Chapter 4

The effect of torsion on the Schwarzschild Metric and light deflection due to gravitation

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Abstract

The effect of torsion on the Schwarzschild metric and light deflection due to gravitation is calculated straightforwardly using the tetrad method at the root of Einstein Cartan Evans (ECE) unified field theory. Consideration of torsion changes several of the assumptions at the root of standard model cosmologies such as Big Bang, and torsion is shown to affect the deflection of light due to gravitation. Thus, any deviations from Einstein Hilbert theory may be explained by the presence of torsion.

Keywords: Einstein Cartan Evans (ECE) unified field theory, Schwarzschild metric, light deflection due to gravitation, effect of torsion on standard model cosmologies.

4.1 Introduction

Light deflection due to gravitation is a famous prediction of gravitational general relativity, and is based on the Einstein Hilbert (EH) field equation published independently by Einstein and Hilbert in 1916 as is well known. The phenomenon of light deflection by the sun can now be measured to an accuracy of one part in one hundred thousand (NASA Cassini) and even more accurate tests are being prepared by NASA. It is shown in Section 4.3 that any small deviations from the EH result that may become observable can be understood straightforwardly as being due to space-time torsion in general relativity. The Cartan torsion is of key importance to the recently inferred [1]– [16] Einstein Cartan Evans (ECE) unified field theory because the electromagnetic field is Cartan torsion within a factor $cA^{(0)}$ with the units of volts and thus referred to as the primordial voltage. The EH equation is well known to produce twice the Newtonian result for the deflection angle of light grazing a mass, such as the mass of the sun. In Section 4.2 this result is derived straightforwardly using the tetrads appropriate to the Schwarzschild metric (SM). The latter was used in the original and famous test by Eddington and co-workers and is used here to illustrate the effect of torsion. More generally in ECE field theory metrics must be calculated in the presence of Cartan torsion, which changes many of the basic assumptions of standard model cosmology. In the presence of Cartan torsion the Ricci cyclic equation is no longer true, the Riemann tensor is no longer anti-symmetric in its first two indices, the symmetric metric and symmetric Ricci tensor are true only if the central part of torsion affected motion is considered, and the symmetric Christoffel symbol must be replaced by a more general and asymmetric gamma connection. The neglect of Cartan torsion in cosmologies such as Big Bang is arbitrary. Without Cartan torsion the gravitational field cannot be unified with the electromagnetic field, which as originally inferred by Cartan himself, is the Cartan torsion within $cA^{(0)}$ [1]– [16]. Attempts to interpret astronomical data in terms of a purely central cosmology such as Big Bang are therefore purposeless because torsion is likely to pervade all cosmologies. There is no reason to assert that Cartan curvature is always large in magnitude in comparison with Cartan torsion. This EH assumption appears to be true for the sun, but may not be true for other cosmological objects.

4.2 Calculation of gravitational light deflection using the tetrad method

The SM is well known to be the first solution to the Einstein Hilbert field equation, and was inferred in 1916. The SM metric is the static solution for a spherically symmetric space-time and produces a deflection of light twice that expected from Newtonian theory. For light deflection from the sun this result of the SM has been verified by NASA Cassini to one part in one hundred thousand. So for the sun, EH theory is adequate to this accuracy. For other systems however, this may not be the case at all, because there is no reason to assume that Cartan torsion is small in magnitude compared with Cartan curvature for all cosmological objects [1]– [16]. The SM $g_{\mu\nu}$ is necessarily symmetric in its indices:

$$g_{\mu\nu} = g_{\nu\mu} \tag{4.1}$$

and defines the square of the line element:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{4.2}$$

where x^μ is the four-coordinate:

$$x^\mu = (ct, x, y, z)\,. \tag{4.3}$$

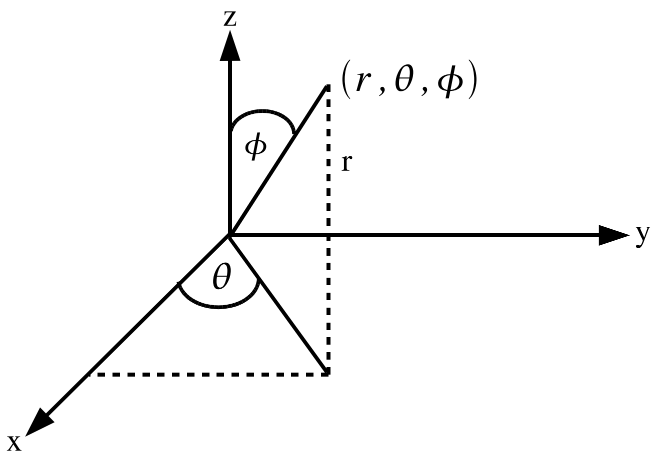

Figure 4.1: Spherical polar system

This symmetric metric is defined in terms of the tetrad of ECE theory [1]– [16] by:

$$g_{\mu\nu} = q^a{}_\mu q^b{}_\nu \eta_{ab} \tag{4.4}$$

where η_{ab} is the Minkowski metric of flat space-time. The latter is defined by:

$$\eta_{ab} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{4.5}$$

In spherical polar coordinates the line element is:

