Chapter 5

The resonant Coulomb Law from ECE Theory: Application to the hydrogen atom

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Abstract

The Einstein Cartan Evans (ECE) generally covariant unified field theory is applied to the Coulomb law by defining the electric field in terms of a scalar potential and a spin connection vector. The resulting equation modifies the Poisson equation into a linear inhomogeneous differential equation with resonant solutions. When applied to the hydrogen (H) atom as a model, the resonances thus produced by the spin connection can cause amplified oscillations, ionization and the production of a free electron. The latter may be used in a circuit to produce electric power.

Keywords: Einstein Cartan Evans (ECE) unified field theory, resonant Coulomb law, hydrogen atom, electric power from space-time.

5.1 Introduction

The Einstein Cartan Evans (ECE) field theory [1]– [16] is a generally covariant unified field theory based on standard Cartan geometry. As first suggested by Cartan himself, the electromagnetic field form (F^a) is the Cartan torsion form (T^a) within a factor $A^{(0)}$. Here $cA^{(0)}$ is the scalar potential in volts. The electromagnetic potential form (A^a) is the tetrad form (q^a) within the same factor $A^{(0)}$. The ECE field theory has been applied to many of the major areas of contemporary physics [1]– [16] and has been accepted as mainstream physics [17]. This is unsurprising in view of the fact that it is a logical extension to all fields of the theory of general relativity first applied by Einstein and Hilbert to the gravitational field. ECE theory has been tested in a number of ways [1]– [16] both for mathematical self consistency and against experimental data and has been shown to be a rigorous quantum field theory in which the tetrad is both the fundamental field and the wave-function.

In Section 5.2 the ECE theory is applied straightforwardly to the Coulomb law, and the Poisson equation is developed into a linear inhomogeneous differential equation with resonant solutions due to the spin connection of ECE theory [1]– [16]. In the simplest approximation, and considering only the scalar potential, the spin connection modifies the relation between the electric field strength (E in volt/m) and the scalar potential (ϕ in volts) to:

$$\mathbf{E} = -\left(\boldsymbol{\nabla} + \boldsymbol{\omega}\right)\phi \tag{5.1}$$

where $\boldsymbol{\omega}$ is a vector valued component of the complete spin connection form ${\omega^a}_{\mu b}$. In the standard model (Maxwell Heaviside field theory of special relativity) the equivalent of Eq.(5.1) is well known to be:

$$\mathbf{E} = -\boldsymbol{\nabla}\phi \tag{5.2}$$

so the term $\boldsymbol{\omega}$ introduces an extra potential from the ECE space-time of general relativity. In ECE theory the Coulomb law in the simplest approximation has the same form as the Coulomb law in the standard model:

$$\boldsymbol{\nabla}\cdot\mathbf{E} = \frac{\rho}{\epsilon_0} \tag{5.3}$$

but the relation between $\mathbf{E}$ and ϕ in ECE theory is always given by Eq.(5.1). Therefore it follows that:

$$\nabla^2\phi + \boldsymbol{\omega}\cdot\boldsymbol{\nabla}\phi + \left(\boldsymbol{\nabla}\cdot\boldsymbol{\omega}\right)\phi = -\frac{\rho}{\epsilon_0} \tag{5.4}$$

This is a linear inhomogeneous differential equation with the well known resonant solutions first inferred by the two Bernoulli's in 1739 and reported by Euler in 1743. Thus ECE gives the well known results of the Coulomb law, known to be a very precise law of physics, but also gives the possibility of extracting electric field strength from space-time (represented by the spin connection vector $\boldsymbol{\omega}$). Repeatable experimental results [18] indicate that E can be extracted at resonance from Eq.(5.4).

In Section 5.3 this phenomenon [18] is modeled with the H atom, used only as a simple thought model. It is shown that at resonance form Eq.(5.4) amplified oscillations develop and the H atom is ionized into a free electron and proton. The free electron can be used in a circuit to provide electric power from space-time. This model illustrates theoretically what has already been shown experimentally [18]to be possible. There is no standard model explanation for the results of ref. [18] because in the standard model:

$$\nabla^2\phi = -\frac{\rho}{\epsilon_0} \tag{5.5}$$

and there are no resonant solutions of this (Poisson) equation.

