Chapter 9

The resonant Coulomb Law of Einstein Cartan Evans Field Theory

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Abstract

Einstein Cartan Evans (ECE) field theory is used to show that in general relativity the structure of the laws of electricity, magnetism and electromagnetism is changed fundamentally. This result is demonstrated analytically and numerically with the Coulomb Law. In ECE theory the electromagnetic field is spinning space-time, characterised by the spin connection of Cartan geometry. The spin connection is shown in this paper to change the Poisson equation into a differential equation capable of giving resonance. Off resonance, the standard Poisson equation is observed, and the standard Coulomb Law. At space-time resonance the scalar potential in volts is greatly amplified with fundamental consequences in the natural, engineering and life sciences.

Keywords: Einstein Cartan Evans (ECE) field theory, generally covariant unified field theory, electricity, magnetism, electromagnetism, classical electrodynamics, Coulomb Law, space-time resonance.

9.1 Introduction

The need for objectivity in the natural, engineering and life sciences means that all the fundamental laws of physics must be laws of general relativity, where objectivity is represented by geometry. This includes the laws of classical electrodynamics, the laws of electricity, magnetism and electromagnetism. General

relativity means that these laws must be generally covariant, must retain their form under any type of coordinate transformation. They must therefore be laws of a generally covariant unified field theory. Recently [1]– [19] the Einstein Cartan Evans (ECE) theory has been developed along these well known guidelines. Objectivity in ECE field theory is maintained through the use of standard Cartan geometry [20]. The electromagnetic field is represented as the Cartan torsion within a scalar factor $A^{(0)}$. Here $cA^{(0)}$ is a primordial voltage. This procedure follows a well known suggestion by Cartan to Einstein that the electromagnetic field be the Cartan torsion form, a vector valued two-form of differential geometry [1]– [20]. ECE theory applies Cartan's suggestion systematically to all the laws of physics.

The well known Maxwell Heaviside (MH) field theory is used in the standard model [21] to represent the laws of electricity, magnetism and electromagnetism. The MH theory is a nineteenth century theory of special relativity and is neither generally covariant nor unified with other fundamental fields such as gravitation. It is a Lorentz covariant theory in which the electromagnetic field is considered to be an entity separate from the frame of reference in a Minkowski space-time. The latter is often referred to as flat space-time, because it has neither curvature nor torsion. It is also a static space-time. The laws of gravitation on the other hand are described in the standard model by the Einstein Hilbert (EH) field theory of general relativity [1]– [20] and the gravitational field is the frame itself, not something separate from the frame as in MH theory. EH space-time has curvature but no torsion, and is a dynamic space-time. Objectivity in EH field theory is based on Riemann geometry with a Christoffel connection. This assumption implies a zero torsion tensor [20] and means that gravitation cannot be unified with electromagnetism in EH theory. As we have argued, electrodynamics cannot be unified with gravitation in MH theory. In ECE theory [1]– [19] a unified description of all fields has been developed straightforwardly using ECE space-time in which curvature and torsion are simultaneously non-zero. The generally covariant unified field of ECE theory is the frame itself, as required by general relativity and objectivity. Gravitation is described by curvature, the electromagnetic, weak and strong fields by torsion using the appropriate representation spaces (respectively $O(3)$, $SU(2)$ and $SU(3)$). The two Cartan structure equations and the two Bianchi identities of differential geometry control all the laws of physics [1]– [19], including those of quantum mechanics through the tetrad postulate [20]. The latter is the fundamental requirement that the complete vector field in n dimensions be independent of the components and basis elements chosen to represent it. Thus ECE theory has unified quantum mechanics with general relativity and provides a generally covariant unified field theory. ECE theory has therefore been accepted as mainstream physics [22].

In Section 9.2, the Coulomb law of electricity is developed with the spin connection incorporated as required by general relativity, by the fact that the complete electromagnetic field is spinning space-time. A spinning of space-time means a spinning of the frame of reference itself. This means that the spin connection must always be non-zero.

The Coulomb Law is derived in ECE theory [1]– [19] from the first Cartan structure equation and the first Bianchi identity. Use of vector notation and some simplifying assumptions [1]– [19] lead to the initial equations of Section 9.2. These give a resonance equation whose properties are developed analytically

to give the required resonance solution. The latter is fundamentally important in the natural, engineering and life sciences because the Coulomb law is the basis of all quantum chemistry, and therefore the basic law of computational quantum chemistry.

In Section 9.3, numerical solutions of the resonance Coulomb law are developed to illustrate a novel space-time resonance spectrum. Off resonance the standard Coulomb Law is recovered. The standard Coulomb law is well known [21] to be among the most precise laws of physics, so it must be recovered from ECE theory in a given limit. This is achieved by identifying the radial spin connection as the one that gives the standard Coulomb law off resonance. In the off resonant condition the spin connection effectively doubles the value of the electric field, so its presence cannot be detected experimentally. At space-time resonance however the scalar potential in volts of the Coulomb Law is greatly amplified, leading to a surge in voltage that cannot be explained by MH theory. This phenomenon has been reported experimentally by several independent groups [23]. If this resonant amplification of the scalar potential occurs inside an atom or molecule, electrons may be released by ionization. In this Section the process is illustrated by the radial wave-functions of the hydrogen atom in anticipation of the systematic development of density functional code incorporating the resonant Coulomb law of ECE theory. A short review of density functional methods is also given in this section. Finally a discussion is given of how to induce space-time resonance in circuits and materials.