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2 \tag{4.6}$$

where

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2 \tag{4.7}$$

and the SM in spherical polar coordinates and complete S.I. units is well known to be:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2. \tag{4.8}$$

Here G is the Newton gravitational constant, M is the mass of the object responsible for the light deflection (e.g. the sun), c is the speed of light and where r is the radial coordinate of the spherical polar system defined in Fig. 4.1: The SM reduces to the Minkowski result in the limit of large r or small M as is well known. In Cartesian coordinates the Minkowski metric is found from:

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 \tag{4.9}$$

and in spherical polar coordinates it is:

$$\eta_{ab} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \phi \end{bmatrix}. \tag{4.10}$$

The SM in spherical polar coordinates is:

$$g_{\mu\nu} = \begin{bmatrix} -\left(1 - \frac{2GM}{c^2 r}\right) & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2GM}{c^2 r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \phi \end{bmatrix}. \tag{4.11}$$

Therefore from a comparison of the diagonal elements in Eqs.(4.10) and (4.11) the tetrads of the SM may be found straightforwardly. The non-zero Minkowski elements in spherical polar coordinates are:

$$\eta_{00} = -1, \quad \eta_{11} = 1, \quad \eta_{22} = r^2, \quad \eta_{33} = r^2 \sin^2 \phi \tag{4.12}$$

and the non-zero SM elements in the same coordinates are:

$$g_{00} = -\left(1 - \frac{2GM}{c^2 r}\right), \quad g_{11} = \left(1 - \frac{2GM}{c^2 r}\right)^{-1}, \quad g_{22} = r^2, \quad g_{33} = r^2 \sin^2 \phi \tag{4.13}$$

where in general:

$$g_{00} = q^a{}_0 q^b{}_0 \eta_{ab} \\ \vdots \tag{4.14}$$

$$g_{33} = q^a{}_3 q^b{}_3 \eta_{ab} \tag{4.15}$$

Considering only the diagonal elements Eqs.(4.14) to (4.15) simplify to:

$$g_{00} = g^0{}_0 q^0{}_0 \eta_{00} \\ \vdots \tag{4.16}$$

$$g_{33} = g^3{}_3 q^3{}_3 \eta_{33} \tag{4.17}$$

Therefore the required tetrad elements of the SM are:

$$q^0{}_0 = \left(1 - \frac{2GM}{c^2 r}\right)^{1/2} \tag{4.18}$$

$$q^1{}_1 = \left(1 - \frac{2GM}{c^2 r}\right)^{1/2} \tag{4.19}$$

$$q^2{}_2 = 1 \tag{4.20}$$

$$q^3{}_3 = 1. \tag{4.21}$$

In the limit of large r or small M these reduce to the correct Minkowski elements:

$$g_{00} \to \eta_{00} \quad \text{etc.} \tag{4.22}$$

so Eqs.(4.16) – (4.17) are correctly compatible with this limit. According to the ECE Lemma [1]– [16]

$$\Box q^0{}_0 = R_0 q^0{}_0 \tag{4.23}$$

$$\Box q^1{}_1 = R_1 q^1{}_1 \tag{4.24}$$

so scalar curvatures R_0 and R_1 are generated by two of the tetrad elements of the SM. There are no ECE scalar curvatures produced by the Minkowski metric, and this result is compatible with the fact that that metric describes a flat space-time with no curvature. The four tetrads of the Minkowski metric are all unity. In spherical polar coordinates:

$$r = \left(x^2 + y^2 + z^2\right)^{1/2} \tag{4.25}$$

so Eqs.(4.23) and (4.24) reduce to:

$$\nabla^2 q^0{}_0 = -R_0 q^0{}_0 \tag{4.26}$$

$$\nabla^2 q^1{}_1 = -R_1 q^1{}_1 \tag{4.27}$$

compatible with the fact that the SM is a static solution of the EH field equation for a spherically symmetric spacetime.