5.2 Resonant Coulomb Law

This law is deduced [1]– [16] from the first Cartan structure equation [19]:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{5.6}$$

and the first Bianchi identity:

$$d \wedge T^a := R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \tag{5.7}$$

where $R^a{}_b$ is the Riemann form of Cartan geometry. The fundamental ECE assumptions are:

$$A^a = A^{(0)} q^a, \quad F^a = A^{(0)} T^a, \tag{5.8}$$

so Eq.(5.6) produces the relation between F^a and A^a:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{5.9}$$

and Eq.(5.7) gives the homogeneous field equation [1]– [16]:

$$d \wedge F^a = \mu_0 j^a = A^{(0)} \left(R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \right) \tag{5.10}$$

where μ_0 is the vacuum S.I, permeability and j^a is the homogeneous current. The Hodge dual of Eq.(5.10) is:

$$d \wedge \widetilde{F}^a = \mu_0 J^a = A^{(0)} \left(\widetilde{R}^a{}_b \wedge q^b - \omega^a{}_b \wedge \widetilde{T}^b \right) \tag{5.11}$$

and defines the inhomogeneous field equation where J^a is the inhomogeneous current.

The Coulomb Law is part of Eq.(5.11), which is a linear, inhomogeneous differential equation of Cartan geometry with resonant solutions. In vector notation Eq.(5.9) is developed into two equations:

$$\mathbf{E}^a = -\frac{\partial \mathbf{A}^a}{\partial t} - c \boldsymbol{\nabla} A^{0a} - c \omega^{0a}{}_b \mathbf{A}^b + c \boldsymbol{\omega}^a{}_b A^{0b}, \tag{5.12}$$

$$\mathbf{B}^a = \boldsymbol{\nabla} \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b, \tag{5.13}$$

where the superscripts a and b derive from Cartan geometry [1]– [16]. Here $\mathbf{E}^a$ is the electric field strength (volt/m) and $\mathbf{B}^a$ the magnetic flux density (tesla). The scalar potential is:

$$\phi^a = c A^{0a} \tag{5.14}$$

and the vector potential is A^a. The spin connection form is a four vector with scalar ($\omega^{0a}{}_b$) and vector ($\boldsymbol{\omega}^a{}_b$) components [1]– [16]. In vector notation Eqs. (5.10) and (5.11) become four field equations of ECE theory. These are the generally covariant Gauss law of magnetism :

$$\nabla \cdot \mathbf{B}^a = \mu_0 \widetilde{j}^{0a}, \tag{5.15}$$

the generally covariant Faraday law of induction:

$$\boldsymbol{\nabla} \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mu_0 \widetilde{\mathbf{j}}^a, \tag{5.16}$$

the generally covariant Coulomb law:

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a = c\mu_0 \widetilde{J}^{0a} = \frac{\rho^a}{\epsilon_0}, \tag{5.17}$$

and the generally covariant Amperé-Maxwell law:

$$\boldsymbol{\nabla} \times \mathbf{B}^a - \frac{1}{c^2}\frac{\partial \mathbf{E}^a}{\partial t} = \frac{\mu_0}{c}\widetilde{\mathbf{J}}^a \tag{5.18}$$

The various resonance equations of ECE field theory [1]– [16] are found by substituting Eqs. (5.12) and (5.13) into Eqs.(5.15) to (5.18). The four currents are defined by:

$$\widetilde{j}^{a\nu} = \left(\frac{1}{c}\widetilde{j}^{a0}, \widetilde{\mathbf{j}}^a\right) \tag{5.19}$$

and

$$\widetilde{J}^{a\nu} = \left(\frac{1}{c}\widetilde{J}^{a0}, \widetilde{\mathbf{J}}^a\right) \tag{5.20}$$