9.2 The Resonant Coulomb Law

In the simplest instance [1]– [19] the Coulomb law in ECE theory is given by:

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \tag{9.1}$$

where

$$\mathbf{E} = -\left(\boldsymbol{\nabla} + \boldsymbol{\omega}\right)\phi. \tag{9.2}$$

Here ϕ is the scalar potential in volts, $\boldsymbol{\omega}$ is the vector spin connection in inverse meters, $\mathbf{E}$ is the electric field strength in volts m^{-1}, ρ is the charge density in Cm^{-3}, and ϵ_0 is the S. I. vacuum permittivity:

$$\epsilon_0 = 8.854 \times 10^{-12} J^{-1} C^2 m^{-1}. \tag{9.3}$$

Thus:

$$\boldsymbol{\nabla} \cdot \left(\left(\boldsymbol{\nabla} + \boldsymbol{\omega}\right)\phi\right) = -\frac{\rho}{\epsilon_0} \tag{9.4}$$

i.e.

$$\nabla^2 \phi + \boldsymbol{\nabla} \cdot \left(\phi \boldsymbol{\omega}\right) = -\frac{\rho}{\epsilon_0} \tag{9.5}$$

If there is no spin connection, Eq.(9.5) is the Poisson equation [21] of the standard model. Otherwise:

$$\nabla^2 \phi + \boldsymbol{\omega} \cdot \boldsymbol{\nabla} \phi + \left(\boldsymbol{\nabla} \cdot \omega\right)\phi = -\frac{\rho}{\epsilon_0} \tag{9.6}$$

which is an equation capable of giving resonant solutions [1]– [19], [24] from the spin connection vector. The Poisson equation does not give resonant solutions.

Eq.(9.6) is first developed in one (Z) dimension of the Cartesian coordinate system, and then for the radial component of the spherical polar coordinate system [25].

In one Z dimension Eq.(9.6) becomes:

$$\frac{\partial^2\phi}{\partial Z^2}+\omega_Z\frac{\partial\phi}{\partial Z}+\left(\frac{\partial\omega_Z}{\partial Z}\right)\phi=-\frac{\rho}{\epsilon_0} \tag{9.7}$$

The spin connection in Eq.(9.7) must be:

$$\omega_Z=\frac{2}{Z} \tag{9.8}$$

in order to recover the standard Coulomb law off resonance. This is because:

$$\phi=\frac{-e}{4\pi\epsilon_0 Z},\quad \frac{\partial\phi}{\partial Z}=\frac{e}{4\pi\epsilon_0 Z^2}=-\frac{\omega_z}{2}\phi \tag{9.9}$$

in the off resonant condition, giving Eq.(9.8). In the off resonant condition the role of the spin connection is to change the sign of the electric field according to Eq.(9.9). The way in which the field $\mathbf{E}$ and potential ϕ are related is changed, but this has no experimental effect since $\mathbf{E}$ is effectively replaced by $-\mathbf{E}$. With the spin connection (9.8) Eq.(9.7) becomes:

$$\frac{\partial^2\phi}{\partial Z^2}+\frac{2}{Z}\frac{\partial\phi}{\partial Z}-\frac{2}{Z^2}\phi=-\frac{\rho}{\epsilon_0} \tag{9.10}$$

Now assume that the charge density is initially oscillatory:

$$\rho=\rho\left(0\right)\cos\left(\kappa Z\right) \tag{9.11}$$

where κ is a wave-number. Thus:

$$\frac{\partial^2\phi}{\partial Z^2}+\frac{2}{Z}\frac{\partial\phi}{\partial Z}-\frac{2}{Z^2}\phi=-\rho\left(0\right)\cos\left(\kappa Z\right) \tag{9.12}$$

Since ϕ depends only on Z the partial derivatives can be replaced by total derivatives to give an ordinary differential equation [24], [26]:

$$\frac{\partial^2\phi}{\partial Z^2}+\frac{2}{Z}\frac{\partial\phi}{\partial Z}-\frac{2}{Z^2}\phi=-\rho\left(0\right)\cos\left(\kappa Z\right) \tag{9.13}$$

using the well known Euler method [24], [26] this equation can be reduced to an undamped oscillator equation that has resonant solutions. Define a change of variable [24], [26] by:

$$\kappa Z=e^{i\kappa x} \tag{9.14}$$

Thus:

$$\frac{dx}{dZ}=-\frac{i}{\kappa Z} \tag{9.15}$$

Now use:

$$\frac{d\phi}{dZ}=\frac{d\phi}{dx}\frac{dx}{dZ}=-\frac{i}{\kappa Z}\frac{d\phi}{dx} \tag{9.16}$$

and construct the second derivative:

$$\frac{d^2\phi}{dZ^2}=\frac{i}{\kappa Z^2}\frac{d\phi}{dx}-\frac{i}{\kappa Z}\frac{d}{dZ}\left(\frac{d\phi}{dx}\right) \tag{9.17}$$

Isotropy means that:

$$\frac{d}{dZ}\left(\frac{d\phi}{dx}\right)=\frac{d^2\phi}{dZdx}=\frac{d^2\phi}{dxdZ}=\frac{d}{dx}\left(\frac{d\phi}{dZ}\right) \tag{9.18}$$

so

$$\frac{d^2\phi}{dZ^2}=\frac{i}{\kappa Z^2}\frac{d\phi}{dx}-\frac{1}{\kappa^2 Z^2}\frac{d^2\phi}{dx^2}. \tag{9.19}$$

