq.(1.78). It follows that there are dynamical effects in ECE theory that do not exist in EH theory, and so do not exist in classical non-relativistic rotational dynamics. When rotational and translational motions are mutually influential, the generally covariant torque interacts with a gravitational field through the COMPLETE first Bianchi identity (1.21) to give:

$$\begin{aligned} d \wedge N^a + \omega^a{}_b \wedge N^b &= cJ^{(0)} R^a{}_b \wedge q^b \\ &= cR^a{}_b \wedge J^b \end{aligned} \tag{1.84}$$

i.e.

$$D \wedge N^a = cR^a{}_b \wedge J^b \tag{1.85}$$

In the absence of a gravitational field:

$$R^a{}_b = 0 \tag{1.86}$$

so we recover Eq.(1.85) for generally covariant rotational dynamics:

$$D \wedge N^a = 0 \tag{1.87}$$

On the other hand, the source of the classical non-relativistic [17] torque (1.83) is not the first Bianchi identity (1.21), but the second Bianchi identity:

$$D \wedge R^a{}_b = 0 \tag{1.88}$$

which reduces to Newtonian dynamics as argued already in this section. The reason is that the classical non-relativistic torque is derived through Eq.(1.32) from the Newtonian force, and the latter derives from the Riemann curvature Eq.(1.20), a curvature which obeys the second Bianchi identity. The Riemann or curvature form enters into the first Bianchi identity if and only if there is interaction between rotation and translation. If there is no such interaction the motion is either pure rotational Eq.(1.87) or pure translational as in the Einstein Hilbert theory of 1916.

1.4 The generally covariant field equations of rotational dynamics and electrodynamics

In analogy with the Riemann form Eq.(1.54), the torsion form may also be developed as follows into an anti-symmetric tensor in four dimensions with orbital and spin components:

$$T^{\mu\nu} = \begin{bmatrix} 0 & -\frac{T_L^1}{c} & -\frac{T_L^2}{c} & -\frac{T_L^3}{c} \\ \frac{T_L^1}{c} & 0 & -T_S^3 & T_S^2 \\ \frac{T_L^2}{c} & T_S^3 & 0 & -T_S^1 \\ \frac{T_L^3}{c} & -T_S^2 & T_S^1 & 0 \end{bmatrix} \tag{1.89}$$

The factor c has been introduced into the definition of the orbital components for ease of comparison with generally covariant ECE electromagnetic theory [2]–[14]. Therefore there are orbital and intrinsic (or spin) components of torque in general relativity. The orbital component is proportional to the electric field and the spin component is proportional to the magnetic field in generally covariant electrodynamics. In vector notation:

$$\mathbf{T}_L^a = -\frac{\partial \mathbf{q}^a}{\partial t} - c\nabla q^{0a} - c\omega^{0a}{}_b \mathbf{q}^b + c\boldsymbol{\omega}^a{}_b q^{0b} \tag{1.90}$$

$$\mathbf{T}_S^a = \nabla \times \mathbf{q}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{q}^b \tag{1.91}$$

The generally covariant equation of rotational motion is obtained using:

$$\mathbf{J}^a = J^{(0)}\mathbf{q}^a \tag{1.92}$$

$$\mathbf{N}^a = cJ^{(0)}\mathbf{T}^a \tag{1.93}$$

thus defining the generally covariant angular momentum $\mathbf{J}^a$ and the generally covariant torque $\mathbf{N}^a$. Therefore the generally covariant orbital torque is:

$$\mathbf{N}_L^a = -\frac{\partial \mathbf{J}^a}{\partial t} - c\nabla J^{0a} - c\omega^{0a}{}_b \mathbf{J}^b + c\boldsymbol{\omega}^a{}_b J^{0b} \tag{1.94}$$

and the generally covariant intrinsic or spin torque is:

$$\mathbf{N}_S^a = c\left(\nabla \times \mathbf{J}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{J}^b\right) \tag{1.95}$$

Eqs.(1.94) and (1.95) are the generally covariant equations of rotational dynamics. They are valid in any reference frame moving arbitrarily with respect to any other reference frame, as required by the principle of relativity and thus of objective physics. The Eulerian limit of classical non-relativistic dynamics is obtained as follows:

$$\mathbf{N}_L^a \longrightarrow -\frac{\partial \mathbf{J}^a}{\partial t}, \quad \mathbf{N}_S^a \longrightarrow -c\boldsymbol{\omega}^a{}_b \times \mathbf{J}^b \tag{1.96}$$

