Chapter 7

Space-time resonance in the Coulomb Law

(Paper 61)
by
Myron W. Evans, and H. Eckardt
Alpha Institute for Advanced Study (AIAS).
(emyrone@aol.com, horsteck@aol.com, www.aias.us,
www.atomicprecision.com)

Abstract

The Coulomb law is derived from general relativity applied to classical electro-dynamics within Einstein Cartan Evans (ECE) unified field theory. The radial component of the spin connection is modeled to be of the form $1/r$, where r is the radial component of the spherical polar coordinate system. The Coulomb potential so obtained may be amplified by space-time resonance. If this resonant Coulomb potential is used in a computation of the radial orbitals of the H atom, for example, the latter ionizes if the kinetic energy inputted from space-time at resonance exceeds the ionization potential energy (13.6 eV). The free electrons so released may be used as a novel source of electric power.

Keywords: Einstein Cartan Evans (ECE) field theory, resonant Coulomb law, radial orbitals of the H atom, free electrons from resonance, source of electric power from space-time.

7.1 Introduction

The theory of general relativity was developed for the gravitational field, as is well known, and has recently been tested in the solar system [1] to one part in one hundred thousand with the NASA Cassini experiments. It is therefore logical to extend general relativity to other areas of physics, notably classical electro-dynamics, thereby developing a generally covariant unified field theory [2]– [18] for the natural, engineering and life sciences. In the standard model, classical

electrodynamics is a theory of special relativity - the Maxwell Heaviside (MH) field theory [19]. The well known Coulomb law is part of the MH field theory and is usually regarded as one of the most precise laws in physics [19], [20]. The Coulomb law is the basis for the quantum theory of atomic and molecular spectra for example, and is used in many of the advanced computational techniques employed in this area of physics and chemistry. When the MH theory is extended from special to general relativity [2]– [18] with the Einstein Cartan Evans (ECE) theory, important new features develop in all the basic laws of classical electrodynamics, including the Coulomb law. These features emanate from the spin connection of ECE space-time. The Minkowski space-time of the MH theory is the well known flat space-time [19] of special relativity, but ECE space-time is characterized by the presence of both curvature and torsion [2]– [18]. In general relativity (ECE theory) the electromagnetic field is spinning space-time and the gravitational field is curving space-time. The spinning and curving may interact through standard Cartan geometry [21] and therefore the electromagnetic and gravitational fields may interact as verified experimentally in the well known bending of light by gravity. This phenomenon has been observed with great precision in the recent NASA Cassini experiments. ECE theory has been accepted [22] as the first classical explanation of this phenomenon [2]– [18]. The original well known inference of this effect by Einstein and others is based on a semi-classical approach, where the photon mass gravitates with the mass of the sun according to the Einstein Hilbert (EH) field theory of gravitation published in 1916. A classical explanation was not possible prior to ECE theory because standard model electrodynamics is special relativity un-unified with gravitational general relativity. ECE theory [2]– [18] provides a relatively simple and practical unified field theory based on the fundamental and well known principle of general covariance [21]. Unification occurs on both classical and quantum levels, and so ECE theory has been accepted as unifying general relativity with quantum mechanics, a major aim of physics throughout the twentieth century.

In Section 7.2 the Coulomb law is developed within the context of ECE field theory using a simple model of the spin connection, which is assumed to have a $1/r$ radial dependence, where (r, θ, ϕ) is the spherical polar coordinate system [23]. The result is that the Poisson equation is extended to a second order differential equation through which the scalar potential may be amplified at resonance according to well known mathematical principles [24]. This capacity for resonance is due to the presence of the spin connection of ECE space-time itself. Resonance of this type is not possible in a flat space-time, because in a flat space-time there is no spin connection. The latter indicates that the electromagnetic field is spinning space-time. The latter inference is indicated independently by several other phenomena [2]– [18], notably the magnetization of matter by electromagnetic radiation (the inverse Faraday effect) and the presence of the ECE spin field ($\boldsymbol{B}^{(3)}$ [25]) in all types of electromagnetic radiation. The inverse Faraday effect is magnetization due to the $\boldsymbol{B}^{(3)}$ spin field. The latter originates [2]– [18] in the spin connection, which works its way into other observable phenomena throughout the whole of the natural, engineering and life sciences.

In Section 7.3 some graphical results are given from the resonant Coulomb law, and it is shown how this produces free electrons from the H atom by ionizing the latter with kinetic energy inputted from space-time at resonance. The H

atom is used here as a simple model material. The release of free electrons at space-time resonance has been observed recently [26] and shown to be a repeatable phenomenon. The material and circuit designs used in this series of experiments [26] are much more complicated than H, but the latter serves as a model to illustrate the theoretical principles at work - those of general relativity applied to classical electrodynamics with ECE theory.

7.2   The ECE resonance Coulomb Law

The law is given [2]– [18] from the first Cartan structure equation:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{7.1}$$

and the first Bianchi identity:

$$d \wedge T^a + \omega^a{}_b \wedge T^b = R^a{}_b \wedge q^b \tag{7.2}$$

with the ECE Ansatz:

$$A^a = A^{(0)} q^a, \quad F^a = A^{(0)} T^a \tag{7.3}$$

Here T^a is the torsion form, $R^a{}_b$ is the Riemann or curvature form, q^a is the tetrad form, is the spin connection form, A^a is the electromagnetic potential form, $cA^{(0)}$ is the primordial voltage, and F^a is the electromagnetic field form. The Ansatz was first proposed by Cartan in well known correspondence with Einstein in the first part of the twentieth century, but was not developed into ECE theory until the spring of 2003 [2]– [18]. Eqs. (7.1) to (7.3) lead to [2]– [18]:

$$\mathbf{E}^a = -\frac{\partial \mathbf{A}^a}{\partial t} - \boldsymbol{\nabla}\phi^a - c\omega^{0a}{}_b \mathbf{A}^b + \phi^b \boldsymbol{\omega}^a{}_b \tag{7.4}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E}^a = c\mu_0 \widetilde{J}^{0a} \tag{7.5}$$

in vector notation. Here $\mathbf{E}^a$ is the electric field strength (volts per meter), μ_0 Is the vacuum S.I. permeability, $\widetilde{J}^{oa}$ is the time-like component of the inhomogeneous four-current of ECE theory, c is the vacuum speed of light, $\mathbf{A}^a$ is the vector potential, ϕ^b is the scalar potential, $\omega^{0a}{}_b$ is the time-like part of the spin connection four-vector, and $\widetilde{\omega}^a{}_b$ is the space-like part of the spin connection four-vector. The indices a and b originate in Cartan geometry [2]– [18], [21] and are the indices of the tangent space-time at a point P in the base manifold. These indices indicate polarization states of electromagnetic radiation in ECE theory [2]– [18]. Eq.(7.5) may be written for each index a as:

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \tag{7.6}$$

where ρ is the charge density and where ϵ_0 is the S.I. vacuum permittivity. Therefore for each index a, Eq.(7.5) has the same mathematical structure as the standard model Coulomb law [19], [20]. However, the electric field in ECE theory must always be defined by Eq.(7.4), which always involves the spin connection. The electric field is part of spinning space-time.

If attention is restricted to the scalar potential, then for each index a, Eq.(7.4) is:

$$\mathbf{E} = -\boldsymbol{\nabla}\phi + \phi^b\boldsymbol{\omega}_b \tag{7.7}$$

Here ϕ^b is interpreted as a scalar quantity indexed or labeled by b, indicating that the scalar potential applies to this state of polarization of electromagnetic radiation. For a given b index, Eq.(7.7) is:

$$\mathbf{E} = -\boldsymbol{\nabla}\phi + \phi\boldsymbol{\omega} \tag{7.8}$$

Summation over repeated b indices in Eq.(7.7) is implied (Einstein convention) but for the sake of simplicity it has been assumed in Eq.(7.8) that there is only one index and one state of polarization. Therefore we have reduced the complicated Eq.(7.8) to its simplest form (7.4). The result is that the familiar definition of the electric field in the standard model Coulomb law:

$$\mathbf{E} = -\nabla\phi \tag{7.9}$$

is supplemented by a term in the vector part of the spin connection, the vector $\boldsymbol{\omega}$. Eqs.(7.6) and (7.8) give the second order differential equation:

$$\nabla^2\phi - \boldsymbol{\nabla}\cdot(\phi\boldsymbol{\omega}) = -\frac{\rho}{\epsilon_0} \tag{7.10}$$

which compares with the standard model Poisson equation [19], [20]:

$$\nabla^2\phi = -\frac{\rho}{\epsilon_0} \tag{7.11}$$

Eq.(7.10) is an equation of general relativity. Eq.(7.11) is an equation of special relativity. The mathematical properties of Eq.(7.10) include the ability to give resonance, whereas Eq.(7.11) has no resonance solutions. This is a key difference. Resonance is the key to the production of free electrons from ECE space-time, providing a new source of electric power for engineering.