Thus:

$$Z\frac{d\phi}{dZ}=-\frac{i}{\kappa}\frac{d\phi}{dx}, \tag{9.20}$$

$$Z^2\frac{d^2\phi}{dZ^2}=\frac{i}{\kappa}\frac{d\phi}{dx}-\frac{1}{\kappa^2}\frac{d^2\phi}{dx^2}. \tag{9.21}$$

Now substitute Eqs.(9.20) and (9.21) in Eq.(9.13) to give:

$$\frac{d^2\phi}{dx^2}+2\kappa^2\phi=\frac{\rho\left(0\right)}{\epsilon_0}\mathrm{Real}\left(e^{2i\kappa x}\cos\left(e^{i\kappa x}\right)\right) \tag{9.22}$$

which the undamped oscillator equation [24], [26]:

$$\begin{aligned}\frac{d^2\phi}{dx^2}+2\kappa^2\phi=\frac{\rho\left(0\right)}{\epsilon_0}&\left(\cos\left(2\kappa x\right)\cos\left(\cos\left(\kappa x\right)\right)\cosh\left(\sin\left(\kappa x\right)\right)\right.\\&\left.+\sin\left(2\kappa x\right)\sin\left(\cos\left(\kappa x\right)\right)\sinh\left(\sin\left(\kappa x\right)\right)\right).\end{aligned} \tag{9.23}$$

Assume that the particular integral of this equation is:

$$\phi_p\left(x\right)=\frac{A_\rho\left(0\right)}{\epsilon_0}\left(\frac{\cos\left(\kappa' x\right)}{2\kappa^2-\kappa'^2}\right) \tag{9.24}$$

where κ' is defined by the identity:

$$A\cos\left(\kappa' x\right):=\mathrm{Real}\ \left(e^{2i\kappa x}\cos\left(e^{i\kappa x}\right)\right) \tag{9.25}$$

where A is a function of κ and x in general.

From Eqs.(9.22) to (9.25) it is found that:

$$\frac{d^2f}{dx^2}+2\kappa^2 f=\cos\kappa' x \tag{9.26}$$

where:

$$f=\frac{\cos\kappa' x}{2\kappa^2-\kappa'^2} \tag{9.27}$$

Eq.(9.27) is, self-consistently, the solution of Eq.(9.26), Q.E.D.

Therefore Eq.(9.24) is a valid particular integral of Eq.(9.23) if κ' is defined by:

$$\kappa'=\frac{1}{x}\cos^{-1}\left(\frac{1}{A}\mathrm{Real}\ \left(e^{2i\kappa x}\cos\left(e^{i\kappa x}\right)\right)\right) \tag{9.28}$$

At resonance:

$$2\kappa^2=\kappa'^2 \tag{9.29}$$

and ϕ_p theoretically infinite. For a given A, resonance in ϕ occurs when x is defined by:

$$\begin{aligned} A\cos\left(\sqrt{2}\kappa x\right) = &\cos\left(2\kappa x\right)\cos\left(\cos\left(\kappa x\right)\right)\cosh\left(\sin\left(\kappa x\right)\right) \\ &+\sin\left(2\kappa x\right)\sin\left(\cos\left(\kappa x\right)\right)\sinh\left(\sin\left(\kappa x\right)\right) \end{aligned} \tag{9.30}$$

It is seen that A can be greater than unity because $\cosh y$ and $\sinh y$ can be greater than unity.

Secondly consider the radial component r in three dimensions of the spherical polar coordinate system. In this system [24]:

$$\left.\begin{aligned} \nabla^2\phi &= \frac{\partial^2\phi}{\partial r^2}+\frac{2}{r}\frac{\partial\phi}{\partial r}, \\ \boldsymbol{\omega}\cdot\boldsymbol{\nabla}\phi = \omega_r\frac{\partial\phi}{\partial r}, &\quad \left(\boldsymbol{\nabla}\cdot\boldsymbol{\omega}\right)\phi = \frac{\phi}{r^2}\frac{\partial}{\partial r}\left(r^2\omega_r\right) \end{aligned}\right\} \tag{9.31}$$

so Eq.(9.6) becomes:

$$\frac{\partial^2\phi}{\partial r^2}+\left(\frac{2}{r}+\omega_r\right)\frac{d\phi}{dr}+\frac{\phi}{r^2}\left(2r\omega_r+r^2\frac{d\omega_r}{dr}\right)=-\frac{\rho}{\epsilon_0} \tag{9.32}$$

Now choose a radial spin connection:

$$\omega_r=-\frac{1}{r} \tag{9.33}$$

to obtain:

$$\frac{\partial^2\phi}{dr^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}-\frac{1}{r^2}\phi=-\frac{\rho}{\epsilon_0} \tag{9.34}$$

This equation has the same mathematical structure as Eq.(9.13), and since ϕ is a function only of r can be written as an ordinary differential equation:

$$\frac{\partial^2\phi}{dr^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}-\frac{1}{r^2}\phi=-\frac{\rho}{\epsilon_0} \tag{9.35}$$