The corresponding equations of the standard model (the Maxwell Heaviside field theory of special relativity) are well known to be:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \boldsymbol{\nabla}\phi \tag{5.21}$$

and:

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} \tag{5.22}$$

The Gauss law of magnetism of the standard model is:

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{5.23}$$

The Faraday law of induction is:

$$\boldsymbol{\nabla} \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \tag{5.24}$$

The Coulomb law is:

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \tag{5.25}$$

and the Ampére Maxwell law is:

$$\boldsymbol{\nabla} \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \frac{\mu_0}{c}\widetilde{\mathbf{J}} \tag{5.26}$$

Eqs.(5.12) to (5.18) are equations of a generally covariant unified field theory, Eqs.(5.21) to (5.26) are equations of special relativity, where the electromagnetic field is not unified with other fields such as the gravitational field.

The ECE Coulomb law is obtained by substituting Eq.(5.12) into Eq.(5.17):

$$\nabla^2 A^{a0} + \frac{1}{c}\frac{\partial}{\partial t}\left(\boldsymbol{\nabla} \cdot \mathbf{A}^a\right) + \boldsymbol{\nabla} \cdot \left(\omega^{0a}{}_b \mathbf{A}^b\right) - \boldsymbol{\nabla} \cdot \left(\boldsymbol{\omega}^a{}_b A^{0b}\right) = -\mu_0 \widetilde{J}^{0a} \tag{5.27}$$

Eq.(5.27) is a driven damped oscillator equation [19] with resonance solutions. On the right hand side appears a small "driving force". On the left hand side there are Hooke's law and damping terms. At resonance the scalar and vector potentials can be greatly amplified for an initially small driving charge current density, a process which is well known to obey Noether's Theorem under all circumstances and thus to conserve energy/momentum and charge/current density.

The standard model equivalent of Eq.(5.27) is the Poisson equation:

$$\boldsymbol{\nabla} \cdot \left(\frac{\partial \mathbf{A}}{\partial t} + \boldsymbol{\nabla} \phi \right) = -\frac{\rho}{\epsilon_0} \tag{5.28}$$

which does not give resonant solutions because the spin connection terms are missing in the flat or Minkowski space-time of Maxwell Heaviside field theory. Resonance is due to the spin connection, and so resonance is due to space-time itself. The simplest type of resonance equation is [19]:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = \alpha \cos \omega t \tag{5.29}$$

where there is a damping term proportional to $\dot{x}$, a Hooke's law term proportional to x and a driving force on the right hand side. The particular integral [19] is:

$$x_p(t) = \frac{\alpha \cos(\omega t - \delta)}{\left(\left(\omega_0^2 - \omega^2 \right)^2 + 4\omega^2\beta^2 \right)^{1/2}} \tag{5.30}$$

where:

$$\delta = \tan^{-1} \left(\frac{2\omega\beta}{\omega_0^2 - \omega^2} \right) \tag{5.31}$$

and the resonance frequency is:

$$\omega_0 = \omega. \tag{5.32}$$

At resonance:

$$\frac{d}{d\omega} \left(\frac{\alpha}{\left(\left(\omega_0^2 - \omega^2 \right)^2 + 4\omega^2\beta^2 \right)^{1/2}} \right) \Bigg|_{\omega = \omega_r} = 0. \tag{5.33}$$

The quality factor of resonance is defined by:

$$Q = \frac{\omega_R}{2\beta} \tag{5.34}$$

and is infinite for an undamped oscillator. On the left hand side of Eq.(5.29) there is a double derivative in time, a single derivative in time, and a term linear in x. The Coulomb law (5.27) can be rewritten as:

$$\begin{aligned} \nabla^2 A^{a0} + \frac{1}{c} \boldsymbol{\nabla} \cdot \frac{\partial \mathbf{A}^a}{\partial t} + \left(\omega^{0a}{}_b \boldsymbol{\nabla} \cdot \mathbf{A}^b - \omega^a{}_b \cdot \boldsymbol{\nabla} A^{0b} \right) + \\ \left(\boldsymbol{\nabla} \omega^{0a}{}_b \cdot \mathbf{A}^b - A^{0b} \boldsymbol{\nabla} \cdot \boldsymbol{\omega}^a{}_b \right) = -\mu_0 \widetilde{J}^{0a} \end{aligned} \tag{5.35}$$