The spin connection vector in Cartesian and spherical polar coordinates is:

$$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$
$$= \omega_r\mathbf{e}_r + \omega_\phi\mathbf{e}_\phi + \omega_\theta\mathbf{e}_\theta \tag{7.12}$$

where ω_r is the radial component of $\boldsymbol{\omega}$. If the latter is assumed to be purely radial, for simplicity of argument, then:

$$\boldsymbol{\omega} = \omega_r\mathbf{e}_r \tag{7.13}$$

and in spherical polar coordinates [23]:

$$\boldsymbol{\omega}\cdot\boldsymbol{\nabla}\phi = \omega_r\frac{\partial\phi}{\partial r} \tag{7.14}$$

$$\phi\boldsymbol{\nabla}\cdot\boldsymbol{\omega} = \frac{\phi}{r^2}\frac{\partial}{\partial r}\left(r^2\omega_r\right) \tag{7.15}$$

$$\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) = \frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r} \tag{7.16}$$

118

The dimensions of $\boldsymbol{\omega}$ are inverse meters 2-18, so the simplest model of the vector spin connection is:

$$\omega_r = \frac{A}{r} \qquad (7.17)$$

where A is a dimensionless scaling factor. Eqs.(7.13) to (7.17) give the result:

$$\frac{\partial^2 \phi}{\partial r^2} + (2 - A) \frac{1}{r} \frac{\partial \phi}{\partial r} - \frac{A\phi}{r^2} = -\frac{\rho}{\epsilon_0} \qquad (7.18)$$

in spherical polar coordinates. Eq.(7.18) contains second and first order partial derivatives in the scalar potential ϕ. In the special case

$$A = 2 \qquad (7.19)$$

Eq.(7.18) becomes:

$$\frac{\partial^2 \phi}{\partial r^2} - \frac{2\phi}{r^2} = -\frac{\rho}{\epsilon_0} \qquad (7.20)$$

in which the second term on the left hand side is a REPULSION term. This means that the familiar Coulomb attraction between a proton and an electron in an H atom develops a repulsive component due to the presence of the spin connection vector. Eq.(7.18) has a similar structure to the well known one-dimensional Schrödinger equation for motion in an effective potential with repulsive centrifugal term [20] in the H atom. So the spin connection may be interpreted similarly. If the repulsion term in Eq.(7.20) becomes strong enough, the H atom ionizes, releasing a free electron. Eq.(7.18) is similar to the well-known [24] class of linear inhomogeneous differential equations that give resonance - the damped driven oscillator equations. Eq.(7.20) is a special case - the undamped driven oscillator. In order to induce resonance, the charge density *rho* must be initially oscillatory [24]. In the H atom model we are considering the source of this small original oscillation may be considered to be zitterbewegung (jitterbugging) from quantum electrodynamics [20]. In a molecule it could be a rotational frequency or vibrational bond frequency. At space-time resonance the initially small oscillation is greatly amplified [2]– [18], [24] and kinetic energy is absorbed into the atom or molecule from ECE space-time. If this energy is greater than the ionization potential energy of H (13.6 eV) the electron breaks free of the proton and may be used in a circuit to produce electric power from space-time through the intermediacy of the H atom. This concept may be generalized to any material which contains electrons which are easily released by ionization. The skill in material design revolves around this need. The engineering skill consists in devising a design to induce the resonance and this has been accomplished recently in a repeatable manner [26]. The output power in such experiments [26] may exceed the input power by as much as a factor of one hundred thousand, an amplification that illustrates dramatically the resonance of the spin connection in classical electrodynamics. Care has been taken to ensure that this experiment is repeatable and the apparatus has been observed independently [26] in different laboratories. Every effort has been made to eliminate artifact, and reproducible amplification by five orders of magnitude is unlikely to be artifact. The standard model (MH theory) has no explanation for this phenomenon, even on a qualitative level. Its explanation in general relativity (ECE theory) relies on resonating the spin connection as described already.

In summary of this section therefore the Poisson equation of the standard model (Eq.(7.11)) is modified in the simplest instance to the following ECE equation of general relativity:

$$\frac{\partial^2 \phi}{\partial r^2} = \frac{2\phi}{r^2} - \frac{\rho}{\epsilon_0} \tag{7.21}$$

introducing a repulsive term:

$$\rho_{\text{eff}} = 2\epsilon_0 \frac{\phi}{r^2} \tag{7.22}$$

If the charge density ρ is very small, Eq.(7.21) takes on the approximate mathematical form:

$$\frac{\partial^2 \phi}{\partial r^2} \sim \frac{2\phi}{r^2} \tag{7.23}$$

which has an analytical solution:

$$\phi \sim \frac{\beta}{r} + \alpha r^2 \tag{7.24}$$

where α and β are constants. When r is very small, the potential ϕ becomes very large and a large amount of POSITIVE potential energy may be inputted into the H atom from the spin connection, depending on the value of β. If:

$$\beta \geq \frac{e}{4\pi\epsilon_0} \tag{7.25}$$

then the positive repulsion potential becomes equal to or greater than the negative attraction potential, releasing the electron from the proton. The standard model inverse square Coulomb law is very precise in the vast majority of experiments in macroscopic classical electrodynamics [19] but the recent experiments carried out in ref. [26] indicate that it does not hold in general.

The ECE theory reduces straightforwardly to the standard Coulomb law as follows. In ECE theory the electric field is defined in the simplest instance by:

$$\mathbf{E} = -\boldsymbol{\nabla}\phi + \phi\boldsymbol{\omega} \tag{7.26}$$

and in the standard Coulomb law it is defined by:

$$\mathbf{E} = -\boldsymbol{\nabla}\phi \tag{7.27}$$

Therefore if:

$$\boldsymbol{\nabla}\phi = -\phi\boldsymbol{\omega} \tag{7.28}$$

the mathematical form of the standard Coulomb law is obtained:

$$\mathbf{E} := -2\boldsymbol{\nabla}\phi \tag{7.29}$$

This simply means that the scalar potential is defined by:

$$\Phi := 2\phi \tag{7.30}$$

This makes no difference to the observable force (inverse square law). If:

$$\Phi := \frac{e}{4\phi\epsilon_0 Z} \tag{7.31}$$

and

$$\nabla\Phi = -\Phi\boldsymbol{\omega} \tag{7.32}$$

then:

$$\omega_A = \frac{1}{Z} \tag{7.33}$$

The important conclusion is reached that a spin connection of the type (7.33) is ALWAYS observed in the Coulomb law, which becomes a law of general relativity as required by objectivity in physics. Therefore any experimental departure from the inverse square Coulomb law would indicate that the spin connection is no longer given by Eq.(7.33)In general relativity (ECE theory) the electric field must always be defined according to Eq.(7.26) and the vast majority of experimental data have confirmed the inverse square law of Coulomb for over two hundred years. In general relativity this means that the data show that the spin connection must be of the form (7.33) experimentally. This type of spin connection, conversely, gives the inverse square law of Coulomb. Eq.(7.28) is similar to the operator equivalence of quantum mechanics, and means that:

$$\boldsymbol{\omega} \rightarrow -\boldsymbol{\nabla} \tag{7.34}$$

The operator equivalence is:

$$\mathbf{p} \rightarrow -i\hbar\boldsymbol{\nabla} \tag{7.35}$$

Therefore in general relativity the electric field can be defined equivalently in two ways:

$$\mathbf{E} = -\boldsymbol{\nabla}\Phi = \Phi\boldsymbol{\omega} \tag{7.36}$$

and this is the fundamental definition of the electric field in general relativity. These considerations confirm that ECE theory is correct to very high precision, and give a simple meaning to the spin connection. The result of general relativity, Eq.(7.26), is preferred to the result of special relativity, Eq.(7.27), on several grounds, notably that other aspects of electrodynamics such as the inverse Faraday effect and Eddington effect require a generally covariant unified field theory for their objective interpretation on the classical level.

Dramatically new results such as those by the Mexican group are also accounted for by ECE theory by considerations of resonance as in previous work.

It is significant that spin connections of the type (7.33) also occur as Christoffel connections of the Schwarzschild metric of spherically symmetric space-time. These are well known to indicate a dynamic space-time. Electrodynamics is also now known to be a phenomenon of dynamic space-time, and similarly for the natural, engineering and life sciences. The two simplest Christoffel connections in a spherically symmetric space-time are [2]– [18]:

$$\Gamma^2{}_{12} = \Gamma^3{}_{13} = \frac{1}{r} \tag{7.37}$$

and these are similar to the connection (7.33) of the Coulomb law in ECE theory. For any spherically symmetric space-time the non-vanishing Christoffel

connections are:

$$\Gamma^2{}_{12} = \Gamma^3{}_{13} = \frac{1}{r}, \quad \Gamma^0{}_{00} = \frac{1}{c}\frac{\partial\alpha}{\partial t}, \quad \Gamma^0{}_{01} = \frac{\partial\alpha}{\partial X}, \quad \Gamma^1{}_{01} = \frac{1}{c}\frac{\partial B}{\partial t},$$

$$\Gamma^1{}_{11} = \frac{\partial\beta}{\partial X}, \quad \Gamma^0{}_{11} = e^{2(\beta-\alpha)}\frac{1}{c}\frac{\partial\beta}{\partial t}, \quad \Gamma^1{}_{22} = -re^{-2\beta}, \tag{7.38}$$

$$\Gamma^1{}_{33} = \Gamma^2{}_{33} = \Gamma^3{}_{23} = f(\theta),$$

$$ds^2 = -e^{2\alpha}dt^2 + e^{2\beta}dr^2 + r^2 d\Omega^2$$

In the particular case of the Schwarzschild metric [2]– [18]:

$$e^{2\alpha} = \left(1 - \frac{2GM}{c^2 r}\right), \tag{7.39}$$

$$e^{2\beta} = \left(1 - \frac{2GM}{c^2 r}\right)^{-1}, \tag{7.40}$$

and in spherical polar coordinates:

$$\Gamma^0{}_{01} = \partial_1\alpha, \quad \Gamma^0{}_{11} = e^{2(\beta-\alpha)}\partial_0\beta,$$

$$\Gamma^1{}_{00} = e^{2(\alpha-\beta)}\partial_1\alpha, \quad \Gamma^1{}_{11} = \partial_1\beta. \tag{7.41}$$

However, the ECE Coulomb law is derived from the Cartan torsion, while the Christoffel connections are for the Cartan curvature in the absence of Cartan torsion.

