Chapter 8

Application of the ECE Lemma to the fermion and electromagnetic fields

(Paper 62)
by
Myron W. Evans
Alpha Institute for Advanced Study (AIAS).
(emyrone@aol.com, www.aias.us, www.atomicprecision.com)

Abstract

The Einstein Cartan Evans (ECE) Lemma is illustrated with reference to the fermion field (for example a Dirac electron) and the electromagnetic field. In both cases the scalar curvature R of the Lemma must be identically non-zero in a generally covariant unified field theory. In the case of the electromagnetic field this result means that the mass of the photon must be identically non-zero. If not, the electric and magnetic components of the electromagnetic field vanish.
Keywords: Einstein Cartan Evans (ECE) Lemma, fermion field, electromagnetic field, generally covariant unified field theory, photon mass.

8.1 Introduction

The Einstein Cartan Evans (ECE) field theory [1]– [18] is a generally covariant unified field theory as required by objectivity in the natural sciences. ECE theory unifies quantum mechanics and general relativity using the ECE Lemma and ECE wave equation. The former is derived straightforwardly (Section 8.2) from the well known [19] tetrad postulate of Cartan's geometry (differential geometry). In this paper the ECE Lemma is illustrated with respect to the fermion and electromagnetic fields (Section 8.3) and found to be self-consistent in its generally covariant description of both fields. In general relativity (ECE field theory) the photon mass must be identically non-zero and this result is derived self-consistently from the ECE Lemma through the finding that its eigenvalues, the scalar curvatures R, must be identically non-zero. The ECE Lemma shows that if the photon mass is identically zero, the electric and magnetic components of the electromagnetic field disappear. In the limit of zero Cartan torsion, ECE theory reduces to Einstein Hilbert (EH) field theory, the original and well known theory of gravitational general relativity published in 1916. The EH theory explains the bending of light by gravity to a precision of one part in 100,000 in the Solar System. This semi-classical explanation self-consistently depends on identically non-zero photon mass, without which the bending of light does not occur semi-classically.

8.2 Derivation of the ECE Lemma from the tetrad postulate

The derivation of the ECE Lemma [1]– [18] from the tetrad postulate is straightforward and is given here for convenience of reference. Begin with the tetrad postulate [19]:

$$D_\mu q^a{}_\nu = 0 \tag{8.1}$$

where D_μ denotes covariant derivative and $q^a{}_\nu$ is the tetrad form. The latter is a mixed index tensor, a vector-valued one-form of differential geometry [1]– [19]. The basic rules of covariant differentiation of a rank two mixed index tensor are given in ref. [19] and applied self-consistently in ECE field theory [1]– [18]. These rules mean that:

$$D_\mu q^a{}_\lambda = \partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu_{\mu\lambda} q^a{}_\nu = 0 \tag{8.2}$$

where $\Gamma^\nu_{\mu\lambda}$ is the general gamma connection of Riemann geometry and where $\omega^a{}_{\mu b}$ is the spin connection of Cartan geometry. Summation over repeated contravariant-covariant indices is implied as usual (the Einstein convention). This summation applies to both Greek and Latin repeated indices wherever they occur in an equation. The ECE Lemma is an identity:

$$D^\mu (D_\mu q^a{}_\nu) := 0 \tag{8.3}$$

formed by covariant differentiation of the tetrad postulate. For a scalar [1]– [19]:

$$D^\mu \phi = \partial^\mu \phi \tag{8.4}$$

All elements in Eq.(8.1) are zero. Applying covariant differentiation to each element:

$$D^\mu 0 = \partial^\mu 0 = 0 \tag{8.5}$$

Therefore Eq.(8.3) becomes:

$$\partial^\mu \left(\partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu_{\mu\lambda} q^a{}_\nu \right) = 0 \tag{8.6}$$

This result has been checked for internal self consistency [1]– [18] by applying the rules [19] for covariant differentiation of a rank three mixed index tensor as follows:

$$\begin{aligned} D_\mu (D^\mu q^a{}_\nu) &= \\ \partial_\mu (D^\mu q^a{}_\nu) + \Gamma^\mu_{\mu\lambda} D^\lambda q^a{}_\nu &+ \omega^a{}_{\mu b} D^\mu q^b{}_\nu - \Gamma^\lambda_{\mu\nu} D^\mu q^a{}_\lambda \\ &= \partial_\mu (D^\mu q^a{}_\nu) \end{aligned} \tag{8.7}$$

using the Lemma (8.3) in Eq.(8.7) produces:

$$\partial^\mu (D_\mu q^a{}_\nu) = 0 \tag{8.8}$$

which is Eq.(8.6) again, Q.E.D. Various other cross-checks of the Lemma are given in the literature [1]– [18]. Eq.(8.6) can be rewritten as:

$$\Box q^a{}_\lambda = \partial^\mu \left(\Gamma^\nu_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda \right) \tag{8.9}$$

where

$$\Box := \partial^\mu \partial_\mu \tag{8.10}$$

is the d'Alembertian operator. Now define the scalar curvature by:

$$R q^a{}_\lambda := \partial^\mu \left(\Gamma^\nu_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda \right) \tag{8.11}$$