The spherical polar coordinates and Cartesian coordinates are related by:

$$\left.\begin{array}{c} x = r\sin\phi\cos\theta \\ y = r\sin\phi\sin\theta \\ z = r\cos\phi \end{array}\right\} \tag{4.28}$$

so:

$$x^2 + y^2 + z^2 = r^2. \tag{4.29}$$

The infinitesimal elements are defined [17] by:

$$\left.\begin{array}{c} dx = -r\sin\phi\sin\theta d\theta + r\cos\phi\cos\theta d\phi + \sin\phi\cos\theta dr \\ dy = r\sin\phi\cos\theta d\theta + r\cos\phi\sin\theta d\phi + \sin\phi\sin\theta dr \\ dz = -r\sin\phi d\theta + \cos\phi dr \end{array}\right\} \tag{4.30}$$

so the square of the line element is:

$$ds^2 = dx^2 + dy^2 + dz^2 = dr^2 + r^2 d\phi^2 + r^2\sin^2\phi d\theta^2. \tag{4.31}$$

The space-like metric elements in curvilinear coordinates are the squares of the scale factors [17]:

$$g_{11} = h^2{}_1, \quad g_{22} = h^2{}_2, \quad g_{33} = h^2{}_3. \tag{4.32}$$

The scale factors in spherical polar coordinates [17] are:

$$h_1 = h_r = 1, \quad h_2 = h_\phi = r, \quad h_3 = h_\theta = r\sin\phi \tag{4.33}$$

in Euclidean space-time. The surface of a sphere is:

$$S = \int_0^{2\pi} d\theta \int_0^{\pi} r^2 \sin\phi d\phi = 4\pi r^2 \tag{4.34}$$

and the volume of a sphere is:

$$V = \int_0^r S dr = \frac{4}{3}\pi r^3. \tag{4.35}$$

The Euclidean unit vectors of the spherical polar coordinate system are [17]:

$$\left.\begin{aligned} \mathbf{e}_r &= \sin\phi\cos\theta\mathbf{i} + \sin\phi\sin\theta\mathbf{j} + \cos\phi\mathbf{k} \\ \mathbf{e}_\phi &= \cos\phi\cos\theta\mathbf{i} + \cos\phi\sin\theta\mathbf{j} - \sin\phi\mathbf{k}. \\ \mathbf{e}_\theta &= -\sin\theta\mathbf{i} + \cos\theta\mathbf{j} \end{aligned}\right\} \tag{4.36}$$

where **i**, **j** and **k** are the unit vectors of the Cartesian system. The Euclidean vector field in spherical polar coordinates is therefore:

$$\begin{aligned} \mathbf{V} &= V_r\mathbf{e}_r + V_\phi\mathbf{e}_\phi + V_\theta\mathbf{e}_\theta \\ &= V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}. \end{aligned} \tag{4.37}$$

In Cartan geometry [1]– [16] [18], the governing equations of the EH equation and the SM are torsion-less:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b = 0 \tag{4.38}$$

$$R^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{4.39}$$

$$R^a{}_b \wedge q^b = 0 \tag{4.40}$$

$$d \wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b - R^a{}_c \wedge \omega^c{}_b = 0. \tag{4.41}$$

Here T^a is the Cartan torsion form, q^a is the Cartan tetrad form, $\omega^a{}_b$ is the spin connection, and $\omega^a{}_b$ is the curvature or Riemann form of Cartan geometry. The elements of the tetrad of the SM are diagonal as shown already, and the non-vanishing elements of the Riemann tensor of the SM are:

$$R^0{}_{101}, R^0{}_{202}, R^0{}_{303}, R^0{}_{212}, R^0{}_{313}, R^1{}_{212}, R^1{}_{313}, R^2{}_{323}. \tag{4.42}$$

The Riemann form and Riemann tensor are related by [1]– [16] [18]:

$$R^a{}_{b\mu\nu} = q^a{}_\rho q^a{}_b R^\rho{}_{\sigma\mu\nu}. \tag{4.43}$$

In the presence of the Cartan torsion, equations (4.38) to 4.41 become:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{4.44}$$

$$R^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b \tag{4.45}$$

$$d \wedge T^a + \omega^a{}_b \wedge T^b := R^a{}_b \wedge q^b \tag{4.46}$$

$$d \wedge R^a{}_b + \omega^a{}_c \wedge R^c{}_b - R^a{}_c \wedge \omega^c{}_b := 0. \tag{4.47}$$

Eqs.(4.44) and (4.45) are the two Cartan structure equations, and Eqs.(4.46) and (4.47) are the two Bianchi identities. These are well known equations of standard Cartan geometry and form the basis of ECE theory [1]– [16] through the ansatzen:

$$A^a = A^{(0)} q^a \tag{4.48}$$

$$F^a = A^{(0)} T^a \tag{4.49}$$

first proposed by Cartan himself in well known correspondence with Einstein. Here $A^{(0)}$ is the electromagnetic potential form, and $F^{(0)}$ is the electromagnetic field form. In the EH equation and SM there is no consideration given to the interaction of gravitation with other fields such as electromagnetism. In the presence of torsion the familiar Ricci cyclic equation (4.40) of EH theory and the SM is no longer obeyed. In tensor notation the Ricci cyclic equation is:

$$R_{\sigma\mu\nu\rho} + R_{\sigma\rho\mu\nu} + R_{\sigma\nu\rho\mu} = 0 \tag{4.50}$$

but this is not the case in the presence of torsion. The latter means therefore that the Riemann tensor is no longer anti-symmetric in its first two indices, and that the Christoffel connection becomes the general gamma connection no longer symmetric in its lower two indices. Cartan torsion fundamentally changes cosmologies based on the EH equation, for example Big Bang.