On the left hand side there is a double derivative in distance $\nabla^2 A^{a0}$, single derivatives in distance, $(\omega^{0a}{}_b \boldsymbol{\nabla} \cdot - \boldsymbol{\omega}^a{}_b \cdot \boldsymbol{\nabla} A^{0b})$, and terms linear in the potential, $\left(\boldsymbol{\nabla} \omega^{0a}{}_b \right) \cdot \mathbf{A}^b - A^{0b} \boldsymbol{\nabla} \cdot \boldsymbol{\omega}^a{}_b$. The driving term appears on the right hand side of Eq.(5.34). There is also a time dependent term $\frac{1}{c} \boldsymbol{\nabla} \cdot \frac{\partial \mathbf{A}^a}{\partial t}$ on the left hand side.

In the rest of this section Eq.(5.34) is simplified with approximations to the point where it becomes analytically soluble. The latter solution is applied in Section 5.3 to the H atom. To proceed, a quasi electrostatic situation is used in which the vector potential is assumed to be very small. Eq.(5.34) the reduces to an equation in the scalar potential ϕ^a:

$$\nabla^2\phi^a - \boldsymbol{\omega}^a{}_b \cdot \boldsymbol{\nabla}\phi^b - \left(\boldsymbol{\nabla} \cdot \boldsymbol{\omega}^a{}_b\right)\phi^b = -\frac{\rho^a}{\epsilon_0} \tag{5.36}$$

In order to produce resonance amplification, it must be assumed that the charge density is oscillatory, for example:

$$\rho^a = -\rho^a\left(0\right)\cos\left(\kappa Z\right) \tag{5.37}$$

The meaning of the index on the scalar potential is clarified if we consider it to be complex valued, whereby for example:

$$\phi^{(1)} = \phi^{(2)*} = \frac{1}{\sqrt{2}}\left(1-i\right)e^{i(\omega t - \kappa Z)} \tag{5.38}$$

If the scalar potential is real-valued the indices can be omitted. For simplicity of argument define the vector spin connection through only one of its elements, for example:

$$\boldsymbol{\omega} = -\boldsymbol{\omega}^0{}_0 \tag{5.39}$$

Eq.(5.12) then reduces to its simplest form:

$$\mathbf{E} = -\left(\boldsymbol{\nabla} + \boldsymbol{\omega}\right)\phi. \tag{5.40}$$

The term $\boldsymbol{\omega}\phi$ adds a term to the electric field as described in the introduction to this paper. The linear inhomogeneous differential equation in its simplest form is therefore:

$$\nabla^2\phi + \boldsymbol{\omega} \cdot \boldsymbol{\nabla}\phi + \left(\boldsymbol{\nabla} \cdot \boldsymbol{\omega}\right)\phi = \frac{\rho}{\epsilon_0} \tag{5.41}$$

Its particular integral is

$$\phi_p\left(Z\right) = \frac{\rho^{(0)}}{\epsilon_0}\frac{\cos\left(\kappa Z - \delta\right)}{\left(\left(\frac{\partial\omega}{\partial Z} - \kappa^2\right)^2 + \omega^2\kappa^2\right)^{1/2}} \tag{5.42}$$

where:

$$\delta = \tan^{-1}\left(\frac{\omega\kappa}{\frac{\partial\omega}{\partial Z} - \kappa^2}\right) \tag{5.43}$$