Using the rule [19] for tetrad normalization:

$$q^a{}_\lambda q^\lambda{}_a = 4 \tag{8.12}$$

multiply both sides of Eq.(8.11) by $q^\lambda{}_a$ to obtain:

$$R := \frac{1}{4} q^\lambda{}_a \partial^\mu \left(\Gamma^\nu_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda \right) \tag{8.13}$$

With this definition the ECE Lemma is [1]– [18]

$$\Box q^a{}_\lambda = R q^a{}_\lambda \tag{8.14}$$

and is an Eigen-equation or wave equation of generally covariant unified field theory. It unifies quantum mechanics and general relativity. The ECE Lemma is the subsidiary proposition that leads to the ECE wave equation [1]– [18]:

$$(\Box + kT) q^a{}_\lambda = 0 \tag{8.15}$$

where k is Einstein's constant [20] and T is the index contracted canonical energy-momentum density. In ECE theory the Einstein Ansatz:

$$R = -kT \tag{8.16}$$

is applied to the unified field.

8.3 The fermion and electromagnetic fields

The scalar curvature in Eq.(8.13) may be expressed as:

$$R = q^\nu{}_a R^a{}_\nu \tag{8.17}$$

where:

$$R^a{}_\nu = \partial^\mu \left(\Gamma^a_{\mu\nu} - \omega^a{}_{\mu\nu} \right) \tag{8.18}$$

and

$$\begin{aligned} \Gamma^a_{\mu\lambda} &= \Gamma^\nu_{\mu\lambda} q^a{}_\nu \\ \omega^a{}_{\mu\lambda} &= \omega^a{}_{\mu b} q^b{}_\lambda \end{aligned} \tag{8.19}$$

The mixed index tensor R^a_ν is similar to the well known Ricci tensor of gravitational general relativity. Now assume that the tetrad is a traveling wave in the Z axis:

$$q^a{}_\lambda = q^a{}_\lambda(0) e^{i(\omega t - \kappa Z)} \tag{8.20}$$

and let

$$\kappa = \frac{\omega}{v}$$

where v is the phase velocity. Then:

$$\begin{aligned}\Box q^a{}_\lambda &= \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial Z^2}\right) q^a{}_\lambda(0)e^{i(\omega t - \kappa Z)} \\ &= \left(\kappa^2 - \frac{\omega^2}{c^2}\right) q^a{}_\lambda\end{aligned} \tag{8.21}$$

Therefore:

$$R = \kappa^2 - \frac{\omega^2}{c^2} = \omega^2\left(\frac{1}{v^2} - \frac{1}{c^2}\right) \tag{8.22}$$

and

$$R^a{}_\nu = \omega^2\left(\frac{1}{v^2} - \frac{1}{c^2}\right) q^a{}_\nu \tag{8.23}$$

From Eq.(8.18):

$$\begin{aligned}(\Gamma^a - \omega^a)_{30} &= R\int q^a{}_0(0)e^{i(\omega t - \kappa Z)}dZ \\ &= \frac{iR}{\kappa} q^a{}_0(0)e^{i(\omega t - \kappa Z)}\end{aligned} \tag{8.24}$$

whose real part is:

$$Re(\Gamma^a - \omega^a)_{30} = -\frac{R}{\kappa} q^a{}_0(0)sin(\omega t - \kappa Z) \tag{8.25}$$

These results are useful to understand the Dirac equation. The latter is obtained in the following limit of the ECE wave equation [1]– [18]:

$$R = -\left(\frac{mc}{\hbar}\right)^2 \tag{8.26}$$

From Eqs.(8.22) and (8.26):

$$\kappa^2 - \frac{\omega^2}{c^2} = -\frac{m^2c^2}{\hbar^2} \tag{8.27}$$

i.e.