Assuming that:

$$\nabla \times \mathbf{J}^a \longrightarrow \mathbf{0} \tag{1.97}$$

(i.e. that the angular momentum is ir-rotational) the total torque is:

$$\mathbf{N}^a = \mathbf{N}_L^a + \mathbf{N}_S^a = -\frac{\partial \mathbf{J}^a}{\partial t} - c\boldsymbol{\omega}^a{}_b \times \mathbf{J}^b \tag{1.98}$$

and this has the same form as the Euler equation (1.79). However, as argued in Section 1.3, Eq.(1.98) comes from the Cartan torsion form, and the Euler equation comes from the Cartan curvature form. Therefore ECE theory has a great deal more inherent information than the classical and non-relativistic Euler theory. The task is to reveal such information experimentally, using high accuracy experiments in the laboratory or in astronomy. These would amount to rigorous experimental tests of Einsteinian philosophy itself, because ECE theory completes the EH theory of 1916. They would therefore be important experiments.

For pure rotational motion unaffected by the Cartan curvature but defined by the Cartan torsion, it has been shown [2]– [14] that the spin connection is dual to the tetrad and that the form $R^a{}_b$ (torsion) is dual to the Cartan torsion:

$$\omega^a{}_b = -\frac{\kappa}{2}\epsilon^a{}_{bc}q^c \tag{1.99}$$

$$R^a{}_b\,(\text{torsion}) = -\frac{\kappa}{2}\epsilon^a{}_{bc}T^c \tag{1.100}$$

Here κ has the units of inverse meters and can be identified as a scalar wave-number. Eqs.(1.99) and (1.100) are written in the tangent space-time and are valid for all indices of the base manifold. They mean that for pure rotational motion, the spin connection is the two index quantity dual to the one index tetrad. The tangent space-time of Cartan geometry is a Minkowski space-time [2]– [15] so the duality of Eqs.(1.99) and (1.100) occurs through the totally anti-symmetric unit tensor $\epsilon^a{}_{bc}$ in the Minkowski space-time. If we restrict consideration to three space indices and dimensions, this type of duality is analogous to the well known three-dimensional duality between an anti-symmetric tensor and an axial vector [2]– [15]. Note that $R^a{}_b$(torsion) in Eq.(1.100) is not the Riemann curvature form. The latter is not dual to the torsion form because the Cartan curvature (or Riemann) form can be non-zero when the Cartan torsion form is zero and vice-versa. Thus one cannot be the dual of the other. The quantity $R^a{}_b$(torsion) is the two-index representation of the Cartan torsion and the existence of $R^a{}_b$(torsion) had not been inferred or clearly defined prior to the development [2]– [14] of ECE theory. For a complete understanding of generally covariant dynamics, and also of generally covariant electrodynamics, it is necessary to realize that $R^a{}_b$(torsion) exists in Cartan geometry and that it is different from the curvature form $R^a{}_b$. This is also important in devising new tests of relativity, as argued already in this section, and for coding purposes in computation.

The generally covariant angular velocity for pure rotational motion may now be defined as:

$$\Omega^a{}_b = c\omega^a{}_b = -\frac{\omega}{2}\epsilon^a{}_{bc}q^c \tag{1.101}$$

where

$$\Omega^c = \omega q^c, \quad \omega = \kappa c \tag{1.102}$$

So the two index $\Omega^a{}_b$ is the dual of the one index ω^c as follows:

$$\Omega^a{}_b = -\frac{1}{2}\epsilon^a{}_{bc}\omega^c \tag{1.103}$$

It is now possible to define a generally covariant two index quantity with the units of force:

$$F^a{}_b = cJ^{(0)}R^a{}_b\,(\text{torsion}) \tag{1.104}$$

This definition is analogous to Eq.(1.93) for the torque. Therefore $F^a{}_b$ is the two-index dual of the generally covariant torque:

$$F^a{}_b = -\frac{\kappa}{2}\epsilon^a{}_{bc}N^c_L \tag{1.105}$$

Inverting Eq.(1.105) gives an expression for torque:

$$N^c_L = -2r\epsilon^{bc}{}_a F^a{}_b \tag{1.106}$$

in terms of the distance r defined by:

$$r = \frac{1}{\kappa}. \tag{1.107}$$

Eq.(1.106) is the correctly and generally covariant formulation of the classical and non-relativistic definition of orbital torque as being the arm cross multiplied by the Newtonian force:

$$\mathbf{N}_L = \mathbf{r} \times \mathbf{F} \tag{1.108}$$

Eq.(1.106) is again valid in any reference frame moving arbitrarily with respect to any other frame, while Eq.(1.108) is non-relativistic and therefore incomplete. Eq.(1.106) originates in the spinning of space-time while Eq.(1.108) originates in Newtonian concepts of force. In Newtonian dynamics the spin connection does not exist as a concept. The intrinsic or spin torque:

$$\mathbf{N}^a{}_S = -c\boldsymbol{\omega}^a{}_b \times \mathbf{J}^b \tag{1.109}$$

originates again in the spinning of space-time, while its Eulerian predecessor, $\boldsymbol{\omega} \times \mathbf{J}$ derives from considerations of one Cartesian frame moving with respect to another Cartesian frame in absolute three dimensional space, using the separate concept of absolute time in the manner of Newton. In ECE theory these are merged into a four dimensional space-time whose curvature and spin are defined by Cartan geometry (Eqs.(1.1) to (1.5)). In ECE theory there is no absolute frame of reference. This is the major philosophical advance made in dynamics between the seventeenth and twenty first centuries and many of the experimental consequences of this advance remain to be evaluated. Added to this advance is the unification of dynamics with electrodynamics made possible by ECE theory [2]– [15]. It is now known how the gravitational field interacts with the electromagnetic field on both classical and quantum levels. This interaction is governed entirely by geometry in both classical and wave mechanics. For physics, Cartan geometry appears to be sufficient in our current state of knowledge, but in mathematics there are more abstract geometries which may contain more information than Cartan geometry in the same way that Cartan geometry contains more information [2]– [15] than Riemann geometry. These more abstract geometries may turn out to contain physical information, so should be borne in mind.

The orbital torque (1.94) may be simplified using certain approximations, given here as examples only. The most general orbital torque is always Eq.(1.95). If angular momentum is considered [18] to be a space-like property with no time-like component the scalars J^{0a} and J^{0b} vanish because the 0 index is time-like and the a and b indices have been restricted by definition to space-like. Using this assumption, Eq.(1.94) simplifies to:

$$\mathbf{N}_L^a = -\frac{\partial \mathbf{J}^a}{\partial t} - c\omega^{0a}{}_b \mathbf{J}^b \tag{1.110}$$

The components of Eq.(1.110) are:

$$\mathbf{N}_L^1 = -\frac{\partial \mathbf{J}^1}{\partial t} - c\omega^{01}{}_b \mathbf{J}^b \tag{1.111}$$

$$\mathbf{N}_L^2 = -\frac{\partial \mathbf{J}^2}{\partial t} - c\omega^{02}{}_b \mathbf{J}^b \tag{1.112}$$

$$\mathbf{N}_L^3 = -\frac{\partial \mathbf{J}^3}{\partial t} - c\omega^{03}{}_b \mathbf{J}^b \tag{1.113}$$

For rotational motion not affected by central gravitation:

$$\omega^a{}_b = -\frac{\kappa}{2}\epsilon^a{}_{bc} q^c \tag{1.114}$$

$$\epsilon^a{}_{bc} = \eta^{ad}\epsilon_{dbc} \tag{1.115}$$

where η^{ab} is the Minkowski metric [2]– [15], so

$$\omega^1{}_{\mu 2} = -\frac{\kappa}{2} q^3{}_\mu \tag{1.116}$$

$$\omega^3{}_{\mu 1} = -\frac{\kappa}{2} q^2{}_\mu \tag{1.117}$$

$$\omega^3{}_{\mu 2} = -\frac{\kappa}{2} q^1{}_\mu \tag{1.118}$$

It follows that:

$$\omega^1{}_{02} = -\frac{\kappa}{2} q^3{}_0 = 0 \tag{1.119}$$

$$\omega^3{}_{01} = -\frac{\kappa}{2} q^2{}_0 = 0 \tag{1.120}$$

$$\omega^3{}_{02} = -\frac{\kappa}{2} q^1{}_0 = 0 \tag{1.121}$$

and:

$$\mathbf{N}_L^a = -\frac{\partial \mathbf{J}^a}{\partial t} \tag{1.122}$$

This equation has the same form as the well known non-relativistic and inertial part of torque:

$$\mathbf{N} = \frac{\partial \mathbf{J}}{\partial t} \tag{1.123}$$

The sign change in Eq.(1.123) is a convention, and the indices a in Eq.(1.122) refer to the Cartesian space-like part of Minkowski space-time. Thus Eqs.(1.122) and (1.123) contain the same physical information but are philosophically different as argued already. The classical result (1.123) is a well defined limit of the generally covariant result (1.94) and the latter contains more observable information. Some of this information may already have been observed and described as “anomalies” which cannot be described by Einstein Hilbert theory or its Newtonian limit. These anomalies are therefore due to Cartan torsion, an important result of ECE theory [2]– [14].

If central gravitation affects rotational motion then Eqs.(1.114) to (1.121) are no longer true in general and there is an additional torque term as in Eq.(1.110). This term leads to additional physical and observable effects on a rotating object in a gravitational field. If for some reason J^{0a} and J^{0b} are non-zero, then all four terms on the right hand side of Eq.(1.94) contribute in general to the generally covariant orbital torque, and there are several effects which may for example affect the orbit of a planet or satellite. In addition there are the effects of the generally covariant intrinsic torque or spin torque, Eq.(1.95). Both orbital and intrinsic torque are present in general in subjects such as astronomy and cosmology as well as in precise laboratory experiments. Prior to this paper such effects have not been realized to exist, and they are not considered in EH theory, because in EH theory there is no torsion at all. These conclusions require an extensive re-evaluation of cosmology, notably planetary cosmology and satellite technology, and Big Bang theory, dark matter theory and so forth. In none of these theories is Cartan torsion adequately considered, or considered at all. Standard model textbooks in general relativity usually give at best only a cursory mention of Cartan torsion and restrict development to Cartan curvature in its Riemann limit (EH theory) in an un-unified field theory of gravitation only. Thus ECE theory open up a large new area of physics hitherto unexplored. ECE theory is a generally covariant unified field theory [2]– [14], the first of its kind, and completes the search of Einstein, Cartan and others for this end.

The generally covariant spin torque is the required objective and generally covariant description of the well known Coriolis and centripetal accelerations of the classical, non relativistic limit of general relativity. In order to introduce these classical accelerations (usually known as the “non-inertial” accelerations) it is convenient firstly to give a summary of the original derivation of Coriolis, (1835), a summary which is based on ref. [17].

Consider two sets of Caretsian coordinate axes. One is fixed (“inertial”) and the other is in arbitrary motion with respect to the first. These are designated ”fixed” and ”rotating” as in the following figure [17]: For any point P:

$$\mathbf{r}' = \mathbf{R} + \mathbf{r} \tag{1.124}$$

as in Fig. 1.1. It is always possible [17] to represent an arbitrary infinitesimal displacement by a pure rotation and an axis called the instantaneous axis of

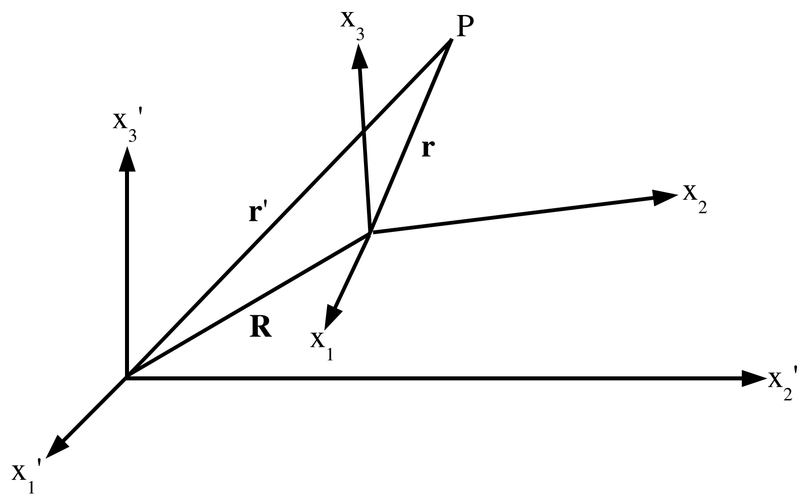

Figure 1.1: Fixed and Rotating Coordinate Axis

rotation. For example, if a disk rolls down an inclined plane, the motion is a rotation about the point of contact of the disk with the plane. Therefore if the x_i system undergoes an infinitesimal rotation $\partial\theta$, corresponding to an arbitrary infinitesimal displacement,

$$(d\mathbf{r})_{\text{fixed}} = d\boldsymbol{\theta} \times \mathbf{r} \tag{1.125}$$