At resonance:

$$\kappa_0^2 = \frac{\partial\omega}{\partial Z} \tag{5.44}$$

and the scalar potential ϕ becomes highly oscillatory. Eq.(5.41) may be rewritten as

$$\phi_p\left(Z\right) = \frac{e}{4\pi\epsilon_0 V_0}\frac{\cos\left(\kappa Z - \delta\right)}{\left(\left(\frac{\partial\omega}{\partial Z} - \kappa^2\right)^2 + \omega^2\kappa^2\right)^{1/2}} \tag{5.45}$$

where e is charge and V_0 is a volume. The potential energy from Eq.(5.44) is:

$$V_E = -e\phi_p\left(Z\right) \tag{5.46}$$

When:

$$\kappa = 0 \tag{5.47}$$

Eq.(5.45) reduces to:

$$V_E = -\frac{e^2}{4\pi\epsilon_0 r} \tag{5.48}$$

if:

$$r := V_0 \frac{\partial \omega}{\partial Z} \tag{5.49}$$

5.3 Application to the H atom

It is well known that the H atom is accurately described in the vast majority of applications [20] by the Schrödinger equation with the Coulomb law limit (5.47):

$$\hat{H}\psi = E\psi \tag{5.50}$$

where the hamiltonian operator is:

$$\hat{H} = -\frac{\hbar^2}{2\mu}\nabla^2 + V_E \tag{5.51}$$

Here μ is the reduced mass:

$$\mu = \frac{m_e m_p}{m_e + m_p} \tag{5.52}$$

where m_p and m_e are masses of proton and electron respectively. These equations are written in S.I. units. The Schrödinger equation of the H atom is therefore:

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi - \frac{e^2}{4\pi\epsilon_0 r}\psi = E\psi \tag{5.53}$$

In spherical polar coordinates [20]:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2} r\psi + \frac{1}{r^2}\Lambda^2\psi + \frac{\mu e^2}{2\pi\epsilon_0\hbar^2 r}\psi = -\frac{2\mu E}{\hbar^2}\psi \tag{5.54}$$

and the wave-function is the product of radial and angular parts:

$$\psi(r,\theta,\phi) = R(r)\,Y(\theta,\phi) \tag{5.55}$$

These are respectively the associated Laguerre polynomials and spherical harmonics with an effective potential:

$$V_{\text{eff}} = -\frac{e^2}{4\pi\epsilon_0 r} + \frac{e\,(l+1)\,\hbar^2}{2\mu r^2} \tag{5.56}$$

These equations define the energy levels of the H atom in terms of the principal quantum number n:

$$E = -\frac{\mu e^4}{32\pi^2\epsilon_0^2\hbar^2} \cdot \frac{1}{n^2} \tag{5.57}$$

The complete atomic orbitals of H are:

$$\psi_{nlm_e}(r,\theta,\phi) = R_{ne}(r)\,Y_{em_e}(\theta,\phi) \tag{5.58}$$

These are the s, p, d, f, ... atomic orbitals defined respectively by $l = 0, 1, 2, 3$, ... In the vast majority of applications these are well known to be defined by the limit (5.47) of the resonant Coulomb law. The complete resonant Coulomb law will change all these orbitals, and at resonance will ionize the H atom by resonance, breaking it into a free electron and a free proton. The energy required to do this is well known [20] to be 13.6 eV:

$$\Delta E = \frac{\mu e^4}{32\pi^2 \epsilon_0^2 \hbar^2} = 13.6 eV = 2.2 \times 10^{-18} J \tag{5.59}$$

This is the potential energy needed for a bound electron to be removed to infinity. If an electron is given more than this energy by tuning as in Eq.(5.43), its state becomes an unbound state, an unquantized continuum state [20]. Another effect of tuning to resonance through Eq.(5.43) is to lift the well known n - fold degeneracy of the H atom. So the spectral lines of the H atom will be affected by resonance.