Restricting attention in this section to the EH field theory, the spin connection of the SM may be obtained from the tetrad of the SM using:

$$d \wedge q^a + \omega^a{}_b \wedge q^b = 0. \tag{4.51}$$

The Riemann form and the spin connection are related by the second Cartan structure equation 4.45:

$$R^a{}_b = d \wedge \omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b. \tag{4.52}$$

In Section 4.3 the equation (4.51) will be perturbed by a small torsion δT^a, to give:

$$d \wedge q^a + \omega^a{}_b \wedge q^b = \delta T^a \tag{4.53}$$

while in the rest of Section 4.2 the light deflection of the SM will be calculated by the tetrad method. This is shown to be much simpler and easier to use and understand than the conventional metric method [18] [19]. Use of the tetrad method also allows the effect of torsion to be calculated via equation (4.53).

The SM written out in spherical polar coordinates (Fig. 4.1) is:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\phi^2 + r^2 \sin^2 \phi d\theta^2. \tag{4.54}$$

Light travels along null paths:

$$ds^2 = 0. \tag{4.55}$$

Now restrict consideration to a single plane through the center of mass:

$$\theta = 0. \tag{4.56}$$

Therefore Eq.(4.54) becomes:

$$c^2 dt^2 = \left(1 - \frac{2GM}{c^2 r}\right)^{-2} dr^2 + r^2 \left(1 - \frac{2GM}{c^2 r}\right)^{-1} d\phi^2. \tag{4.57}$$

The metric corresponding to this equation is:

$$g_{\mu\nu} = \begin{bmatrix} \left(1 - \frac{2GM}{c^2 r}\right)^{-2} & 0 \\ 0 & r^2 \left(1 - \frac{2GM}{c^2 r}\right)^{-1} \end{bmatrix} \tag{4.58}$$

which reduces to the Minkowski metric for large r or small M:

$$\eta_{ab} = \begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix}. \tag{4.59}$$

Therefore using Eq.(4.4) the tetrads are:

$$q_{rr} = \left(1 - \frac{2GM}{c^2 r}\right)^{-1}, \tag{4.60}$$

$$q_{\phi\phi} = \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2}. \tag{4.61}$$

If:

$$c^2 r \gg 2GM \tag{4.62}$$

then:

$$q_{rr} \to 1 + \frac{2GM}{c^2 r} + \cdots \tag{4.63}$$

$$q_{\phi\phi} \to 1 + \frac{GM}{c^2 r} + \cdots \tag{4.64}$$

The tetrad element q_{rr} means that r is not a straight line, it is a curve:

$$\zeta\,(r) = \zeta^{(0)} q_{rr} \tag{4.65}$$

where $\zeta^{(0)}$ is a scalar proportionality factor. By differentiation with respect to r:

$$c^2 \frac{\partial q_{rr}}{\partial r} = -\frac{2GM}{r^2}. \tag{4.66}$$

The Newtonian force between a photon of mass m and the sun of mass M is:

$$F = -\frac{GmM}{r^2}. \tag{4.67}$$

The force from Eq.(4.66) is:

$$F = -mc^2 \frac{\partial q_{rr}}{\partial r} = -\frac{2GmM}{r^2}. \tag{4.68}$$

This is twice the Newtonian force and is $\partial q_{rr}/\partial r$ multiplied by the photon rest energy:

$$E_0 = mc^2 = \hbar\omega_0. \tag{4.69}$$

Eq.4.69 is the Planck / Einstein / de Broglie equation. Using the equivalence of inertial and gravitational mass, the force from Eq.(4.68) is:

$$F = mg = -mc^2 \frac{\partial q_{rr}}{\partial r} \tag{4.70}$$

so the acceleration due to gravity is due to the r derivative of the radial tetrad within a factor c^2:

$$g = -c^2 \frac{\partial q_{rr}}{\partial r}. \tag{4.71}$$

The angle of deflection in the Eddington experiment is defined by Fig. 4.2:

The Newtonian result is:

$$\delta\,(\text{Newton}) = \frac{2MG}{c^2 r_0} \tag{4.72}$$

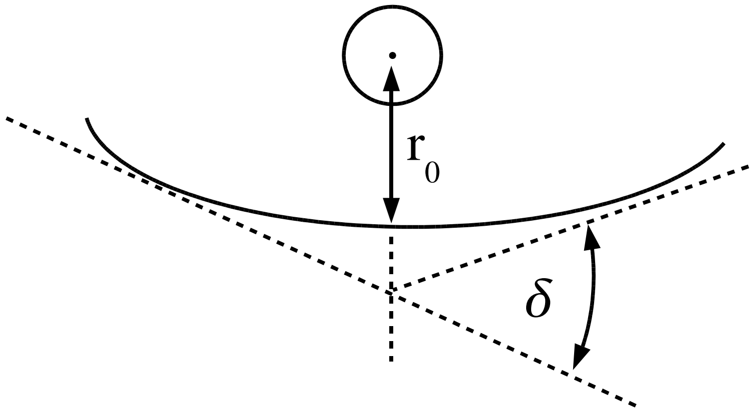

Figure 4.2: The angle of deflection in the Eddington experiment

where r_0 is the distance of closest approach. So the result from the EH theory is twice this from Eq.(4.68):

$$\delta\,(\text{Schwarzschild}) = \frac{4MG}{c^2 r_0} \tag{4.73}$$

Using the tetrad method the effect of Cartan torsion on this result will be calculated in Section 4.3. The tetrad method developed in this Section for the first time, is straightforward, and is ideally suited to calculate the effect of torsion from Eq.(4.53) from standard Cartan geometry. The metric method of calculating the Eddington deflection is much more complicated.

4.3 Torsional perturbation of light deflection due to gravity

The angle of deflection in the absence of torsional perturbation is given from the result in Eq.(4.73) by:

$$\delta = 2\left(q_r r - 1\right)_{r=r_0} \tag{4.74}$$

In the absence of torsion, Eq.(4.51) gives:

$$d \wedge q_{rr,0} = -\omega_0 \wedge q_{rr,0} \tag{4.75}$$

where ω_0 is the spin connection in the absence of torsion. In the presence of a small torsional perturbation Eq.(4.75) becomes:

$$d \wedge q_{rr,T} = -\omega \wedge q_{rr,T} + \delta T \tag{4.76}$$

In a first approximation:

$$\omega \sim \omega_0 \tag{4.77}$$

and:

$$\omega \wedge q_{rr,T} \sim \omega_0 \wedge q_{rr,0} \tag{4.78}$$

so

$$d \wedge q_{rr,T} - d \wedge q_{rr,0} \sim \delta T \tag{4.79}$$

or

$$d \wedge (\delta q_{rr}) \sim \delta T \tag{4.80}$$

and:

$$\Delta\delta \sim 2\,(\delta q_{rr})_{r=r_0} \tag{4.81}$$

From Eqs.(4.80) and (4.81) it is clear that the torsional perturbation δT will change the angle of deflection by $\Delta\delta$. In Cartesian coordinates introduce a perturbation of the type:

$$\delta q_{rr} = \frac{1}{\sqrt{2}}\,(1-i)\,e^{i\phi} \sim \frac{1}{\sqrt{2}}\phi \tag{4.82}$$

for $\phi \ll 1$. Thus:

$$\Delta\delta \sim \frac{2}{\sqrt{2}}\phi \tag{4.83}$$

This is a simple illustration of the effect of torsion on the angle of deflection of light due to gravitation. From experimental data (NASA Cassini) on gravitational lensing within the solar system it is known that ϕ must be very small for the sun photon system because the EH result (torsionless or baseline result) is accurate to one part in one hundred thousand. For other cosmological objects such as rotating pulsars of great mass, the effect of torsion could be much larger. In this illustration the SM has been assumed to be approximately true in the presence of a torsional perturbation. Metrics in a generally covariant unified field theory must however be calculated from the second Bianchi identity of Cartan geometry. The torsionless SM is calculated as a solution of the second Bianchi identity of Riemann geometry, in which torsion is zero.