Note that Eq.(1.125) is the result of geometry and so must be a limit of Cartan geometry, specifically a limit of Cartan torsion. The quantity $d\mathbf{r}$ is measured in the $x_i^{'}$, or fixed, coordinate system. The point P is considered to be at rest with respect to the x_i system, but P is moving with respect to the $x_i^{'}$ system. Now divide Eq.(1.124) by the time interval during which the infinitesimal displacement takes place, using the Newtonian fluxions (differential calculus) to give:

$$\left(\frac{d\mathbf{r}}{dt}\right)_{\text{fixed}} = \frac{d\boldsymbol{\theta}}{dt} \times \mathbf{r} \tag{1.126}$$

Identify the angular velocity as:

$$\omega = \frac{d\boldsymbol{\theta}}{dt} \tag{1.127}$$

so:

$$\left(\frac{d\mathbf{r}}{dt}\right)_{\text{fixed}} = \boldsymbol{\omega} \times \mathbf{r} \tag{1.128}$$

Now make the theory more general by considering P to have a velocity with respect to the x_i system:

$$\left(\mathbf{v} = \frac{d\mathbf{r}}{dt}\right)_{\text{moving}} \tag{1.129}$$

The total velocity is therefore:

$$\left(\frac{d\mathbf{r}}{dt}\right)_{\text{fixed}} = \left(\frac{d\mathbf{r}}{dt}\right)_{\text{moving}} + \boldsymbol{\omega} \times \mathbf{r} \tag{1.130}$$

Eq.(1.130) is valid for any vector $\mathbf{Q}$ [17]:

$$\left(\frac{d\mathbf{Q}}{dt}\right)_{\text{fixed}} = \left(\frac{d\mathbf{Q}}{dt}\right)_{\text{moving}} + \boldsymbol{\omega} \times \mathbf{Q} \tag{1.131}$$

If for example Q denotes linear velocity then:

$$\left(\frac{d\mathbf{v}}{dt}\right)_{\text{fixed}} = \left(\frac{d\mathbf{v}}{dt}\right)_{\text{moving}} + \boldsymbol{\omega} \times \mathbf{v} \tag{1.132}$$

where from Eq.(1.129):

$$\mathbf{v}_{\text{fixed}} = \mathbf{v}_{\text{moving}} + \boldsymbol{\omega} \times \mathbf{r} \tag{1.133}$$

The quantity $\boldsymbol{\omega} \times \mathbf{v}$ is proportional to the non-relativistic Coriolis acceleration. Now differentiate Eq.(1.133) to obtain:

$$\left(\frac{d\mathbf{v}}{dt}\right)_{\text{fixed}} = \left(\frac{d\mathbf{v}}{dt}\right)_{\text{moving}} + \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r} + \boldsymbol{\omega} \times \frac{d\mathbf{r}}{dt} \tag{1.134}$$

From Eq.(1.130):

$$\boldsymbol{\omega} \times \left(\frac{d\mathbf{r}}{dt}\right)_{\text{fixed}} = \boldsymbol{\omega} \times \left(\frac{d\mathbf{r}}{dt}\right)_{\text{moving}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \tag{1.135}$$

and the quantity $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ is proportional to the non-relativistic centripetal acceleration. The third type of "non-inertial" and non-relativistic acceleration which emerges form this analysis is $\dot{\boldsymbol{\omega}} \times \mathbf{r}$. The non-inertial accelerations link a translating object in the fixed frame with the same translating object in the rotating frame. The three non-inertial accelerations are therefore observable in both frames: the observer or fixed frame and the rotating frame. Therefore a link with relativity theory is clearly indicated, but in a general context where both translations and rotations are considered, and where accelerations are considered. In other words general relativity with torsion is needed, and ECE theory.