$$\frac{\omega^2}{c^2} = \kappa^2 + \frac{m^2c^2}{\hbar^2} \tag{8.28}$$

Now use the well known Planck/Einstein and de Broglie quantum relations:

$$E = \hbar\omega\ ,\ p = \hbar\kappa \tag{8.29}$$

and Eq.(8.28) becomes the well known Einstein equation of special relativity:

$$E^2 = p^2c^2 + m^2c^4 \tag{8.30}$$

where E is the total relativistic energy, p is the relativistic momentum and:

$$E_0 = mc^2 \tag{8.31}$$

is the rest energy. The well known Compton wavelength is:

$$\lambda = \frac{\hbar}{mc} \tag{8.32}$$

The Dirac equation is therefore [1]– [18]:

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right) q^a{}_\nu = 0 \tag{8.33}$$

where:

$$\begin{aligned} q^a{}_\nu &= q^a{}_\nu(0)e^{i(\omega t - \kappa Z)} \\ &:= q^a{}_\nu(0)e^{ix_\mu p^\mu} \end{aligned} \tag{8.34}$$

By convention [21] the positive energy plane wave spinor is:

$$q^a{}_\nu = q^a{}_\nu(0)e^{-ix_\mu p^\mu} \tag{8.35}$$

and the negative energy plane wave spinor is:

$$q^a{}_\nu = q^a{}_\nu(0)e^{ix_\mu p^\mu} \tag{8.36}$$

So the Ricci type curvature tensor for the Dirac electron is:

$$R^a{}_\nu = \left(\kappa^2 - \frac{\omega^2}{c^2}\right) q^a{}_\nu \tag{8.37}$$

and the connection element in Eq.(8.25) is given for the Dirac equation and any fermion by using Eq.(8.22) for R.

The Dirac equation is therefore a manifestation of ECE space-time [1]– [18] in the particular case defined by Eq.(8.22) for R and Eq.(8.37) for R^α_ν. The Einstein equivalence principle [20] states that this particular case is the limit when the fermion field has become independent of the gravitational field. This is the limit of special relativity according to Einstein's original equivalence principle. More generally in ECE theory it is the case where the fermion field has become independent of ALL other fields, not only of the gravitational field. In order to apply the ECE Lemma to the electromagnetic field the ECE Ansatz is used [1]– [18]:

$$A^a_\mu = A^{(0)} q^a{}_\mu \tag{8.38}$$

following a suggestion made by Cartan to Einstein in well known correspondence that the electromagnetic field be the Cartan torsion within a factor $A^{(0)}$. Here $cA^{(0)}$ has the units of volts and is referred to in ECE theory as the primordial voltage. Therefore for the electromagnetic field the ECE Lemma is:

$$\Box A^a_\mu = R A^a_\mu \tag{8.39}$$

In the limit of special relativity Eq.(8.39) becomes the Proca equation [1]– [18]:

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right) A^a_\mu = 0 \tag{8.40}$$

where m is the identically non-zero mass of the photon. If it is asserted for the sake of argument that in Eq.(8.39):

$$R = 0 \tag{8.41}$$

it follows that:

$$\partial^\mu \left(\Gamma^a_{\mu\nu} - \omega^a{}_{\mu\nu}\right) = 0 \tag{8.42}$$

so:

$$\Gamma^a_{\mu\nu} = \omega^a{}_{\mu\nu} + \zeta^a_{\mu\nu}(0) \tag{8.43}$$

where $\zeta^a_{\mu\nu}(0)$ is a constant of integration independent of x^μ. It follows that in ECE theory:

$$\Gamma^a_{\mu\nu} = \omega^a{}_{\mu\nu} \tag{8.44}$$

because if $\zeta^a_{\mu\nu}(0)$ is independent of x^μ it cannot present curvature or torsion. Any ECE field must be curvature or torsion or a combination thereof. If there is no curvature or torsion there is no field present and the constant of integration may be set to zero. So:

$$\Gamma^a_{\mu\nu} = \omega^a{}_{\mu\nu} \text{ if } R = 0 \tag{8.45}$$

for identically zero scalar curvature. It follows from Eq.(8.42) that:

$$\partial_\nu q^a{}_\mu = 0 \tag{8.46}$$

in this limit. In gravitational theory the tetrad is the fundamental field and Eq.(8.46) indicates that the tetrad is independent of x^ν, meaning again no curvature and no field. So if the scalar curvature is identically zero there is no gravitational field, a self-consistent result. Similarly for identically zero R the electromagnetic field obeys:

$$\partial_\nu A^a_\mu = 0 \tag{8.47}$$

and A^a_μ is independent of x^ν, meaning that there is no Cartan torsion and no electric or magnetic field present. The absence of R means the absence of photon mass, so the absence of photon mass means in turn the absence of electric and magnetic components of the electromagnetic field. Therefore in a generally covariant unified field theory the photon mass must be identically non-zero. This is again a self-consistent result because without mass in general relativity there is no field. In special relativity (Maxwell Heaviside theory) there is no concept of photon mass, and the Proca equation for identically zero photon mass is used for the free electromagnetic field:

$$\Box A^a_\mu = 0 \tag{8.48}$$

This is however inconsistent with general relativity and is therefore not objective physics. If there are interacting fields (as in the bending of light by gravity) the T term in the ECE wave equation must be constructed from contributions from all fields, including interactive terms, as originally inferred [20] by Einstein himself. In Maxwell Heaviside (MH) theory there is no mechanism for this, even on a conceptual level, because in MH theory the electromagnetic field is an entity superimposed on flat or Minkowski space-time.

Acknowledgements The British Parliament, Prime Minister and Head of State are thanked for the award of a Civil List pension in recognition of distinguished contributions to Britain and the Commonwealth in science. The AIAS intellectual environment is thanked for many interesting and formative discussions.

Bibliography

[1] M. W. Evans, Generally Covariant Unified Field Theory (Abramis, 2005), vol. 1.

[2] M. W. Evans, Generally Covariant Unified Field Theory (Abramis, 2006), vol. 2.

[3] M. W. Evans, Generally Covariant Unified Field Theory (Abramis, 2006), vol. 3.

[4] M. W. Evans, Generally Covariant Unified Field Theory (Abramis, 2007, in press, preprint of papers 55 to 70 on www.aias.us and www.atomicprecision.com), vol. 4.

[5] L. Felker, The ECE Equations of Unified Field Theory (preprint on www.aias.us and www.atomicprecision.com). ; H. Eckardt and L. Felker, papers on these websites.

[6] M. W. Evans, Generally Covariant Dynamics (paper 55 of the ECE series on www.aias.us and www.atomicprecision.com).

[7] M. W. Evans, Geodesics and the Aharonov Bohm Effects (ibid., paper 56).

[8] M. W. Evans, Canonical and Second Quantization in Generally Covariant Quantum Field Theory (ibid., paper 57).

[9] M. W. Evans, The Effect of Torsion on the Schwarzschild Metric and Light Deflection due to Gravitation (ibid., paper 58).

[10] M. W. Evans, The Resonance Coulomb Law from ECE Theory: Application to the Hydrogen Atom (ibid., paper 59).

[11] M. W. Evans, Application of Einstein Cartan Evans (ECE) Theory to Atoms and Molecules: Free Electrons at Resonance (ibid., paper 60).

[12] M. W. Evans and H. Eckardt, Space-time Resonances in the Coulomb Law (ibid., paper 61).

[13] M. W. Evans, papers and letters in Foundations of Physics and Foundations of Physics Letters from 1994 to present on $\boldsymbol{B}^{(3)}$ theory, O(3) electrodynamics and ECE theory.

[14] M. W. Evans (ed.), Modern Non-linear Optics, a special topical issue of I Prigogine and S. A. Rice (Series Eds.), Advances in Chemical Physics (Wiley-Interscience, New York, 2001, 2nd. Ed.), vols. 119(1) to 119(3) (endorsed by the Royal Swedish Academy in honor of the late Jean-Pierre Vigier).

[15] M. W. Evans and S. Kielich (eds.), ibid., first edition (Wiley-Interscience, New York, 1992, 1993, 1997), vols. 85(1) to 85(3), awarded a prize for excellence by the Polish Government.

[16] M. W. Evans and L. B. Crowell, Classical and Quantum Electrodynamics and the $\boldsymbol{B}^{(3)}$ Field (World Scientific, Singapore, 2001).

[17] M. W. Evans and J.-P. Vigier, The Enigmatic Photon (Kluwer, Dordrecht, 1994 to 2002, hardback and softback), in five volumes.

[18] M. W. Evans and A. A. Hasanein, The Photomagneton in Quantum Field Theory (World Scientific, Singapore, 1994).

[19] S. P. Carroll, Space-time and Geometry, an Introduction to General Relativity (Addison-Wesley, New York, 2004).

[20] A. Einstein, The Meaning of Relativity (Princeton Univ. Press, 1921 to 1954).

[21] L. H. Ryder, Quantum Field Theory (Cambridge Univ. Press, 2nd ed., 1996).