4.4 Discussion

Naive unification of the gravitational and electromagnetic fields was first attempted by Reissner [20] and independently by Nordstrom [21], shortly after the discovery of the Schwarzschild metric. Naive unification takes place without any consideration of the Cartan torsion, using the minimal substitution rule:

$$\partial_\mu \rightarrow D_\mu \tag{4.84}$$

The electromagnetic field in naive unification cannot therefore be the Cartan torsion and the effect of electromagnetism is introduced through the addition of an electromagnetic term to the canonical energy momentum of EH field theory. Einstein was dissatisfied with naive unification, and the idea that the electromagnetic field is the Cartan torsion was first suggested by Cartan himself in well known correspondence with Einstein during the twenties and thirties of the last century. Einstein then worked on unification until 1955, as is well known, but did not develop a satisfactory theory. The minimal substitution rule does not produce uniquely defined results [18] [19] and still uses the Christoffel symbol of torsionless EH theory. It was not until the inference of the experimentally observable ECE spin field ($\boldsymbol{B}^{(3)}$) in 1992 [1]– [16] that the general covariance of electromagnetism began to be correctly developed and it was not until 2003 to present that the correct mathematical structure for ECE unification finally emerged from $\boldsymbol{B}^{(3)}$ theory and gauge theory ($O(3)$ electrodynamics [1]– [16]). Naive unification does not produce an ECE spin field, which requires the use of Cartan torsion. The spin field is the direct result of the spinning space-time necessary to describe generally covariant electromagnetism unified in a self-consistent and rigorous geometrical manner with gravitation and the other fundamental fields. With the Christoffel connection of naive unification there is no spinning space-time, only a curving space-time. It is self-inconsistent to add an electromagnetic term to the canonical energy-momentum tensor without spinning space-time. This internal inconsistency is present in all naive unification schemes, such as that of Newman et al. [22] for the Kerr metric. There are several other phenomena [1]– [16] now known to be explicable with ECE but not by naive unification. Misner [23] for example, has used the tetrad method in a gravitational context, but again does not consider Cartan torsion in any relevant detail. Newman and Penrose [24] developed the tetrad method for use with spinors, but again in a restricted gravitational context using the null tetrad. Spinors were discovered by Cartan himself in 1913 [25]. There is some discussion of the method of Newman and Penrose by Barrett [26] but this does not provide even the basis for a generally covariant unified field theory. Throughout the twentieth century, there was difficulty in the development of a generally covariant unified field theory because the ECE spin field was not known. The spin field was inferred only in 1992 [1]– [16]. In the twentieth century, undue reliance continued to be placed on the Maxwell Heaviside (MH) field theory inferred in the nineteenth century. The MH theory is not generally covariant [1]– [16], it is special relativity, and therefore can only be Lorentz covariant. The MH theory does not use a spinning space-time, required for self consistent unification, and for this reason cannot produce an ECE spin field $\boldsymbol{B}^{(3)}$. MH theory must be made generally covariant before it can be unified with gravitation. This is an obvious point, but one which was overlooked for a hundred years or more. In the twentieth century, considerable confusion was caused by the Copenhagen School, especially by the Heisenberg Uncertainty Principle. The latter has no place in physics, which must be an objective and causal subject as recognised by Bacon in the seventeenth century. Several independent experimental refutations of the Uncertainty Principle are now available [1]– [16]. ECE theory produces a rigorous and generally covariant quantum field theory [1]– [16] without using the Copenhagen assertions. In a new twenty first century perspective, Copenhagen is little more than subjective assertion, or pathological science where an idea is not evaluated critically. The Uncertainty Principle deliberately introduces obscure anthropomorphism into science, and for this reason was immediately rejected by the causal realist school of thought led by Einstein, Schrödinger, de Broglie and followers.

The neglect of Cartan torsion restricted twentieth century cosmology to models such as the Big Bang. This model was immediately rejected by Hoyle and followers, as is well known. ECE theory has thrown considerable new light on this twentieth century debate [1]– [16]. An oscillatory cosmological model is favored by ECE theory [1]– [16]. This point may be illustrated in a simple manner as follows. For the general metric in spherically symmetric space-time:

$$ds^2 = -e^{2\alpha(t,r)}c^2dt^2 + e^{2\beta(t,r)}dr^2 + r^2d\Omega^2 \tag{4.85}$$

and therefore the tetrads are:

$$q^0{}_0 = e^\alpha, \quad q^1{}_1 = e^\beta, \quad q^2{}_2 = q^3{}_3 = 1 \tag{4.86}$$

The ECE Lemmas are therefore:

$$\Box e^\alpha = R_0 e^\alpha \tag{4.87}$$

$$\Box e^\beta = R_1 e^\beta \tag{4.88}$$

The differentiations in Eqs.(4.87) and (4.88) are therefore defined by:

$$\Box e^\alpha = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right) e^\alpha \tag{4.89}$$

where from the Leibnitz Theorem:

$$\frac{\partial^2}{\partial t^2}e^\alpha = \frac{\partial}{\partial t}\left(\frac{\partial \alpha}{\partial t}e^\alpha\right) = \left(\frac{\partial^2 \alpha}{\partial t^2} + \left(\frac{\partial \alpha}{\partial t}\right)^2\right) e^\alpha \quad \text{etc.} \tag{4.90}$$