These considerations of the Coulomb law of generally covariant electrostatics can be extended as follows to the generally covariant Ampère Law of magneto-statics. In ECE theory the magnetic field is:

$$\mathbf{B}^a = \boldsymbol{\nabla}\times\mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \tag{7.42}$$

and the Ampère Law of magneto-statics takes on a generally covariant form as follows:

$$\boldsymbol{\nabla}\times\mathbf{B}^a = \frac{\mu_0}{c}\widetilde{\mathbf{J}}^a \tag{7.43}$$

The magnetic field in general relativity must always be defined by Eq.(7.42) with a non-zero spin connection. The latter is always present in general relativity. In magneto-statics we are dealing with rotational motion, so Eq.(7.42) may be written as:

$$\mathbf{B}^a = \boldsymbol{\nabla}\times\mathbf{A}^a + g\mathbf{A}^b \times \mathbf{A}^c \tag{7.44}$$

where the parameter g is defined as [2]– [18]:

$$g = \frac{\kappa}{A^{(0)}} \tag{7.45}$$

For rotational motion the spin connection is dual to the tetrad if it is assumed that the electromagnetic and gravitational fields are independent. In Eq.(7.45) $A^{(0)}$ is a magnitude and κ has the units of inverse meters. Eq.(7.44) is therefore:

$$\mathbf{B}^a = \boldsymbol{\nabla}\times\mathbf{A}^a + \boldsymbol{\omega}^b \times \mathbf{A}^c \tag{7.46}$$

where

$$\boldsymbol{\omega}^b = \frac{\kappa}{A^{(0)}} \mathbf{A}^b \tag{7.47}$$

If gravitation and electromagnetism are inter-dependent, the general equation (7.42) must be used, and it cannot be assumed that $\boldsymbol{\omega}^a{}_b$ is dual to $\mathbf{A}^c$.

If it is assumed that:

$$\boldsymbol{\nabla} \times \mathbf{A}^a = \boldsymbol{\omega}^b \times \mathbf{A}^c \tag{7.48}$$

then:

$$\mathbf{B}^a = 2\boldsymbol{\nabla} \times \boldsymbol{A}^a \tag{7.49}$$

which is the standard model result for each a. The potential is:

$$\mathbf{A}_{\mathrm{mh}} := 2\mathbf{A}_{\mathrm{ECE}} \tag{7.50}$$

and so:

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}_{\mathrm{MH}} \tag{7.51}$$

as is usual in the standard model [2]– [18]. So ECE reduces to the standard model of magneto-statics provided the spin connection obeys Eq.(7.48). This is an important result, because in the vast majority of experiments since the eighteenth century both the Coulomb and Ampère laws hold to very high accuracy. So ECE theory must be able to reduce to these well known results. So both laws are now understood to be very precise laws of general relativity (ECE theory) and not special relativity (Maxwell Heaviside theory). The key advance is that the ECE theory is a generally covariant unified field theory that enables electro-statics and magneto-statics to be unified with all other fields, notably the gravitational field.

The indices a, b and c in Eq.(7.42) originate [2]– [18] in the tangent space of Cartan geometry, and can be defined in the complex circular basis:

$$a, b, c = (1), (2), (3) \tag{7.52}$$

in which the magnetic field is:

$$\mathbf{B}^{(1)*} = \boldsymbol{\nabla} \times \mathbf{A}^{(1)*} - i\boldsymbol{\omega}^{(2)} \times \mathbf{A}^{(3)}$$

$$\mathbf{B}^{(2)*} = \boldsymbol{\nabla} \times \mathbf{A}^{(2)*} - i\boldsymbol{\omega}^{(3)} \times \mathbf{A}^{(1)} \tag{7.53}$$

$$\mathbf{B}^{(3)*} = \boldsymbol{\nabla} \times \mathbf{A}^{(3)*} - i\boldsymbol{\omega}^{(1)} \times \mathbf{A}^{(2)}$$

The complex circular basis is defined by the unit vectors:

$$\mathbf{e}^{(1)} = \frac{1}{\sqrt{2}} (\mathbf{i} - i\mathbf{j})$$

$$\mathbf{e}^{(2)} = \frac{1}{\sqrt{2}} (\mathbf{i} + i\mathbf{j}) \tag{7.54}$$

$$\mathbf{e}^{(3)} = \mathbf{k}$$

with $O(3)$ symmetry:

$$\mathbf{e}^{(1)} \times \mathbf{e}^{(2)} = i\mathbf{e}^{(3)*}$$

$$\mathbf{e}^{(2)} \times \mathbf{e}^{(3)} = i\mathbf{e}^{(1)*} \tag{7.55}$$

$$\mathbf{e}^{(3)} \times \mathbf{e}^{(1)} = i\mathbf{e}^{(2)*}$$

Here $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are the Cartesian unit vectors. The following are self-consisted vector potential solutions of Eq.(7.53):

$$\mathbf{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} \left(\mathbf{i} - i\mathbf{j}\right) e^{-i\kappa Z} \tag{7.56}$$

$$\mathbf{A}^{(2)} = \mathbf{A}^{(1)*} = \frac{A^{(0)}}{\sqrt{2}} \left(\mathbf{i} + i\mathbf{j}\right) e^{i\kappa Z} \tag{7.57}$$

$$\mathbf{A}^{(3)} = A^{(0)}\mathbf{k} \tag{7.58}$$

with vector spin connections:

$$\boldsymbol{\omega}^{(1)} = \frac{\omega^{(0)}}{\sqrt{2}} \left(\mathbf{i} - i\mathbf{j}\right) e^{-i\kappa Z}, \tag{7.59}$$

$$\boldsymbol{\omega}^{(2)} = \frac{\omega^{(0)}}{\sqrt{2}} \left(\mathbf{i} + i\mathbf{j}\right) e^{i\kappa Z}, \tag{7.60}$$

$$\boldsymbol{\omega}^{(3)} = \omega^{(0)}\mathbf{k} \tag{7.61}$$

Using the de Moivre Theorem:

$$\begin{aligned} e^{-i\kappa Z} &= \cos\left(\kappa Z\right) - i\sin\left(\kappa Z\right) \\ e^{i\kappa Z} &= \cos\left(\kappa Z\right) + i\sin\left(\kappa Z\right) \end{aligned} \tag{7.62}$$

Eq.(7.56) has a real and physical component:

$$\mathrm{Real}\mathbf{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} \left(\cos\left(\kappa Z\right)\mathbf{i} + \sin\left(\kappa Z\right)\mathbf{j}\right) \tag{7.63}$$

which is a rotating potential with phase angle:

$$\theta = \kappa Z \tag{7.64}$$

It is seen that at $\theta = 0$, $\mathbf{A}^{(0)}$ is in the $\mathbf{i}$ axis, and if $\theta = \pi/2$, $\mathbf{A}^{(1)}$ is in the $\mathbf{j}$ axis, and so has rotated by 90°. With these definitions it is seen that:

$$\boldsymbol{\nabla} \times \mathbf{A}^{(1)*} = -i\boldsymbol{\omega}^{(2)} \times \mathbf{A}^{(3)}$$
$$\boldsymbol{\nabla} \times \mathbf{A}^{(2)*} = -i\boldsymbol{\omega}^{(3)} \times \mathbf{A}^{(1)} \tag{7.65}$$

so the (1) and (2) magnetic fields are:

$$\begin{aligned} \mathbf{B}^{(1)} &= 2\boldsymbol{\nabla} \times \mathbf{A}^{(1)}, \\ \mathbf{B}^{(2)} &= 2\boldsymbol{\nabla} \times \mathbf{A}^{(2)}, \end{aligned} \tag{7.66}$$

having the same mathematical form as the standard model. However, general relativity (ECE theory) gives a new result:

$$\mathbf{B}^{(3)*} = i\boldsymbol{\omega}^{(1)} \times \mathbf{A}^{(2)} \tag{7.67}$$

which does not occur in special relativity. Eq.(7.65) may be written as:

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}_{\mathrm{MH}} \tag{7.68}$$

From Eq.(7.64) the magnitude of the spin connection is the wave-number:

$$\omega^{(0)} = \kappa \tag{7.69}$$

with units of inverse meters.

It is important to note the existence of the $\mathbf{B}^{(3)}$ field in general relativity, Eq.(7.66). In electrodynamics [2]– [18] this is the ECE spin field of electromagnetic radiation. In magneto-statics, to which the Ampère law applies, a magnetic field may also be defined through the spin connection using Eq.(7.66). Using Eqs.(7.56) to (7.60) the field in Eq.(7.66) is:

$$\mathbf{B}^{(3)} = B^{(0)}\mathbf{k} \tag{7.70}$$

BUT:

$$\boldsymbol{\nabla} \times \mathbf{A}^{(3)} = \mathbf{0} \tag{7.71}$$

In electrodynamics the $\mathbf{B}^{(3)}$ spin field is [2]– [18]:

$$\mathbf{B}^{(3)*} = -i\frac{\kappa}{A^{(0)}}\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \tag{7.72}$$

where

$$\mathbf{A}^{(1)} = \mathbf{A}^{(2)*} = \frac{A^{(0)}}{\sqrt{2}}\left(\mathbf{i} - i\mathbf{j}\right)e^{i(\omega t - \kappa Z)} \tag{7.73}$$

where Ω is the electromagnetic angular frequency. In magneto-statics (Eq.(7.56) the angular frequency Ω is zero. The magneto-static potential rotates (as we have seen), but the electromagnetic potential rotates and also translates along an axis such as Z.