The original derivation by Coriolis is given in ref. [17] as follows. Coriolis considered an observer in a rotating Cartesian coordinate system using the concepts of absolute space and absolute time. Ref. [17] reproduces this 1835 derivation by differentiating Eq.(1.124) to give:

$$\begin{aligned}\left(\frac{d\mathbf{r}'}{dt}\right)_{\text{fixed}} &= \left(\frac{d\mathbf{R}}{dt}\right)_{\text{fixed}} + \left(\frac{d\mathbf{r}}{dt}\right)_{\text{fixed}} \\ &= \left(\frac{d\mathbf{R}}{dt}\right)_{\text{fixed}} + \left(\frac{d\mathbf{r}}{dt}\right)_{\text{moving}} + \boldsymbol{\omega} \times \mathbf{r}\end{aligned} \tag{1.136}$$

Now define:

$$\mathbf{v}_f = \dot{\mathbf{r}}_f = \left(\frac{d\mathbf{r}}{dt}\right)_{\text{fixed}} \tag{1.137}$$

$$\mathbf{V} = \dot{\mathbf{R}}_f = \left(\frac{d\mathbf{R}}{dt}\right)_{\text{fixed}} \tag{1.138}$$

$$\mathbf{v}_r = \dot{\mathbf{r}}_r = \left(\frac{d\mathbf{r}}{dt}\right)_{\text{rotating}} \tag{1.139}$$

so:

$$\mathbf{v}_f = \mathbf{V} + \mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r} \tag{1.140}$$

The Newtonian acceleration is the term [17]:

$$\mathbf{a}_f = m\left(\frac{d\mathbf{v}_f}{dt}\right)_{\text{fixed}} \tag{1.141}$$

$$\mathbf{F} = m\mathbf{a}_f \tag{1.142}$$

while differentiation of Eq.(1.140) gives extra terms:

$$\left(\frac{d\mathbf{v}_f}{dt}\right)_{\text{fixed}} = \left(\frac{d\mathbf{V}}{dt}\right)_{\text{fixed}} + \left(\frac{d\mathbf{v}_r}{dt}\right)_{\text{fixed}} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times \left(\frac{d\mathbf{r}}{dt}\right)_{\text{fixed}} \tag{1.143}$$

Denote:

$$\ddot{\mathbf{R}}_f = \left(\frac{d\mathbf{V}}{dt}\right)_{\text{fixed}} \tag{1.144}$$

The second term in Eq.(1.143) is evaluated by substituting $\mathbf{v}_r$ for $\mathbf{Q}$ in Eq.(1.131):

$$\left(\frac{d\mathbf{v}_r}{dt}\right)_{\text{fixed}} = \left(\frac{d\mathbf{v}_f}{dt}\right)_{\text{rotating}} + \boldsymbol{\omega} \times \mathbf{v}_r = \mathbf{a}_r + \boldsymbol{\omega} \times \mathbf{v}_r \tag{1.145}$$

The last term in Eq.(1.143) is obtained from:

$$\begin{aligned} &\boldsymbol{\omega} \times \left(\frac{d\mathbf{r}}{dt}\right)_{\text{fixed}} \\ =&\boldsymbol{\omega} \times \left(\frac{d\mathbf{v}_f}{dt}\right)_{\text{rotating}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \\ =&\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \end{aligned} \tag{1.146}$$

So:

$$\mathbf{F} = m\mathbf{a}_f = m\ddot{\mathbf{R}}_f + m\mathbf{a}_r + m\dot{\boldsymbol{\omega}} \times \mathbf{r} + m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2m\boldsymbol{\omega} \times \mathbf{v}_r \tag{1.147}$$

To an observer in a rotating coordinate system:

$$\mathbf{F}_r = m\mathbf{a}_r = \mathbf{F} - m\ddot{\mathbf{R}}_f - m\dot{\boldsymbol{\omega}} \times \mathbf{r} - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2m\boldsymbol{\omega} \times \mathbf{v}_r \tag{1.148}$$

The standard centripetal force is thus $-m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$, and is directed outward from the center of rotation. The Coriolis force is $-2m\boldsymbol{\omega} \times \mathbf{v}_r$. Thus:

$$\mathbf{F}_r = m\mathbf{a}_f + \text{non-inertial terms.} \tag{1.149}$$

The classical derivation by Coriolis is incomplete because it is not relativistic, i.e. it is not generally covariant. The latter theory of the Coriolis and centripetal forces must be based on the Cartan structure equation:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{1.150}$$

and the first Bianchi identity:

$$d \wedge T^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \tag{1.151}$$

In an inertial or central treatment such as that of Einstein and Hilbert there is no Cartan torsion and:

$$T^a = 0, \quad R^a{}_b \wedge q^b = 0 \tag{1.152}$$

However in the required ECE theory there are non-zero terms:

$$R^a{}_b \wedge q^b = d \wedge T^a + \omega^a{}_b \wedge q^b \neq 0 \tag{1.153}$$

$$T^a \neq 0, \tag{1.154}$$

which come from the rotational motion or spinning of space-time itself. It is important to realize that this spinning motion is not that of one frame with respect to another, it is a generally covariant motion with no preferred frame of reference. The space-time ITSELF is both spinning and curving. Analogously, in EH theory, the space-time itself is curving but not spinning.