Assume now that the initial charge is oscillatory as follows:

$$\rho=\rho\left(0\right)\cos\left(\kappa_r r\right) \tag{9.36}$$

and use the change of variable:

$$\kappa_r r=e^{i\kappa_r R} \tag{9.37}$$

to obtain the undamped oscillator equation:

$$\frac{d^2\phi}{dR^2}+\kappa_r^2\phi=\frac{\rho\left(0\right)}{\epsilon_0}\text{Real}\,\left(e^{2i\kappa_r R}\cos\left(e^{i\kappa_r R}\right)\right) \tag{9.38}$$

where

$$R=\frac{1}{\kappa_r}\cos^{-1}\left(\kappa_r r\right) \tag{9.39}$$

Now define:

$$A\cos\left(\kappa' R\right):=\text{Real}\,\left(e^{2i\kappa_r R}\cos e^{i\kappa_r R}\right) \tag{9.40}$$

to obtain the particular integral:

$$\phi_\rho(R) = \frac{A_\rho}{\epsilon_0}\frac{\cos(\kappa' R)}{\kappa_r^2 - \kappa'^2} \tag{9.41}$$

Resonance occurs at:

$$\kappa_r = \kappa' \tag{9.42}$$

where:

$$\begin{aligned} A\cos(\kappa_r R) = &\cos(2\kappa_r R)\cos(\cos(\kappa_r R))\cosh(\sin(\kappa_r R)) \\ &+ \sin(2\kappa_r R)\sin(\cos(\kappa_r R))\sinh(\sin(\kappa_r R)) \end{aligned} \tag{9.43}$$

The particular integral of Eq.(9.38) may be obtained by first assuming that the solution has the form:

$$\phi = \frac{A_\rho(0)}{\epsilon_0}\text{Real}\left(e^{2i\kappa_r R}\cos\left(e^{i\kappa_r R}\right)\right) \tag{9.44}$$

where A is to be determined. Substituting Eq.(9.44) in Eq.(9.38) gives:

$$\begin{aligned} A &= \frac{\text{Real}\left(e^{2i\kappa_r R}\cos\left(e^{i\kappa_r R}\right)\right)}{\kappa^2\left(-3e^{2i\kappa_r R}\cos\left(e^{i\kappa_r R}\right) + 5e^{3i\kappa_r R}\sin\left(e^{i\kappa_r R}\right) + e^{4i\kappa_r R}\cos\left(e^{i\kappa_r R}\right)\right)} \\ &= \frac{r^2\cos(\kappa_r r)}{\kappa_r^4 r^4\cos(\kappa_r r) + 5\kappa_r^3 r^3\sin(\kappa_r r) - 3\kappa_r^2 r^2\cos(\kappa_r r)} \end{aligned} \tag{9.45}$$

Therefore the particular integral is:

$$\phi = \frac{\rho(0)}{\epsilon_0}\left(\frac{\kappa_r^2 r^4\cos^2(\kappa_r r)}{\kappa_r^4 r^4\cos(\kappa_r r) + 5\kappa_r^3 r^3\sin(\kappa_r r) - 3\kappa_r^2 r^2\cos(\kappa_r r)}\right) \tag{9.46}$$

which has the correct S.I. units of volts $= JC^{-1}$. Resonance occurs in the scalar potential in volts of Eq.(9.46) when:

$$\kappa_r^4 r^4\cos(\kappa_r r) + 5\kappa_r^3 r^3\sin(\kappa_r r) = 3\kappa_r^2 r^2\cos(\kappa_r r) \tag{9.47}$$

If

$$x := \kappa_r r \tag{9.48}$$

the structure of Eq.(9.47) is as follows:

$$\left.\begin{array}{c} \left(x^2 - 3\right)\cos x + 5x\sin x = 0, \\ \text{i.e.} \quad \tan x = \dfrac{3 - x^2}{5x} \end{array}\right\} \tag{9.49}$$

and in general will show peaks as a function of x. The analytical solution (9.46) also shows many sharp peaks (Section 9.3), all of which denote surges in voltage (scalar potential). These peaks in voltage can be used in the equivalent circuits of Eqs.(9.35) or (9.38) to produce new power.

$$\phi \longrightarrow \infty \quad \text{or maximized} \tag{9.50}$$

These equations are analyzed numerically in Section 9.3. To obtain this result it has been assumed that the initial driving charge density oscillates according

to a cosinal function on the right hand side of Eq.(9.36). A more complicated initial driving function may be used according to circuit design or similar. The important result is that resonance occurs in the voltage, and this surge in voltage is caused by the spin connection of space-time. The voltage obtained in this way may be used for new energy.

In the limit:

$$r \rightarrow \infty, \tag{9.51}$$

$$\kappa = \text{constant}, \tag{9.52}$$

Eq.(9.35) reduces to the Poisson equation:

$$\frac{d^2\phi}{dr^2} = -\frac{\rho}{\epsilon_0} \tag{9.53}$$

used in the standard model. Eq.(9.46) may be rewritten as:

$$\phi = \frac{\rho\left(0\right)}{\epsilon_0} \frac{\cos^2\left(\kappa r\right)}{\left(\kappa^2 \cos\left(\kappa r\right) + \frac{5\kappa}{r}\sin\left(\kappa r\right) - \frac{3}{r^2}\cos\left(\kappa r\right)\right)} \tag{9.54}$$

and in infinite r limit this equation becomes:

$$\phi \quad \overrightarrow{r \rightarrow \infty} \quad \frac{\rho\left(0\right)}{\epsilon_0}\left(\frac{\cos\left(\kappa r\right)}{\kappa^2}\right) \tag{9.55}$$

so that:

$$\frac{d^2\phi}{dr^2} = \frac{-\rho\left(0\right)}{\epsilon_0}\cos\left(\kappa r\right) = \frac{-\rho}{\epsilon_0} \tag{9.56}$$