The electrodynamic spin field $\mathbf{B}^{(3)}$ is observed by its magnetization of matter in the inverse Faraday effect [2]– [18]. This observation shows that classical electrodynamics and non-linear optics are manifestations of general relativity. The spin connection of the inverse Faraday effect is:

$$\boldsymbol{\omega}_{\mathrm{IFE}}^{(1)} = \frac{\kappa}{A^{(0)}}\mathbf{A}^{(1)} \tag{7.74}$$

and without the spin connection there is no inverse Faraday effect. Since all physics must be independent of observer influence (must be objective and co-variant under the general coordinate transformation), all physics, including electrodynamics, must be general relativity.

This means that the electromagnetic field under any circumstance must originate in a spinning space-time described by Cartan torsion [2]– [18]. In turn this means that the spin connection is non-zero under any circumstance, as emphasized in this section for the electro-statics and magneto-statics. If the spin connection is non-zero the $\mathbf{B}^{(3)}$ spin field is always non-zero. In Maxwell Heaviside field theory on the other hand the spin connection is zero because the electromagnetic field is philosophically different, it is an entity superimposed on Minkowski space-time, and in this space-time there is no $\mathbf{B}^{(3)}$ field, contrary to observation.

Having shown that the spin connection has a $1/Z$ dependence for the standard model Coulomb law, this type of spin connection may now be used in the

resonance equation (7.10) for the potential in general relativity. Therefore in the resonance equation:

$$\nabla^2 \phi - \boldsymbol{\omega} \cdot \boldsymbol{\nabla} \phi - (\boldsymbol{\nabla} \cdot \boldsymbol{\omega}) \phi = -\frac{\rho}{\epsilon_0} \qquad (7.75)$$

a vector spin connection of the following type may be used self-consistently

$$\boldsymbol{\omega} = \frac{A}{Z} \mathbf{k} \qquad (7.76)$$

where A is a scaling factor. The initial driving charge density may be defined for convenience as:

$$\rho = -\rho_0 \cos(\kappa Z) \qquad (7.77)$$

In the H atom this cosinusoidal dependence may be assumed to originate in the jitterbugging motion (zitterbewegung) that has its rigorous origins in quantum electrodynamics. In a molecule such as water it may be assumed to originate in a rotational or vibrational frequency of the molecule. Therefore the resonance equation becomes:

$$\nabla^2 \phi - \frac{A}{Z} \frac{\partial \phi}{\partial Z} + \frac{A}{Z^2} \phi = \frac{\rho_0}{\epsilon_0} \cos(\kappa Z) \qquad (7.78)$$

The simplest example [2]– [18] of a resonance equation is the linear inhomogeneous differential equation:

$$\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = \alpha \cos \omega t \qquad (7.79)$$

This is a forced damped oscillator with driving term $\alpha \cos \omega t$. The damping term is $2\beta \dot{x}$ and the Hooke's law term is $\omega_0^2 x$. The frequency resonance from Eq.(7.78) is well known [2]– [18] to occur at:

$$\omega_R = \left(\omega_0^2 - 2\beta^2\right)^{1/2} \qquad (7.80)$$

and the kinetic energy resonance occurs at:

$$\omega_E = \omega_0 \qquad (7.81)$$

Therefore at some fixed values:

$$\frac{A}{Z} = \frac{A}{Z_0}, \quad \frac{A}{Z^2} = \frac{A}{Z_0^2} \qquad (7.82)$$

Eq.(7.77) becomes:

$$\nabla^2 \phi - \frac{A}{Z_0} \frac{\partial \phi}{\partial Z} + \frac{A}{Z_0^2} \phi = \frac{\rho_0}{\epsilon_0} \cos(\kappa Z) \qquad (7.83)$$

Thus wave-number resonance occurs from Eq.(7.79) at:

$$\kappa_R = \frac{A}{\sqrt{2}} \frac{1}{Z_0} \qquad (7.84)$$

and kinetic energy resonance at a wave-number:

$$\kappa_E = \frac{A^{1/2}}{Z_0} \qquad (7.85)$$

At resonance the particular (or transient) solution for the potential is [2]– [18]:

$$\phi\left(\kappa\right) = \frac{1}{\epsilon_0} \frac{\rho_0 \cos\left(\kappa Z - \delta\right)}{\left(\left(\frac{A}{Z_0^2} - \kappa^2\right)^2 + \frac{A^2}{Z_0^2}\kappa^2\right)^{1/2}} \tag{7.86}$$

where

$$\delta = \tan^{-1}\left(\frac{A\kappa/Z_0}{\kappa^2 - A/Z_0^2}\right) \tag{7.87}$$

7.3   Graphical results and discussion

To study the effects of spacetime resonance in Hydrogen on a quantitative level, appropriate numerical code has been developed. Energy levels and radial wave functions are obtained from the solution of the Schrödinger equation. For the Coulomb potential of standard theory a well-known analytical solution exists. Resonance effects, however, require the resonance potential of equation (7.85) to be added to the standard (non-resonant) Coulomb potential. Then an analytical solution of the Schrödinger equation is no longer possible and the equation has to be solved numerically. So the overall numerical approach is to modify the standard Coulomb potential by the analytical form given in equations (7.86), (7.87) and compute the energies and radial wave functions by the numerical code. The latter has been developed as described in detail in [27]. Throughout the presentation of the results atomic units are used, i.e. length is measured in Bohr radius and energy in Rydberg units ($1Ryd = 13.605eV$).

In all calculations equation (7.86) has been multiplied by an exponential function $e(-r/1.25)$ in order to bring the charge oscillations to zero in the limit $r \to \infty$. Otherwise there were unphysical oscillations in the potential at large radii which would lead to unbound free-space solutions of the Schrödinger equation.

The first question to answer is why the Coulomb law of standard theory is valid to high precision although the spin connection is present. One answer is given by equations (7.28), (7.29): half of the Coulomb potential in the general relativistic (ECE) description consists of the spin connection. If we allow for a varying strength of the spin connection, (variable A in equation (7.17)) we can study this effect numerically. For this purpose equation (7.18), which describes the radial part of the Coulomb potential, was programmed numerically and solved for several values of A. In the case $\rho = 0$ and $A = 0$ the $1/r$ potential is obtained to high precision. To see how A effects deviations from the $1/r$ form of the Coulomb potential we have chosen the same initial conditions as for the case $A = 0$ and integrated the equation from a radius $r = r_{max}$ down to a radius near to $r = 0$. The results for different A's are presented in Table 7.1 for certain radius values. Significant deviations are only visible for very small radii ($r \approx 0.1$). At $r = 1$ the deviation from the $1/r$ potential is only 1 part in $10,000$ for relevant A values (up to $A = 2$). Even for $A = 10$ changes are remarkable only near to the center. For experiments this means that the spin connection has no measurable effect in off-resonance except very near to the position of a charge.

The resonance curve of the atomic energy levels is shown in Fig. 7.1. The wave number κ has been varied as indicated on the x axis, the other parameters

r	-1/r	VC(r), A=0	A=1	A=2	A=10
0.1	-1.000000E+01	-1.000000E+01	-9.965750E+00	-9.945510E+00	-9.897040E+00
1.0	-1.000000E+00	-1.000000E+00	-9.999656E-01	-9.999449E-01	-9.998938E-01
3.0	-3.333333E-01	-3.333333E-01	-3.333321E-01	-3.333314E-01	-3.333296E-01
10.0	-1.000000E-01	-1.000000E-01	-9.999999E-02	-9.999998E-02	-9.999995E-02

Table 7.1: Values of ECE Coulomb potential at certain radii in dependence of A (atomic units)

were fixed: $A = 1$, $Z_0 = 1.5$, $\rho_0 = 0.1$. For large κ one obtains the off-resonance case with the well-known energy levels of H. For decreasing κ (and on setting resonance effects) the degeneracy of the l quantum number disappears, resulting in a splitting between l levels of the same principal quantum number. The 1s energy is greatly lifted, the maximum occurs at the resonance wave number $\kappa = 0.75$. Interestingly there is a sharp peak to negative energy values directly below the kinetic energy resonance so that the resonance shows up a pole-like behaviour. The middle of both extrema lies at $\kappa = 0.67$ which is the kinetic energy resonance given by equation (7.85) for the parameters mentioned above. The width of the pole is determined by the damping term of the governing differential equation (7.83). Obviously the width depends on the quantum number, it is smaller for the 2p state than for the 1s state. Besides the 1s state, also the 2s and 3s state are lifted. This leads to an inversion of s and p energy levels compared to multi-electron atoms where the s states have lower energies than the p states. The same holds for the 3d states.

In the limit $\kappa \to 0$ there is again an increase of the energy levels, but this is due to the fact that the resonance potential in this case tends to a constant positive value (see discussion of Fig. 7.7 below). So this is an artifact of the variation parameter of the graph.