Thus:

$$\Box e^\alpha = \left(\Box\alpha + \frac{1}{c^2}\left(\frac{\partial \alpha}{\partial t}\right)^2 - \left(\frac{\partial \alpha}{\partial x}\right)^2 - \left(\frac{\partial \alpha}{\partial y}\right)^2 - \left(\frac{\partial \alpha}{\partial z}\right)^2\right) e^\alpha \tag{4.91}$$

and the scalar curvatures are:

$$R_0 = \Box\alpha + \frac{1}{c^2}\left(\frac{\partial \alpha}{\partial t}\right)^2 - \left(\frac{\partial \alpha}{\partial x}\right)^2 - \left(\frac{\partial \alpha}{\partial y}\right)^2 - \left(\frac{\partial \alpha}{\partial z}\right)^2 \tag{4.92}$$

$$R_1 = \Box\beta + \frac{1}{c^2}\left(\frac{\partial \beta}{\partial t}\right)^2 - \left(\frac{\partial \beta}{\partial x}\right)^2 - \left(\frac{\partial \beta}{\partial y}\right)^2 - \left(\frac{\partial \beta}{\partial z}\right)^2 \tag{4.93}$$

All spherically symmetric space-time solutions of the ECE theory obey this result. Eqs. (4.87) and (4.88) are equations of classical and causal physics. If it were possible to find complex valued solutions:

$$\alpha = \alpha^{'} + i\alpha^{"} \tag{4.94}$$

$$\beta = \beta^{'} + i\beta^{"} \tag{4.95}$$

then Eqs.(4.87) and (4.88) would become eigen-equations via the imaginary components $i\alpha^{"}$ and $i\beta^{"}$. For real valued α and β however there is only one R_0 and only one R_0. In standard model (twentieth century) cosmology, the existence of the ECE Lemma is not known, and Big Bang for example depends on torsionless solutions of the EH equation. The latter is only a limit of ECE cosmology. Wave cosmologies for example can be developed in ECE theory by considering the tetrads to be defined by:

$$q^0{}_0 q^0{}_0 = e^{2\alpha} \tag{4.96}$$

$$q^1{}_1 q^1{}_1 = e^{2\beta} \tag{4.97}$$

If these tetrads are complex valued:

$$q^0{}_0 = e^{\alpha' + i\alpha''}, \quad q^0{}_0{}^* = e^{\alpha' - i\alpha''}, \tag{4.98}$$

$$q^1{}_1 = e^{\beta' + i\beta''}, \quad q^1{}_1{}^* = e^{\beta' - i\beta''} \tag{4.99}$$

then:

$$q^0{}_0 q^0{}_0{}^* = e^2\alpha \tag{4.100}$$

$$q^1{}_1 q^1{}_1{}^* = e^2\beta \tag{4.101}$$

where $*$ denotes complex conjugate. Eqs.(4.98) to (4.101) have solutions:

$$\alpha = \alpha', \quad \beta = \beta' \tag{4.102}$$

for all α''. Therefore:

$$q^0{}_0 = e^{\alpha} e^{i\alpha''}, \quad q^1{}_1 = e^{\beta} e^{i\beta''} \tag{4.103}$$

and the transformations:

$$q^0{}_0 \rightarrow e^{i\alpha''} q^0{}_0 \tag{4.104}$$

$$q^1{}_1 \rightarrow e^{i\beta''} q^1{}_1 \tag{4.105}$$

leave the metric elements unchanged:

$$g_{00} \rightarrow e^{-i\alpha''} g_{00} e^{i\alpha''} \tag{4.106}$$

$$g_{11} \rightarrow e^{-i\beta''} g_{00} e^{i\beta''} \tag{4.107}$$

The oscillatory or wave cosmologies of ECE can therefore be defined by the eigenequations:

$$\square e^{i\alpha''} = R_0'' e^{i\alpha''} \tag{4.108}$$

$$\square e^{i\beta''} = R_1'' e^{i\beta''} \tag{4.109}$$

where for one eigenfunction there are many eigenvalues R_0'' and R_1''. Big Bang severely restricts what is actually available in cosmology to a uniformly expanding universe represented by Eqs.(4.87) and (4.88). More generally, ECE theory gives wave cosmologies described by Eqs.(4.108) and (4.109). Most generally, ECE gives cosmologies in which torsion and curvature play an equal role.