Q.E.D. In this limit is known that the scalar potential is [21]:

$$\phi = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\left(\mathbf{r}'\right)}{\left|\mathbf{r}-\mathbf{r}'\right|} d^3r' \tag{9.57}$$

so:

$$\left(\frac{\cos\left(\kappa r\right)}{\kappa^2}\right) \quad \overrightarrow{r \rightarrow \infty} \quad \frac{1}{4\pi\rho\left(0\right)}\int \frac{\rho\left(\mathbf{r}'\right)}{\left|\mathbf{r}-\mathbf{r}'\right|} d^3r' \tag{9.58}$$

This is a mathematical check on the self-consistency of the analytical solution (9.46) of the resonance equation (9.35). Physically however the spin connection cannot vanish unless r becomes the radius of the universe. This is because the electromagnetic field is always spinning space-time in ECE theory. Similarly, the gravitational field is always curving space-time. MH theory (standard model) has no conception of the spin connection.

For multi electron systems and in three space dimensions, consider the equation:

$$\mathbf{E} = -\boldsymbol{\nabla}\phi = -\boldsymbol{\omega}\phi \tag{9.59}$$

The electric field from this equation is [21]:

$$\mathbf{E}\left(\mathbf{r}\right) = -\frac{1}{4\pi\epsilon_0}\boldsymbol{\nabla}\int \frac{\rho\left(\mathbf{r}'\right)}{\left|\mathbf{r}-\mathbf{r}'\right|} d^3r' \tag{9.60}$$

and the scalar potential is:

$$\phi = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\left(\mathbf{r}'\right)}{\left|\mathbf{r}-\mathbf{r}'\right|} d^3r' \tag{9.61}$$

Thus:

$$\int \boldsymbol{\nabla}\phi d^3r' = \int \phi\boldsymbol{\omega} d^3r' \tag{9.62}$$

where:

$$-\nabla\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) = \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} \tag{9.63}$$

so:

$$\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} = -\frac{\boldsymbol{\omega}}{|\mathbf{r}-\mathbf{r}'|} \tag{9.64}$$

and the three dimensional spin connection for an n electron system is:

$$\boldsymbol{\omega} = -\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2} \tag{9.65}$$

The resonance equation for n electrons is therefore:

$$\nabla^2\phi + \boldsymbol{\omega}\cdot\boldsymbol{\nabla}\phi + (\boldsymbol{\nabla}\cdot\boldsymbol{\omega})\,\phi = -\frac{\rho}{\epsilon_0} \tag{9.66}$$

where the spin connection is given by Eq.(9.65) and where the charge density is defined in terms of the three dimensional Dirac delta function [21] as follows:

$$\rho(\mathbf{r}) = \sum_{i=1}^{n} q_i\delta(\mathbf{r}-\mathbf{r}_i) \tag{9.67}$$

The electric field is [21]:

$$\begin{aligned}\mathbf{E}(\mathbf{r}) &= \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n} q_i\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} \\ &= \frac{1}{4\pi\epsilon_0}\int \rho(\mathbf{r}')\,\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}d^3r'\end{aligned} \tag{9.68}$$

If Δq is the charge in a small volume

$$d^3r = \Delta x\Delta y\Delta z \tag{9.69}$$

then:

$$\Delta q = \rho(\mathbf{r}')\,\Delta x\Delta y\Delta z \tag{9.70}$$

so the three dimensional resonance equation for n electrons is:

$$\nabla^2\phi + \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}\cdot\boldsymbol{\nabla}\phi + \left(\boldsymbol{\nabla}\cdot\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}\right)\phi = -\frac{1}{\epsilon_0}\left(\sum_{i=1}^{n} q_i\delta(\mathbf{r}-\mathbf{r}_i)\right) \tag{9.71}$$

where the spin connection is:

$$\omega_i = -\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2} \tag{9.72}$$

Therefore each electron and proton in an atom or molecule has its spin connection.

In order to clarify the meaning of these equations prior to coding in density function packages (Section 9.3), some detail is given as follows. To prove the vector equation [21]:

$$-\boldsymbol{\nabla}\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right)=\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} \tag{9.73}$$

write it as:

$$-\boldsymbol{\nabla}\left(|\mathbf{r}-\mathbf{a}|\right)^{-1}=\frac{\mathbf{r}-\mathbf{a}}{|\mathbf{r}-\mathbf{a}|^3} \tag{9.74}$$

where

$$|\mathbf{r}-\mathbf{a}|=\left((x-a_x)^2+(y-a_y)^2+(z-a_z)^2\right)^{1/2} \tag{9.75}$$

and

$$-\nabla\left(\frac{1}{|\mathbf{r}-\mathbf{a}|}\right)=-\frac{\partial}{\partial x}\left(\frac{1}{|\mathbf{r}-\mathbf{a}|}\right)\mathbf{i}-\frac{\partial}{\partial y}\left(\frac{1}{|\mathbf{r}-\mathbf{a}|}\right)\mathbf{j}-\frac{\partial}{\partial z}\left(\frac{1}{|\mathbf{r}-\mathbf{a}|}\right)\mathbf{k} \tag{9.76}$$