The question is what happens to the atomic wave functions (radial functions) in case of resonance. This is explained in Figs. 7.2,7.3,7.4. There some orbitals are shown for three κ values, and additionally the orbitals for the atom without a resonance potential (denoted by 0). The κ values are 0.6 (lower resonance peak, A), 0.75 (upper resonance peak, B) and 2 (off-resonance, C). As can be seen for the three graphed quantum states ($1s, 2s, 2p$), the charge density is shifted inwards to the core for the lowered energy and shifted outwards for the lifted energy levels. The resonance effect is quite drastic for the 1s state because the energy shift is highest in this case. A second local maximum occurs for $r \approx 4$ which is not present in off-resonance. The characteristic of the atomic state is altered completely. Nevertheless it is a valid eigenstate for the angular momentum $l = 0$ because there is no passing through zero. A corresponding result holds for the 2s state.

The lower energy resonance of the 2p state (Fig. 7.4) deserves particular attention. It shows that, at this point of resonance, states are significantly more localized than in the undisturbed atom. The localization radius here coincides with the value of $Z_0 = 1.5$. This is not so obvious for the other states.

The reason for the energy and wavefunction shifts can be found by looking at the potentials. In Fig. 7.5 the resonance potential according to equation (7.86) is shown. The negative resonance appears due to a negative bump in the potential while at the positive resonance the potential becomes significantly

repulsive near to the core. In off-resonance the potential is more pure oscillatory, averaging the impact on the energies and wavefunctions. It can nicely be seen also that the oscillation has slown down for increasing radii by means of the additional exponential damping.

Fig. 7.6 presents the total potential including the standard Coulomb term. A repulsive maximum is seen in the positive resonant case. For the negative resonance the potential is lowered in a certain range. The switching between both states takes place in a very narrow region of κ values as has been seen in Fig. 7.1. The reason is the phase jump of the resonance potential being described by equation (7.87). According to the well-known theory of forced oscillations, the phase changes rapidly by 180 degrees in this region. This can further be seen from Fig. 7.7 where the maximum resonance amplitude has been plotted in dependence of κ. There is a sign change at $\kappa = 0.67$. In addition the integral over the resonance potential

$$\int_0^{r_{\max}} \phi(\kappa, Z) dZ \qquad (7.88)$$

is shown. There is an even more significant jump in this curve at the same κ. It is important to note that the integral tends to zero for large wavenumbers. This means that the net contribution to the charge density is zero as is required for charge neutrality. In other words, the exponential damping function has no detrimental effect on charge neutrality.

In the following diagrams several further resonance curves are given. The effect of the strength of spin connection is visible from Fig. 7.8. According to equation (7.85) there is a connection between the resonant κ, the spin connection strength A, and the radius parameter Z_0:

$$\kappa = \frac{A^{1/2}}{Z_0} \qquad (7.89)$$

With $\kappa = 0.75$ and $Z_0 = 1.5$ this gives a resonant A value of

$$A_E = \kappa_E^2 Z_0^2 = 1.27 \qquad (7.90)$$

The maximum of the $1s$ energy, however, occurs at $A = 1$, which corresponds to $\kappa = 0.67$, the resonance value of Fig. 7.1.

Variation of the fixed radius Z_0 is shown in Fig. 7.9. According to the choice of the other parameters the maximum 1s energy is at $Z_0 = 1.5$ as expected. It should be noted that for $Z_0 \to 0$ the energy splitting for the main quantum numbers disappears. This is plausible from equations (7.86), (7.87) where Φ and δ tend to zero for $Z_0 \to 0$. Consequently, one obtains the original energy levels of the H atom in this case.

The next two figures (Figs. 7.10 and 7.11) reveal the characteristics of the resonance itself for varying parameter configurations. For fixed $A = 1$ the value of Z_0 was varied, while at the same time the value of κ was taken to be at resonance according to equation (7.85). Since the energy values are quite different at wavenumbers near to the resonance (the behaviour is pole-like as explained earlier) we have drawn two curves, one for the lower energies at a value

$$\kappa = \frac{A^{1/2}}{Z_0} - 0.05 \qquad (7.91)$$

(see Fig. 7.10) and one for the upper energies at

$$\kappa = \frac{A^{1/2}}{Z_0} + 0.05 \qquad (7.92)$$

(Fig. 7.11). From Fig. 7.10 it can be seen that the energy minima come to lie at different Z_0 values in dependence of the orbital. Because the latter differ in their radial charge distribution, they are affected in an individual way. The energy inversion of s, p and d states has already been mentioned.

The corresponding curve for the resonance maxima (Fig. 7.11) looks differently. There is no maximum energy in the range of Z_0 values. For increasing Z_0 the wave functions are shifted more and more outwards, leading to a continuous increase of energies.This may be considered as a transition to the ionization process where the electron is stripped off of the atom.

The effect of the oscillation strength ρ_0 is studied in Figs. 7.12,7.13. Similar as in Figs. 7.10,7.11 we have chosen κ values at the resonance minimum and maximum: $\kappa = 0.61$ and $\kappa = 0.72$. Switching on the oscillation leads to a mostly linear decrease of energy levels for the minimum (Fig. 7.12), while energies increase to a maximum value in the other case (left half of Fig. 7.13). For $\rho_0 > 0.1$ there is no further change since the wave functions have shifted to the outer region where changes in the repulsion potential near to the core have no effect. Again there is an inversion of the angular momentum dependence. For $\kappa_0 \to 0$ one obtains the original orbital energies of the H atom as expected.

Finally we make some general remarks on the resonance effect in Hydrogen. The numerical results have shown that there is a resonance effect as predicted by the theory and found experimentally in solid materials by the Mexican group. The mechanism of raising the binding energy of the valence electron by resonance has been demonstrated in several resonance diagrams. The ionization effect itself has not been modeled because this requires the inclusion of non-normalizable continuum states. In addition to the raising of energy, there is always a drop of the binding energy at the opposite side of the resonant wavenumber.

If the frequency of excitation charge density ρ has a certain bandwidth, it is plausible that two effects are initiated at the same time: production of conduction electrons by raising energies and production of very deeply bound electrons by lowering energies. Both effects are a consequence of resonant interaction with spacetime. The lowering of electronic states reduces thermal vibration amplitudes which may result in a macroscopic temperature reduction. Thus the experimental finding can be explained that effects of spacetime resonances are often accompanied by a decrease of temperature, an effect, which seems to contradict thermodynamics if one tries to explain it by standard theory.

After the spacetime resonance of the H atom has been studied, the next steps are to investigate these effects in many-electron atoms, molecules and solids. For this, the covariant form of the Coulomb law (equation (7.10)) has to be used in numerical methods which are available today for calculating electronic properties of these materials.
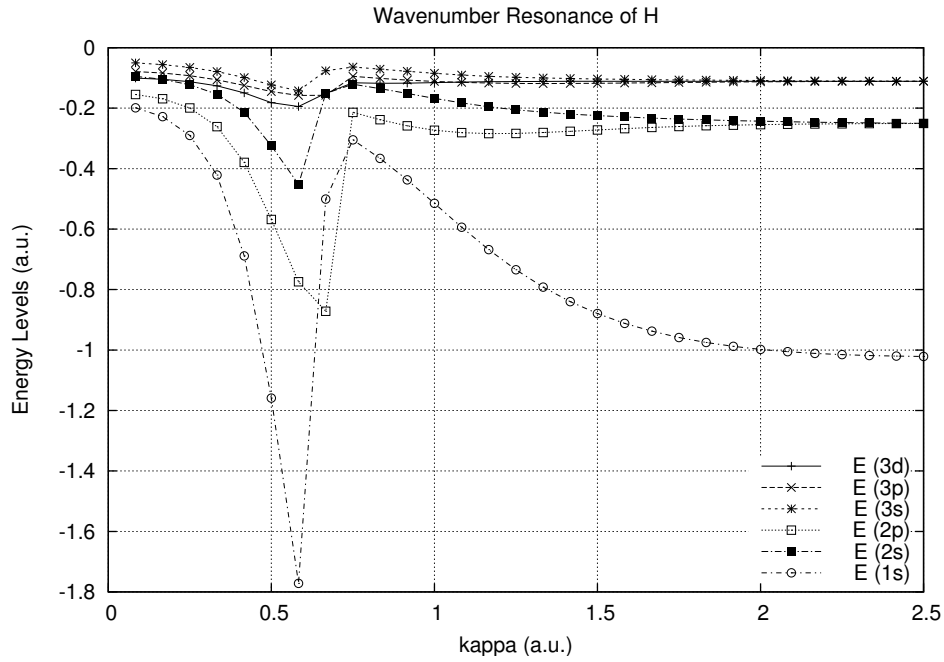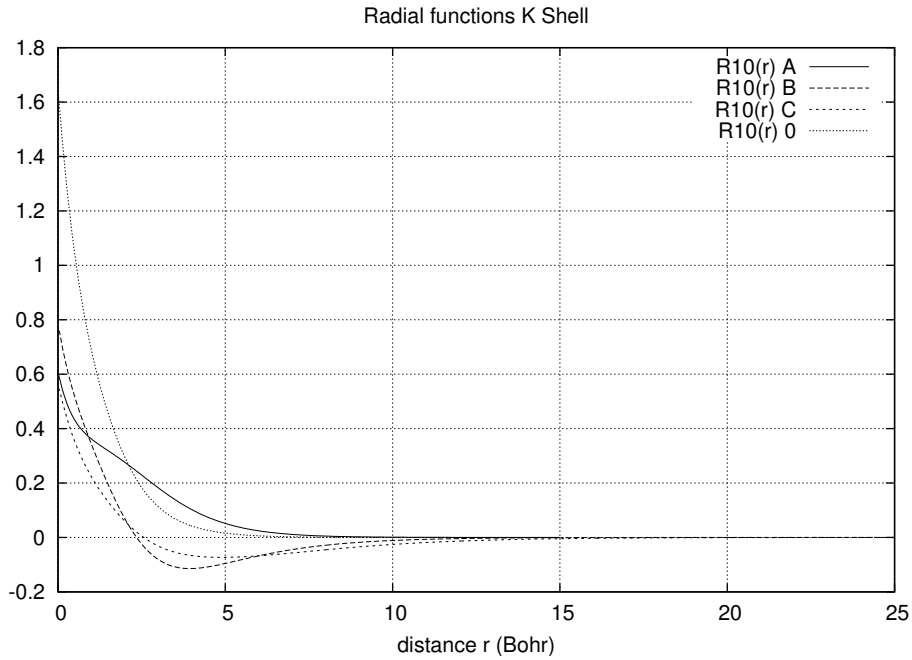
Figure 7.1: Wavenumber resonance of H