Now define the generally covariant version of the vector **Q**. This is the tetrad within a dimensional scalar $Q^{(0)}$ Invariant under frame transformation. Thus:

$$Q^a{}_\mu = Q^{(0)} q^a{}_\mu \tag{1.155}$$

For example there is a position tetrad and velocity tetrad:

$$r^a{}_\mu = r^{(0)} q^a{}_\mu, \quad V^a{}_\mu = V^{(0)} q^a{}_\mu \tag{1.156}$$

Thus:

$$\begin{aligned} R^a{}_b \wedge r^b =& d \wedge \left(d \wedge r^a + \omega^a{}_b \wedge r^b\right) \\ & + \omega^a{}_b \wedge \left(d \wedge r^b + \omega^b{}_c \wedge r^c\right) \end{aligned} \tag{1.157}$$

$$\begin{aligned} R^a{}_b \wedge V^b =& d \wedge \left(d \wedge V^a + \omega^a{}_b \wedge V^b\right) \\ & + \omega^a{}_b \wedge \left(d \wedge V^b + \omega^b{}_c \wedge V^c\right) \end{aligned} \tag{1.158}$$

From Eq.(1.157) we find the generally covariant centripetal term:

$$\left(R^a{}_b \wedge r^b\right)_{\text{centripetal}} = \omega^a{}_b \wedge \left(\omega^b{}_c \wedge r^c\right) \tag{1.159}$$

and from Eq.(1.158) the generally covariant Coriolis term:

$$\left(R^a{}_b \wedge V^b\right)_{\text{coriolis}} = d \wedge \left(\omega^a{}_b \wedge V^b\right) + \omega^a{}_b \wedge \left(\omega^b{}_c \wedge V^c\right) \tag{1.160}$$

When there is no space-time spin:

$$\omega^a{}_b = 0 \tag{1.161}$$

$$\omega^a{}_b \wedge \left(\omega^b{}_c \wedge r^c\right) = 0 \tag{1.162}$$

$$\omega^a{}_b \wedge V^b = 0 \tag{1.163}$$

$$d \wedge q^a = 0 \tag{1.164}$$

$$d \wedge V^a = d \wedge r^a = 0 \tag{1.165}$$

and so:

$$T^a = 0 \tag{1.166}$$

In this limit the Newtonian acceleration is recovered from a limit of the second Bianchi identity as shown in Section 1.3. In the generally covariant development of the non-inertial accelerations there are several terms and phenonema not present in the non-relativistic development of Coriolis. The computational task is to solve for these in order to compare with data from novel or known experiments.

In summary therefore the equations of generally covariant rotational dynamics are:

$$d \wedge T^a = j^a \tag{1.167}$$

$$d \wedge \widetilde{T}^a = J^a \tag{1.168}$$

where

$$j^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \tag{1.169}$$

$$J^a = \widetilde{R}^a{}_b \wedge q^b - \omega^a{}_b \wedge \widetilde{T}^b \tag{1.170}$$

In tensor notation Eqs.(1.167) and (1.168) become:

$$\partial_\mu \widetilde{T}^{a\mu\nu} = \widetilde{j}^{a\nu} \tag{1.171}$$

$$\partial_\mu T^{a\mu\nu} = \widetilde{J}^{a\nu} \tag{1.172}$$

where

$$\widetilde{j}^{a\nu} = \widetilde{R}^a{}_b{}^{\mu\nu} - \omega^a{}_{\mu b} \widetilde{T}^{b\mu\nu} \tag{1.173}$$

$$\widetilde{J}^{a\nu} = R^a{}_b{}^{\mu\nu} - \omega^a{}_{\mu b} T^{b\mu\nu} \tag{1.174}$$