Consider terms such as:

$$\begin{aligned}&\frac{\partial}{\partial x}\left((x-a_x)^2+(y-a_y)^2+(z-a_z)^2\right)^{1/2}\\&=-\frac{(x-a_x)}{\left((x-a_x)^2+(y-a_y)^2+(z-a_z)^2\right)^{3/2}}\end{aligned} \tag{9.77}$$

to find Eqs.(9.73) and (9.74), Q.E.D. In the resonance equation (9.71) there occurs the term:

$$f(\mathbf{r})=\boldsymbol{\nabla}\cdot\frac{(\mathbf{r}-\mathbf{r}_i)}{|\mathbf{r}-\mathbf{r}_i|^2} \tag{9.78}$$

and this may be developed as follows. Write:

$$f(\mathbf{r})=\boldsymbol{\nabla}\cdot\frac{(\mathbf{r}-\mathbf{a})}{|\mathbf{r}-\mathbf{a}|^2} \tag{9.79}$$

where:

$$|\mathbf{r}-\mathbf{a}|^2=(x-a_x)^2+(y-a_y)^2+(z-a_z)^2 \tag{9.80}$$

and

$$\mathbf{r}-\mathbf{a}=(x-a_x)\,\mathbf{i}+(y-a_y)\,\mathbf{j}+(z-a_z)\,\mathbf{k} \tag{9.81}$$

so:

$$\begin{aligned}f(\mathbf{r})&=\frac{\partial}{\partial x}\frac{(\mathbf{x}-\mathbf{a}_x)}{|\mathbf{r}-\mathbf{a}|^2}+\frac{\partial}{\partial y}\frac{(\mathbf{y}-\mathbf{a}_y)}{|\mathbf{r}-\mathbf{a}|^2}+\frac{\partial}{\partial z}\frac{(\mathbf{z}-\mathbf{a}_z)}{|\mathbf{r}-\mathbf{a}|^2}\\&=\frac{\partial}{\partial x}\frac{(x-a_x)}{(x-a_x)^2+(y-a_y)^2+(z-a_z)^2}+\cdots\end{aligned} \tag{9.82}$$

Using the rules of differentiation:

$$\begin{aligned}f(\mathbf{r})&=\left((x-a_x)^2+(y-a_y)^2+(z-a_z)^2\right)^{-2}\\&\quad-\frac{2(x-a_x)^2}{\left((x-a_x)^2+(y-a_y)^2+(z-a_z)^2\right)^2}+\cdots\\&=\frac{1}{|\mathbf{r}-\mathbf{a}|^2}-\frac{2}{|\mathbf{r}-\mathbf{a}|^2}\\&=\frac{-1}{|\mathbf{r}-\mathbf{a}|^2}\end{aligned} \tag{9.83}$$

Therefore:

$$\boldsymbol{\nabla} \cdot \frac{(\mathbf{r}-\mathbf{r}_i)}{|\mathbf{r}-\mathbf{r}_i|^2} = -\frac{1}{|\mathbf{r}-\mathbf{r}_i|^2} \tag{9.84}$$

so the resonance equation is:

$$\nabla^2\phi + \frac{(\mathbf{r}-\mathbf{r}_i)}{|\mathbf{r}-\mathbf{r}_i|^2} \cdot \boldsymbol{\nabla}\phi - \frac{1}{|\mathbf{r}-\mathbf{r}_i|^2}\phi = -\frac{1}{\epsilon_0}\left(\sum_{i=1}^{n} q_i \delta(\mathbf{r}-\mathbf{r}_i)\right) \tag{9.85}$$

This can be written as:

$$|\mathbf{r}-\mathbf{r}_i|^2 \nabla^2\phi + (\mathbf{r}-\mathbf{r}_i)\cdot\boldsymbol{\nabla}\phi - \phi = \frac{-1}{\epsilon_0}|\mathbf{r}-\mathbf{r}_i|^2 \sum_{i=1}^{n} q_i \delta(\mathbf{r}-\mathbf{r}_i) \tag{9.86}$$

and incorporated in density functional code for the Coulomb potential. In Eq.(9.86) the Dirac delta function is defined as usual [21] by:

$$\delta(\mathbf{r}-\mathbf{r}_1) = \delta(x_1-X_1)\,\delta(y_1-Y_1)\,\delta(z_1-Z_1) \tag{9.87}$$

and the charge density is defined by:

$$\rho(\mathbf{r}) = \sum_{i=1}^{n} e_i \delta(\mathbf{r}-\mathbf{r}_i) \tag{9.88}$$

Using:

$$\mathbf{r}-\mathbf{r}_i = (x-x_i)\,\mathbf{i} + (y-y_i)\,\mathbf{j} + (z-z_i)\,\mathbf{k} \tag{9.89}$$

and:

$$|\mathbf{r}-\mathbf{r}_i|^2 = (x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2 \tag{9.90}$$

the spin connection in Eq.(9.72) an be developed as:

$$\boldsymbol{\omega}_i = -\frac{((x-x_i)\,\mathbf{i} + (y-y_i)\,\mathbf{j} + (z-z_i)\,\mathbf{k})}{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} \tag{9.91}$$

With these definitions the resonance equation in three dimensions and for n electrons and protons in an atom or molecule is therefore:

$$\nabla^2\phi + \frac{(\mathbf{r}-\mathbf{r}_i)}{|\mathbf{r}-\mathbf{r}_i|^2} \cdot \boldsymbol{\nabla}\phi - \frac{1}{|\mathbf{r}-\mathbf{r}_i|^2}\phi = -\frac{1}{\epsilon_0}\sum_{i=1}^{n} q_i \delta(\mathbf{r}-\mathbf{r}_i) \tag{9.92}$$