Figure 7.2: Radial 1s wavefunction. A: $\kappa = 0.60$ B: $\kappa = 0.75$ C: $\kappa = 2.0$ O: off-resonance
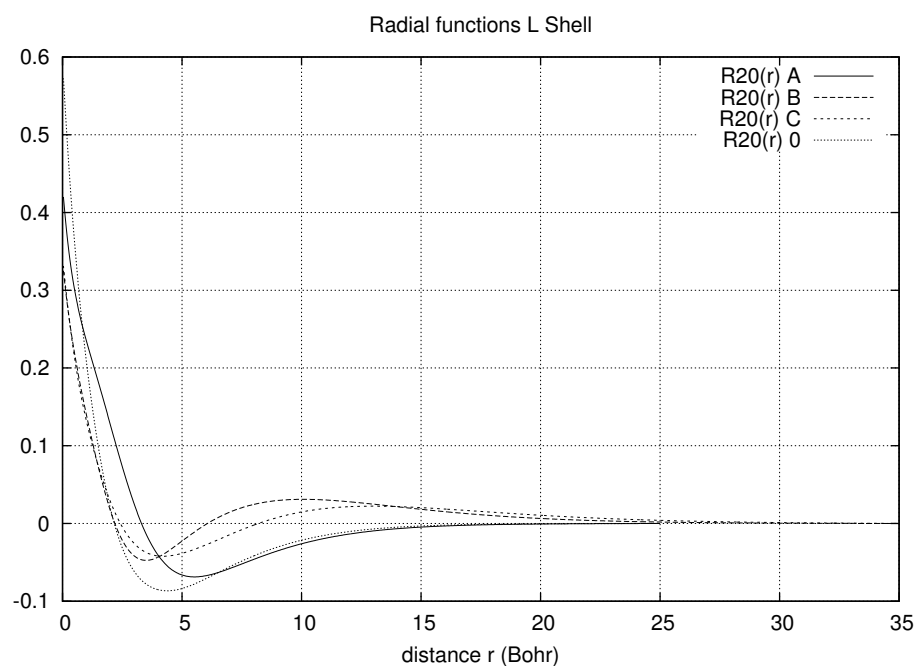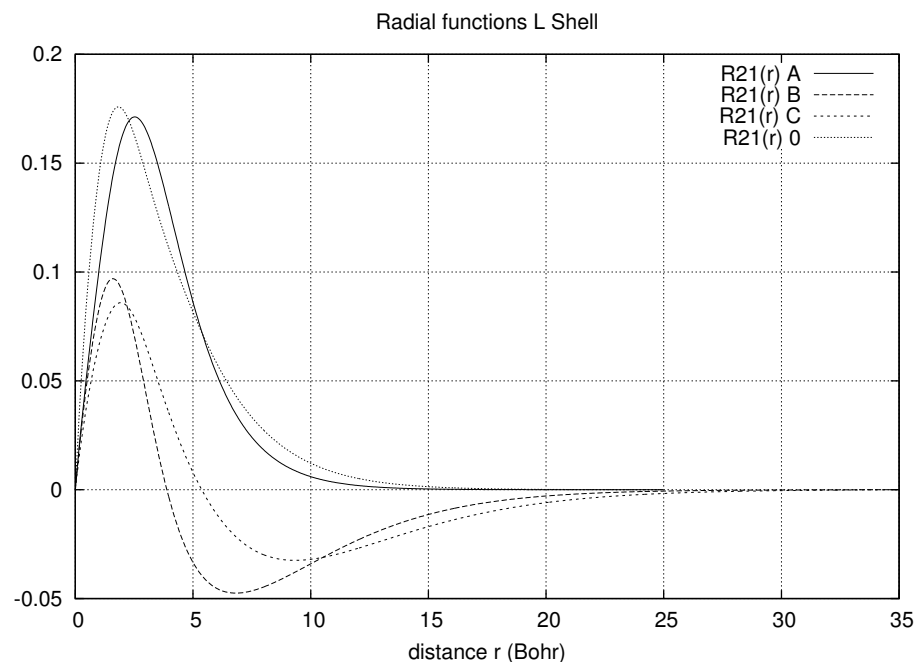
Figure 7.3: Radial 2s wavefunction. A: $\kappa = 0.60$ B: $\kappa = 0.75$ C: $\kappa = 2.0$ O: off-resonance

Figure 7.4: Radial 2p wavefunction. A: $\kappa = 0.60$ B: $\kappa = 0.75$ C: $\kappa = 2.0$ O: off-resonance
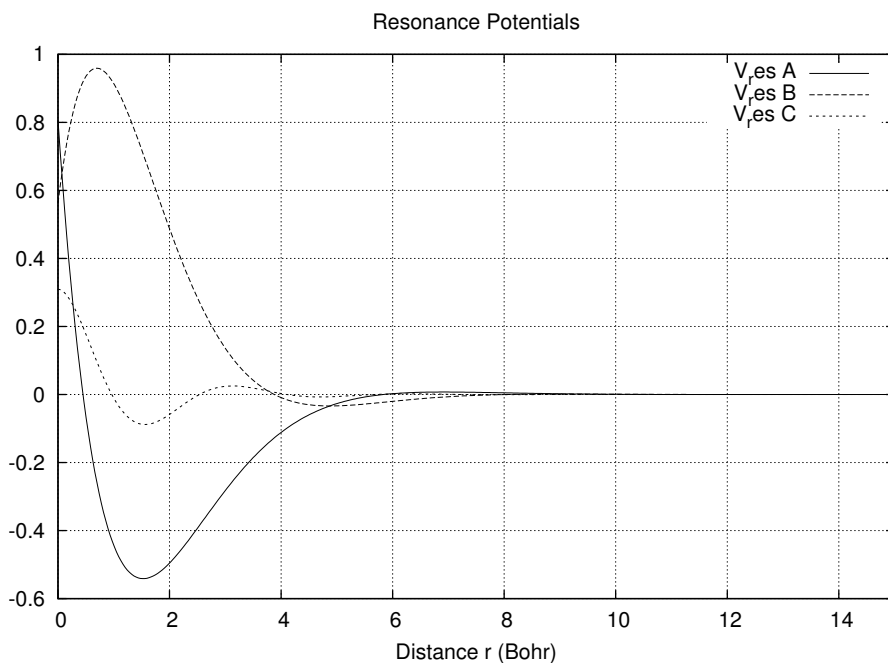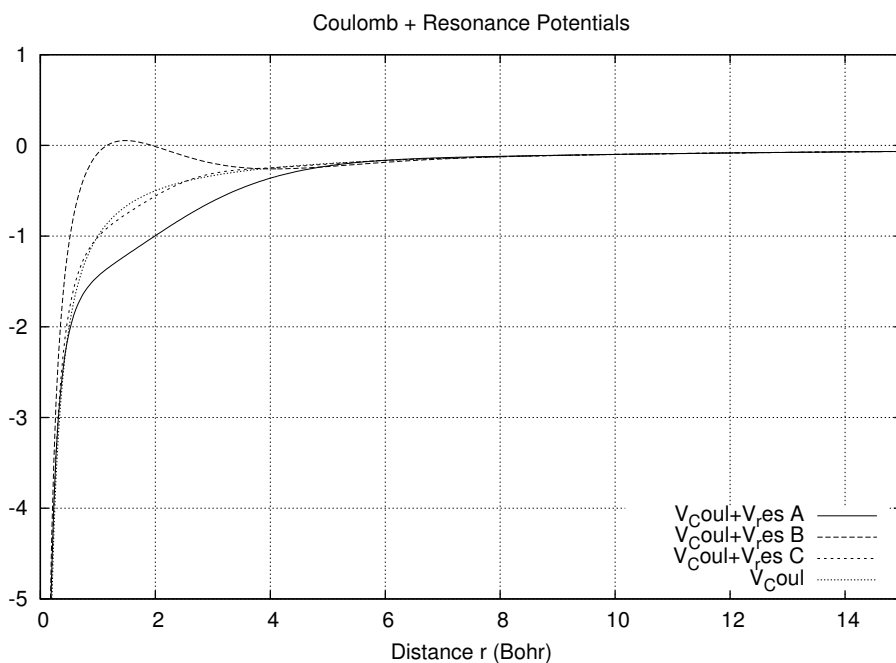
Resonance Potentials