In order to produce the spectrum the potential energy (5.47) must be used in Eq.(5.50) and the resulting Schrödinger equation solved numerically. Experimental methods must be found to produce the resonance (5.43) in the H atom. This thought model is intended only as a plausible mechanism for explaining the results of ref. [18] in the simplest possible approximation. The small driving force oscillations in the H atom can be modeled on the well known phenomenon [20] of zitterbewegung (jitterbugging), known to exist from quantum electrodynamics. If a molecule such as hydrogen deuteride (HD) is considered, the HD bond vibration may be used as the driving force. In more complicated molecules there are several vibrational and rotational frequencies as is well known. It is also well known that a molecule can be ionized by tuning a laser to one of these frequencies. Instead of using a laser, the spin connection of space-time has been used in this paper. The end result is a surge in the scalar potential for a small oscillatory driving charge/current density within the molecule. Thus the output power is much greater than the input power without any violation of the Noether Theorem. In the experimental work of ref. [18] this ratio has been observed in a scientifically repeatable manner to be of the order 100,000 or more. There is no explanation for this observation in the standard model but there is a straightforward explanation in ECE theory. The work in this paper must be made more precise, and applied to engineering situations of practical interest, but this paper illustrates the principle with a thought model based on the H atom.

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Bibliography

[1] M. W. Evans, Generally Covariant Unified Field Theory (Abramis, 2005, softback), vol. One.

[2] M. W. Evans, ibid., vols. 2 - 4 (Abramis, 2006 and 2007, preprints on www.aias.us and www.atomicprecision.com).

[3] L. Felker, The Evans Equations of Unified Field Theory (preprints on www.aias.us and www.atomicprecision.com). ; H. Eckardt and L. Felker, papers on these websites.

[4] M. W. Evans, Generally Covariant Dynamics (paper 55 of the ECE series, preprints on www.aias.us and www.atomicprecision.com).

[5] M. W. Evans, Geodesics and the Aharonov Bohm Effects in ECE Theory (paper 56 of the ECE series, preprints on www.aias.us and www.atomicprecision.com).

[6] M. W. Evans, Canonical and Second Quantization in Generally Covariant Quantum Field Theory (paper 57 of the ECE series, preprints on www.aias.us and www.atomicprecision.com).

[7] M. W. Evans, The Effect of Torsion on the Schwarzschild Metric and Light Deflection due to Gravitation (paper 58 of the ECE series, preprints on www.aias.us and www.atomicprecision.com.).

[8] M. W. Evans et al., papers and letters in Found. Phys. Lett. and Found Phys., 1994 to present.

[9] M. W. Evans (ed.), Modern Non-linear Optics a special topical issue in three parts of I. Prigogine and S. A. Rice (series eds.), Advances in Chemical Physics (Wiley-Interscience, New York, 2001, 2nd. Edition), vols. 119(1) to 119(3).

[10] M. W. Evans and L. B. Crowell, Classical and Quantum Electrodynamics and the $\boldsymbol{B}^{(3)}$ Field (World Scientific, Singapore, 2001).

[11] M. W. Evans and J.-P. Vigier, The Enigmatic Photon (Kluwer, Dordrecht, 1994 to 2002, hardback and softback), vols. 1 - 5.

[12] M. W. Evans and A. A. Hasanein, The Photomagneton in Quantum Field Theory (World Scientific, Singapore, 1994).

[13] M. W. Evans and S. Kielich (eds.), first edition of ref. (9) (Wiley Interscience, New York, 1992, reprinted 1993 and 1997), vols. 85(1) to 85(3).

[14] P. H. Pinter, What is Life? (Abramis, 2006).

[15] M. W. Evans, The Photon's Magnetic Field, Optical NMR Spectroscopy (World Scientific, 1992).

[16] M. W. Evans, Physica B, 182, 227, 237 (1992).

[17] Feedback sites for www.aias.us and www.atomicprecision.com, indicating essentially unanimous acceptance of the ECE theory worldwide.

[18] Mexican group work associated with A.I.A.S, communications and independent observations 2000 to 2006.

[19] J. B. Marion and S. T. Thornton, Classical Dynamics of Particles and Systems (HB Publishers, New York, 1998, 3rd edition) chapter three.

[20] P. W. Atkins, Molecular Quantum Mechanics (Oxford Univ. Press, 1983, 2nd edition).