The interaction of electromagnetism and gravitation (i.e. of torsion and curvature) is of key importance also on the microscopic scale, as well as the macroscopic scale represented by cosmology. ECE theory now allows this fact to be much better defined. The hydrogen (H) atom on a microscopic scale, for example, is made up of an electron bound to a proton. The mass of the H atom is less [18] than the sum of the mass of a proton and an electron. The reason is that there is a negative binding energy. To separate the electron from the proton energy has to be used. There is interaction of electromagnetism with gravitation inside the H atom and this interaction produces the mass deficit referred to already. A standard model text such as ref. [18] deduces that gravitation must interact with all forms of energy and momentum. This is another way of stating the Einstein equivalence principle [18]. The latter means that the equations of general relativity must reduce to those of special relativity in the absence of gravitation. In special relativity and in the non-relativistic limit, the sum of the proton and electron masses would be the same as the mass of the H atom.

So the H atom in ECE theory is described by:

$$\Box q^a{}_\mu = R q^a{}_\mu \tag{4.110}$$

$$R = -kT \tag{4.111}$$

$$R = q^\lambda{}_a \partial^\mu \left(\Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda \right) \tag{4.112}$$

Here $q^a{}_b$ is the wavefunction and also the field [1]– [16], R is the ECE scalar curvature, k is the Einstein constant, T is the index contracted canonical energy-momentum tensor, $\Gamma^\nu{}_{\mu\lambda}$ is the general gamma connection and $\omega^a{}_{\mu b}$ is the spin connection. The Einstein equivalence principle means that:

$$kT \rightarrow \left(\frac{mc}{\hbar} \right)^2 \tag{4.113}$$

in the limit of no gravitation (special relativity). Here m is the mass of the H atom. So it is seen that in the presence of gravitation (Eq.(4.110)) the mass of the H atom is changed from the value given by Eq.(4.113), which is the Dirac equation of the H atom:

$$\left(\Box + \left(\frac{mc}{\hbar} \right)^2 \right) q^a{}_\mu = 0 \tag{4.114}$$

The electromagnetic interaction between the electron and proton in the H atom is described by the ECE field equations:

$$d \wedge F^a = \mu_0 j^a \tag{4.115}$$

$$d \wedge \widetilde{F}^a = \mu_0 J^a \tag{4.116}$$

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{4.117}$$

and thus by a linear inhomogeneous differential equation [1]– [16]:

$$d \wedge \left(d \wedge A^a + \omega^a{}_b \wedge A^b \right) = \mu_0 j^a \tag{4.118}$$

$$j^a = \frac{A^{(0)}}{\mu_0} \left(R^a{}_b \wedge q^b - q^a{}_b \wedge T^b \right) \tag{4.119}$$

At resonance, j^a can be amplified by many orders of magnitude, giving rise to a new source of electric power. This energy is tapped from the H atom, and has recently been observed experimentally [1]– [16]. The H orbitals given by the Dirac equation give no hint of the existence of this energy. The Schrödinger equation is the non-relativistic limit of the Dirac equation and gives even less information about the generally covariant nature of the H atom. The standard model of the H atom is described by the Schrödinger equation with the Coulomb Law. The latter is given by the:

$$j^a = 0 \tag{4.120}$$

limit of the ECE field equations (4.115) to (4.117).

The essence of the ECE theory is the use of the tetrad, which is both the fundamental unified field and also the unified wave-function. In this sense classical and quantum mechanics are unified, they are both manifestations of Cartan geometry, and the needless mysteries of the Copenhagen school are removed from physics. The tetrad may also be used to give a deeper meaning to the Eddington experiment. The relativistic result (4.73) is twice the Newtonian result, and at first sight does not reduce to the Newtonian result. The reason for this is that Eq.(4.73) is derived using Eq.(4.55) for motion infinitesimally close to the speed of light of the photon of mass m, (the lightest particle known in nature). Newtonian dynamics deals with particles moving at $v \ll c$. In this limit the light-like condition (4.55) no longer holds, and the radial metric must be calculated from Eq.(4.8). The relevant tetrad to consider is:

$$q^1{}_1 = \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} \tag{4.121}$$

and when:

$$2GM \ll c^2 r \tag{4.122}$$

this is:

$$q^1{}_1 \rightarrow 1 + \frac{GM}{c^2 r} \tag{4.123}$$

Therefore Eq.(4.65) is replaced by:

$$\zeta(r) = \zeta^{(0)} q^1{}_1 \tag{4.124}$$

so we obtain:

$$c^2 \frac{\partial q^1{}_1}{\partial r} = -\frac{GM}{r^2} \tag{4.125}$$

and the force:

$$F = -\frac{GmM}{r^2} \tag{4.126}$$

This result is the same as the Newtonian force governing the orbit of a mass m around a mass M, so the deflection is Eq.(4.72). This describes the Kepler laws and the orbit of a planet around the sun. However, the interpretation of Eq.(4.126) is different from that of Newton, who derived his inverse square law using an Euclidean space. Time was considered by Newton as a distinct from space. The ECE result (4.126) is derived by considering space and time to be unified into a spacetime with in general curvature and torsion.

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