Figure 7.5: Resonant potential. A: $\kappa = 0.60$ B: $\kappa = 0.75$ C: $\kappa = 2.0$

Coulomb + Resonance Potentials

Figure 7.6: Total potential. A: $\kappa = 0.60$ B: $\kappa = 0.75$ C: $\kappa = 2.0$
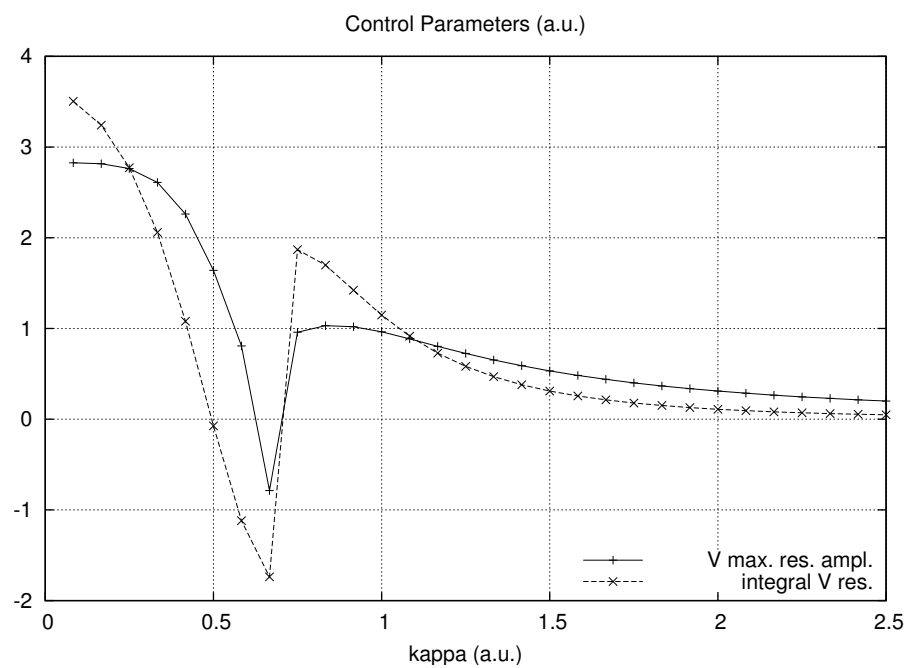
Figure 7.7: Control parameters
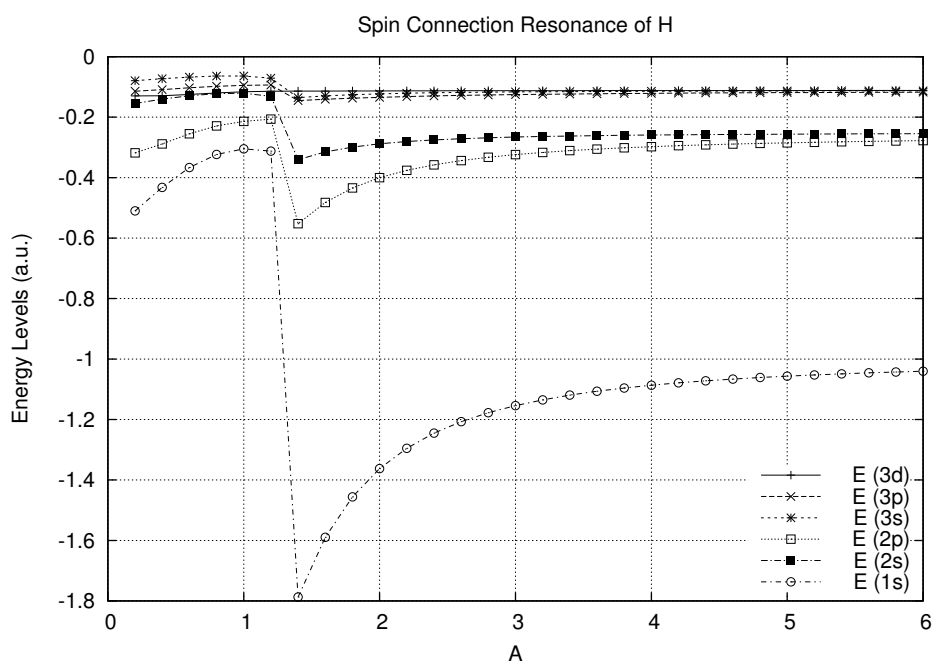
Figure 7.8: Spin connection resonance

134
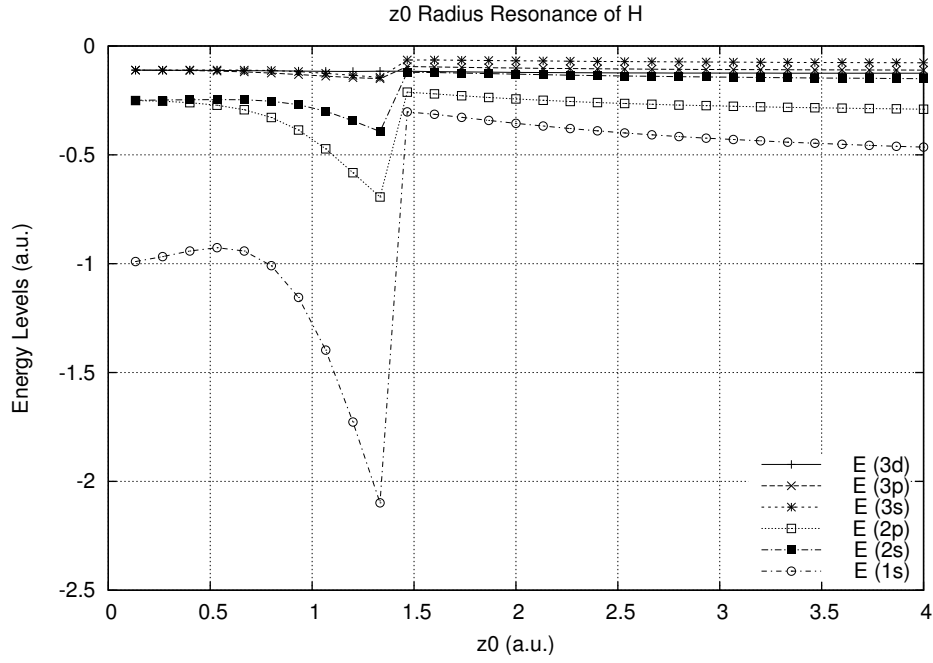
Figure 7.9: z_0 radius resonance
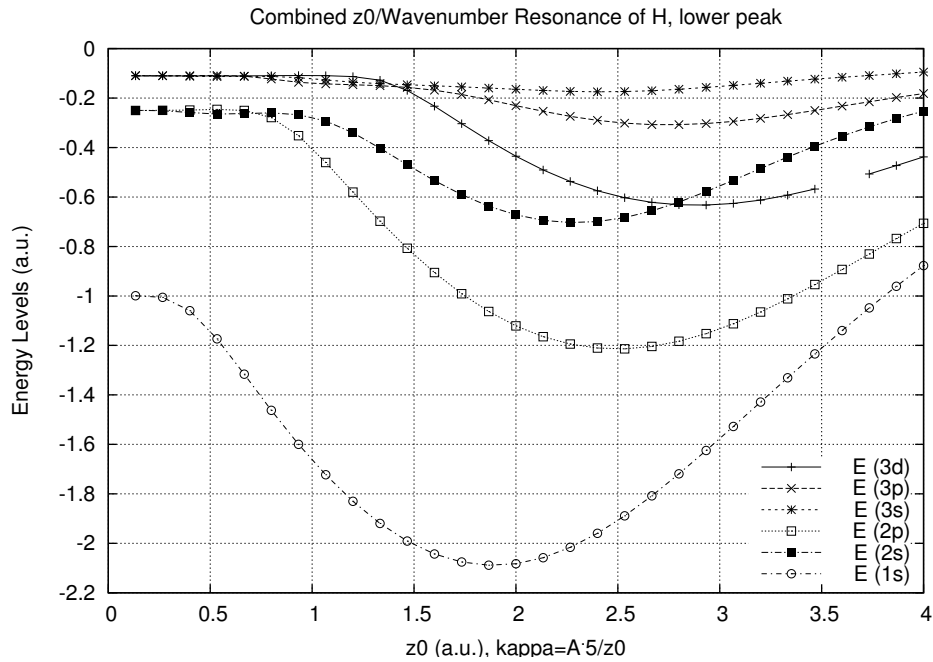
Figure 7.10: Combined z_0/wavenumber resonance, lower energies
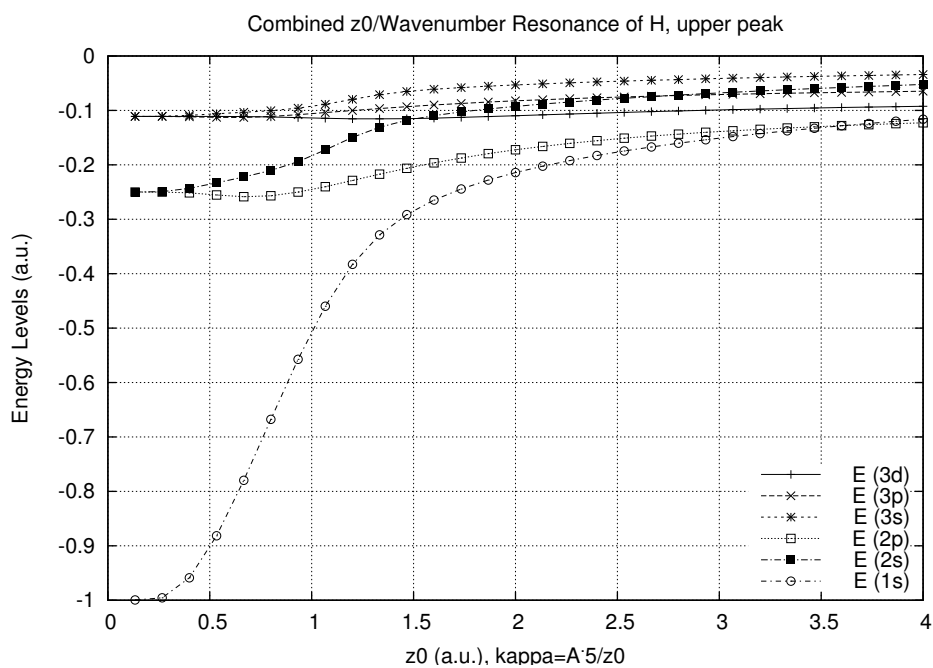
Figure 7.11: Combined z_0/wavenumber resonance, upper energies
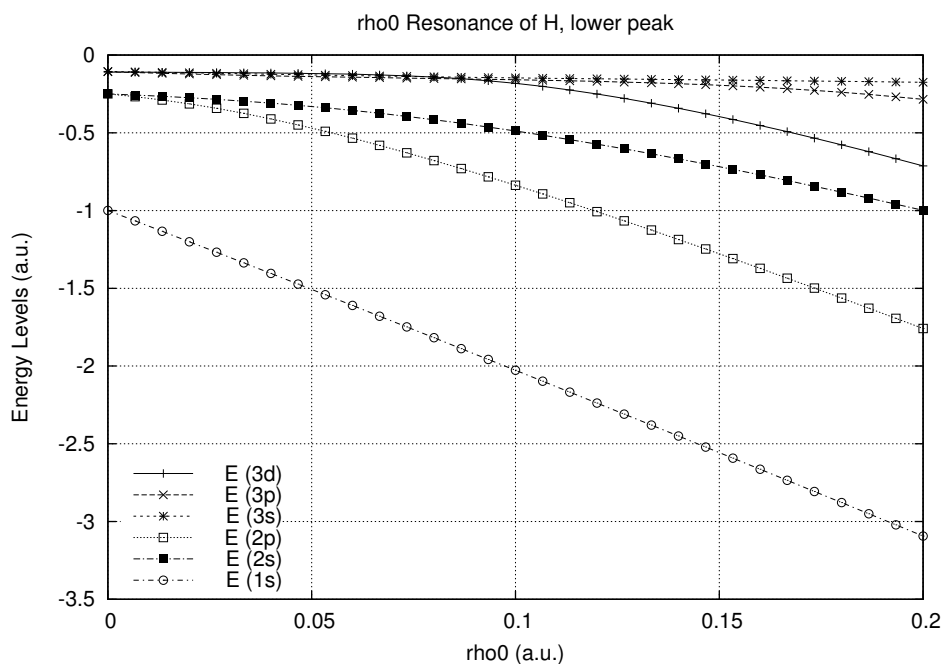
Figure 7.12: Combined ρ_0 resonance, lower energies