In vector notation the currents are defined by

$$\widetilde{j}^{a\mu} = \left(\frac{1}{c}\widetilde{j}^{a0}, \widetilde{\mathbf{j}}^a\right) \tag{1.175}$$

$$\widetilde{J}^{a\mu} = \left(\frac{1}{c}\widetilde{J}^{a0}, \widetilde{\mathbf{J}}^a\right) \tag{1.176}$$

and Eqs.1.171 and 1.172 become four vector equations:

$$\boldsymbol{\nabla} \cdot \mathbf{T}^a_s = \frac{\widetilde{j}^{a0}}{c} \tag{1.177}$$

$$\boldsymbol{\nabla} \times \mathbf{T}^a_L + \frac{1}{c}\frac{\partial \mathbf{T}^a_s}{\partial t} = \widetilde{\mathbf{j}}^a \tag{1.178}$$

$$\boldsymbol{\nabla} \cdot \mathbf{T}^a_L = \widetilde{J}^{a0} \tag{1.179}$$

$$\boldsymbol{\nabla} \times \mathbf{T}^a_s - \frac{1}{c^2}\frac{\partial \mathbf{T}^a_L}{\partial t} = \widetilde{\mathbf{J}}^a \tag{1.180}$$

Analogously, the equations of generally covariant translational dynamics are:

$$d \wedge R^a{}_b = j^a{}_b \tag{1.181}$$

$$d \wedge \widetilde{R}^a{}_b = J^a{}_b \tag{1.182}$$

where

$$j^a{}_b = R^a{}_c \wedge \omega^c{}_b - \omega^a{}_c \wedge R^c{}_b \tag{1.183}$$

$$J^a{}_b = \widetilde{R}^a{}_c \wedge \omega^c{}_b - \omega^a{}_c \wedge \widetilde{R}^c{}_b \tag{1.184}$$

The equations of generally covariant electrodynamics are obtained from the ECE Ansatzen:

$$A^a = A^{(0)} q^a \tag{1.185}$$

$$F^a = A^{(0)} T^a \tag{1.186}$$

In form notation they are:

$$d \wedge F^a = \mu_0 j^a \tag{1.187}$$

$$d \wedge \widetilde{F}^a = \mu_0 J^a \tag{1.188}$$

where:

$$j^a = \frac{A^{(0)}}{\mu_0} \left(R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \right) \tag{1.189}$$

$$J^a = \frac{A^{(0)}}{\mu_0} \left(\widetilde{R}^a{}_b \wedge q^b - \omega^a{}_b \wedge \widetilde{T}^a \right) \tag{1.190}$$

In tensor notation:

$$\partial_\mu \widetilde{F}^{a\mu\nu} = \mu_0 \widetilde{j}^{a\nu} \tag{1.191}$$

$$\partial_\mu F^{a\mu\nu} = \mu_0 \widetilde{J}^{a\nu} \tag{1.192}$$

where:

$$\widetilde{j}^{a\nu} = \frac{A^{(0)}}{\mu_0} \left(\widetilde{R}^a{}_\mu{}^{\mu\nu} - \omega^a{}_{\mu b} \widetilde{T}^{b\mu\nu} \right) \tag{1.193}$$

$$\widetilde{J}^{a\nu} = \frac{A^{(0)}}{\mu_0} \left(R^a{}_\mu{}^{\mu\nu} - \omega^a{}_{\mu b} T^{b\mu\nu} \right) \tag{1.194}$$

and in vector notation:

$$\boldsymbol{\nabla} \cdot \mathbf{B}^a = \mu_0 \widetilde{j}^{a0} \tag{1.195}$$

$$\boldsymbol{\nabla} \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mu_0 \widetilde{\mathbf{j}}^a \tag{1.196}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a = \mu_0 c \widetilde{J}^{a0} \tag{1.197}$$

$$\boldsymbol{\nabla} \times \mathbf{B}^a - \frac{1}{c^2} \frac{\partial \mathbf{E}^a}{\partial t} = \frac{\mu_0}{c} \widetilde{\mathbf{J}}^a \tag{1.198}$$

It is seen that the generally covariant equations of electrodynamics and dynamics have the same overall structure and are inter-influential in general. This means that gravitation may influence electrodynamics as well as rotational dynamics. Many types of novel cross influences of this kind are indicated by ECE theory, on both classical and quantum levels [2]– [18].

Acknowledgements The British Parliament and Head of State, Queen Elizabeth II, are thanked for a Civil List pension. The voluntary staff of AIAS are thanked for many interesting discussions.

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