The potential in volts from this equation can be used to build up the Hartree term in density functional code. This term describes electron electron repulsion through the Coulomb interaction and is (Section 9.3):

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{e^2 n_s(\mathbf{r})}{|\mathbf{r}-\mathbf{r}'|} d^3r' \tag{9.93}$$

where $n_s(\mathbf{r})$ is a number density. Therefore the scalar potential in volts is:

$$\phi_H = \frac{e}{4\phi\epsilon_0}\int \frac{n_s(\mathbf{r})}{|r-r'|} d^2r' \tag{9.94}$$

When there are peaks in ϕ_H, the effect is to greatly amplify the number density and to greatly increase the electron-electron repulsion, resulting in ionization of the atom or molecule into free electrons which can be used for new energy.

The most direct method for acquiring new energy from the spin connection is to construct the circuits equivalent to Eqs.(9.35) or (9.38). The type of circuit needed for Eq.(9.38), whose analytical solution is Eq.(9.46), is that of the undamped oscillator. The simplest undamped oscillator equation [24] is:

$$m\ddot{x} + kx = F \tag{9.95}$$

where m is mass, x is displacement, k is Hooke's constant and F is driving force. Its equivalent circuit equation is [24]:

$$L\ddot{q} + \frac{q}{C} = E \tag{9.96}$$

where q is charge, L is inductance, C is capacitance and E is electromotive force. Eq (9.96) describes an electromotive force in series with a capacitor and induction coil. A material may be incorporated inside the induction coil. Therefore the circuit equivalent to the undamped oscillator (9.38) is the same design, but the electromotive force is synthesized to be the same as the right hand side of Eq.(9.38), i.e. made up of circular and hyperbolic functions. The exact solution (9.46) shows many sharp peaks of voltage (Section 9.3), so the equivalent circuit also shows many sharp peaks of voltage for a small initial electromotive force. These peaks of voltage can be used for new power and fed to the grid from a power plant.

9.3 Numerical results and discussion

In the following we present numerical results for the resonance equations and the application of the resonant Coulomb law to the Hydrogen atom. Then we discuss the general applicability to solid state physics and electrical engineering.

9.3.1 Graphs of resonance effects

As described in Section 9.2, the solution of the ECE Coulomb law (see Eq.(9.12) for Cartesian or Eq.(9.34) for spherical coordinates) shows up resonances. This can be seen in general from the particular integral (9.46). This particular solution is plotted in Fig. 9.1 for four κ values. The higher κ, the more resonances are seen in a certain range of the radial coordinate r. The resonances appear in form of poles of the particular integral, i.e. where the denominator approaches to zero. In general these are the roots of the denominator equation (9.49). These are computed numerically and listed in Table 9.1

Since the variable x in Eq.(9.49) is the product κr, the coordinate r corresponds directly to x in case $\kappa = 1$. This can be seen from Fig. 9.1: For the second κ value the zeros come to lie at the x values of Table 9.1.

The most interesting curve is the resonance behaviour of true solutions of the resonant Coulomb law (9.34). The equation has been transformed by the Euler method to the form of Eq.(9.38) without approximations. This form is easier to solve numerically and it can directly be seen that it is an equation of

n	zero(rad)	zero(degrees)
1	0.660310	37.832983
2	2.807011	160.829901
3	5.502118	315.248163
4	8.409624	481.835945
5	11.416981	654.144835
6	14.474251	829.313494
7	17.558761	1006.042917

Table 9.1: First seven zero values of Eq.(9.49)

a forced oscillation. The "driving term" is the right-hand side of Eq.(9.38). For obtaining the resonance curve the value of κ_r at the left hand side has been kept fixed and the κ value of the driving force has been varied as suggested by Eq.(9.40). The results are shown in Fig. 9.2. The maximum amplitude difference after 15 oscillations has been plotted against the wave number κ of the driving force. In case of the pure cosine term the resonance occurs for $\kappa = \kappa_r$ as expected (we used $\kappa_r = 1$). In principle this is an undamped resonance with infinite resonance amplitude, but since we have restricted the calculation to 15 wavelengths the curve remains bounded. The same holds for the exact driving force. There are two sharp resonances at $\kappa = 0.5$ and $\kappa = 0.25$ and a smaller third at $\kappa = 0.166$. The curve scales with κ_r which was set to unity here. This resonance behaviour is a consequence of the spin connection which is only present in ECE theory, not in standard Maxwell-Heaviside theory.

The resonant increase of the solution Φ of Eq.(9.38) can be studied from Fig. 9.3 where the curve $\Phi(R)$ is presented for four different κ values. The first two κ's were chosen to be the resonance values of Fig. 9.2. Both lead to increasing amplitudes with R. The other two κ values are off resonance, the maximum amplitude does not change with R. The driving force (right-hand side of Eq.(9.38)) is presented in Fig. 9.4. There are two maxima and one minimum per period. Decreasing κ only means a broadening of the form. This driving force is required for experimental construction of equivalent circuits of spin connection resonance, therefore we have calculated the Fourier spectrum (Fig. 9.5). As already seen in Fig. 9.4, changing the κ value does not change the wave form, i.e. the Fourier spectrum remains unaltered for abscissa values of $\kappa = 2\pi/\lambda$, only the right end varies with λ. The three significant peaks of Fig. 9.2 are directly represented in the spectrum. In particular one can see that the first and second resonance peak differ in phase (sign of Fourier coefficient). The phase difference between both amplitudes is 180 degrees as can directly be observed from Fig. 9.3.