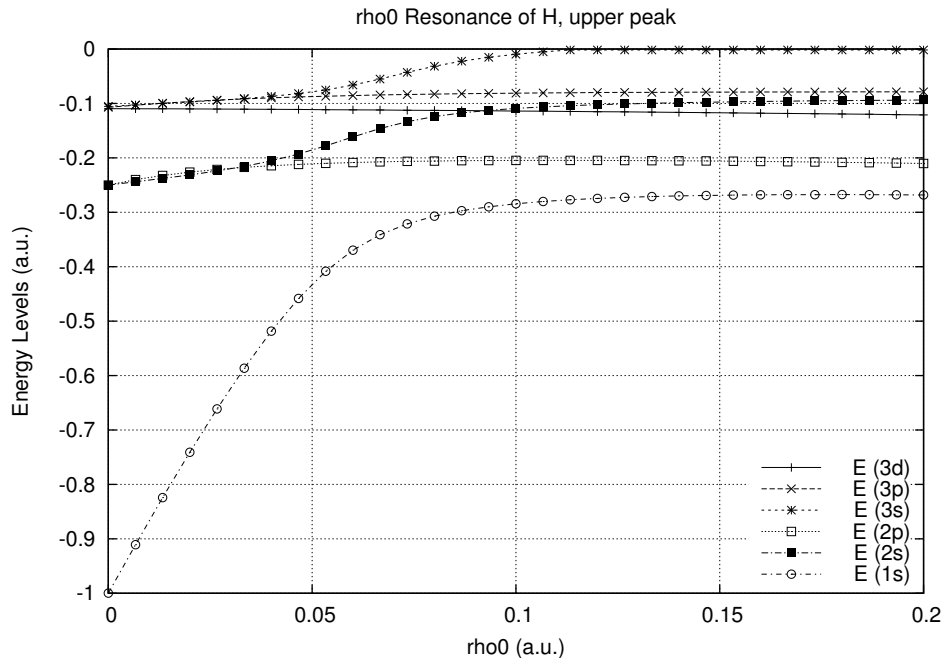Figure 7.13: ρ_0 resonance, upper energies

Bibliography

[1] NASA Cassini experiments (2002 to present).

[2] M. W. Evans, Generally Covariant Unified Field Theory (Abramis 2005), vol. 1.

[3] M. W. Evans, Generally Covariant Unified Field Theory (Abramis, 2006), vol. 2.

[4] M. W. Evans, Generally Covariant Unified Field Theory (Abramis, 2006), vol. 3.

[5] M. W. Evans, Generally Covariant Unified Field Theory (Abramis, 2007, in prep., preprint papers 55 to 70 available to date on www.aias.us and www.atomicprecision.com). , vol. 4.

[6] L. Felker, The Evans Equations of Unified Field Theory (preprint on www.aias.us and www.atomicprecision.com).

[7] H. Eckardt and L. Felker, papers on www.aias.us and www.atomicprecision.com.

[8] M. W. Evans, Generally Covariant Dynamics(paper 55 of the ECE series, www.aias.us and www.atomicprecision.com)

[9] M. W. Evans, Geodesics and the Aharonov Bohm Effects ibid., paper 56).

[10] M. W. Evans, Canonical and Second Quantization in Generally Covariant Quantum Field Theory (ibid., paper 57).

[11] M. W. Evans, The Effect of Torsion on the Schwarzschild Metric and Light Deflection due to Gravitation (ibid., paper 58).

[12] M. W. Evans, The Resonance Coulomb Law from ECE Theory: Application to the Hydrogen Atom. (ibid., paper 59).

[13] M. W. Evans, Application of Einstein Cartan Evans (ECE) Theory to Atoms and Molecules: Free Electrons at Resonance (ibid., paper 60).

[14] M. W. Evans, papers and letters in Foundations of Physics and Foundations of Physics Letters, 1993 to present.

[15] M. W. Evans, (ed.), Modern Non-Linear Optics, a special topical issue in three parts of I. Prigogine and S. A. Rice, (series eds.), Advances in Chemical Physics (Wiley-Interscience, New York, 2001, 2nd. ed.), vols. 119(1), 119(2), 119(3) (endorsed by the Royal Swedish Academy).

[16] M. W. Evans and L. B. Crowell, Classical and Quantum Electrodynamics and the $\boldsymbol{B}^{(3)}$ Field (World Scientific, Singapore, 2001).

[17] M. W. Evans and J.-P. Vigier, The Enigmatic Photon (Kluwer, Dordrecht, 1994 to 2002, hardback and softback), in five volumes.

[18] M. W. Evans and S. Kielich (eds.), first edition of ref. (15) (Wiley Interscience, New York, 1992, 1993, 1997) vols. 85(1), 85(2) and 85(3).

[19] L. H. Ryder, Quantum Field Theory (Cambridge Univ. Press, 2nd. Ed., 1996).

[20] J. D. Jackson, Classical Electrodynamics (Wiley, 1999, 3rd. ed.) chapter one.

[21] S. P. Carroll, Space-time and Geometry, an Introduction to General Relativity (Addison-Wesley, New York, 2004).

[22] Feedback sites to www.aias.us, and www.atomicprecision.com recording essentially complete acceptance of ECE theory worldwide as a workable unified field theory

[23] E. G. Milewski (Chief Ed.), The Vector Analysis Problem Solver (Research and Education Association, New York, 1987).

[24] J. B. Marion and S. T. Thornton, Classical Dynamics of Particles and Systems (HB College Publishers, New York, 1988, 3rd ed.) Chapter 3.

[25] M. W. Evans, Physica B, 182, 227, 237 (1992).

[26] Conference on the Mexican group devices, Mexico City Feb. 2006, and in Europe, observers from the U.S., Canada and Europe, also demonstrated at the U.S. Navy.

[27] S. E. Koonin, Computational Physics, Benjamin/Cummings Publishing Company, 1986, pp. 49-62.