So far we have inspected the numerical solution $\Phi(R)$ in the transformed coordinate R as given by Eq.(9.39). The question is how the original function $\Phi(r)$ behaves. We have to perform the back-transformation $R \rightarrow r$ which is given by

$$r = \frac{1}{\kappa_r} \cos(\kappa_r R) \tag{9.97}$$

Since the value range of the cosine function is restricted to $[-1, 1]$ we have to provide a rule how to obtain r values for $\kappa_r r$ greater than unity. The proposed solution is to shift the values according to the number of full periods $0 \leq \kappa_r R <$

π contained in the argument. Additionally the result is transformed in a way to give a continuous function as is depicted in Fig. 9.6. The back-transformed potential $\Phi(r)$ can be seen in Fig. 9.7 (it corresponds to the left-most part of Fig. 9.3). $\Phi(r)$ has to be interpolated if one wants to have equidistant abscissa values as for example was required for the calculation of the H spectrum. Obviously the cosine function leads to a deformation of Φ where $r(R)$ approaches the horizontal asymptote.

9.3.2 Results for the Hydrogen atom

The energy eigenvalues and radial wavefunctions of atomic Hydrogen have been calculated numerically in presence of a small oscillatory charge density serving as "driving force". The ECE Coulomb potential of the driving force has been added to the proton core potential of the H atom. This is the same proceeding as had already been applied earlier [11]. In that case an approximate method was used to calculate the ECE Coulomb potential with a constant radius parameter. Here we use the numerical solution of Eq.(9.38) as discussed in the preceding section. The value of κ_r was set to 0.5 to obtain a r range in the backtransformation which was broad enough compared to the atomic radius. We used a maximum value of $r = 3.5$. As can be seen from Fig. 9.3, the stationary oscillatory behaviour develops after a certain initial R range which is dominated by initial value effects [24]. To avoid these initial value dependences we have taken the solution $\Phi(R)$ for $R \geq 26.5$ and shifted the range to the coordinate origin of r. For this operation one has to observe that the boundaries of the transformation range shown in Fig. 9.6 are not exceeded in order not to obtain unphysical jumps in Φ. In Fig. 9.8 the total atomic potential is shown for three κ values, two in resonance and one in off-resonance. Because we have chosen $\kappa_r = 0.5$ the resonances come to lie at $\kappa = 0.175$ and $\kappa = 0.225$. In the first resonance the potential is shifted from smaller to greater radii; it takes strongly repulsive values in the second resonance. The third κ value is in off-resonance. The potential leads to the resonance graph presented in Fig. 9.9. There is an oscillating shift of atomic energies with a remarkable lifting of all values at the main resonance peak of $\kappa = 0.225$. This is a more complicated resonance structure compared with our earlier calculation [11] where the analytical form of the particular integral of the resonance equation for a fixed radius was used.

The radial dependence of some orbitals is depicted in Figs. 9.10–9.12. The orbitals are calculated for κ values already discussed with Fig. 9.8. The potential shift in the first case (A) leads to a shift of the $1s$ and $2p$ orbitals to the core because mainly the decrease in potential has an effect for these orbitals. In case of the repulsive resonance (B) all orbitals are significantly pushed outward. In particular the $1s$ orbital is strongly delocalized and is in a transition to an unbound state. This supports the former assumptions on the ionization mechanism. Nevertheless the particular angular momentum character of the orbitals (sign and number of zeros) remains valid.

In Fig. 9.13 some control parameters are shown as was also done in [11]. The resonance potential takes its extrema near to the resonant κ values. The potential integrates out to zero for large r which indicates that charge neutrality is conserved and the calculations are made on a reasonable basis. The overall results are similar to those obtained in [11].

9.3.3 Application to solid state physics and engineering

The resonance effect in Hydrogen was investigated because it serves as a model system for the huge class of atomic, molecular and solid state physics. Resonances in solids are particularly important because the electronic structure of solids is the basis of modern electronic chip design which can be extended to resonant devices on a microstructure scale. The most used method for computation of electronic properties of solids is Density Functional Theory. Therefore we give a short introduction into the theory as contributed by Lothar Fritsche [27]. The method, which in practical applications presupposes the validity of the Born-Oppenheimer approximation (fixed atomic nuclei), is as follows:

1. The interacting N-electron system (ground-state wavefunction $\widehat{\Psi}_0(\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N)$) is reduced to a non-interacting N-particle system (wavefunction $\Phi_0(\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N)$ = Slater determinant)
by adiabatically switching off the electron-electron interaction while keeping the oneparticle density $\rho(\boldsymbol{r})$ fixed. The latter means

$$\begin{aligned}\rho(\boldsymbol{r}) =& N \sum_{\sigma} \int \mid \widehat{\Psi}_0((\boldsymbol{r}, \sigma), \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N \mid^2 d^4x_2 \cdots d^4x_N = \\ & N \sum_{\sigma} \int \mid \phi_0((\boldsymbol{r}, \sigma), \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N \mid^2 d^4x_2 \cdots d^4x_N\end{aligned} \tag{9.98}$$

Note that the coordinate $\boldsymbol{x}$ stands collectively for $(\boldsymbol{r}; \sigma)$ where $\sigma = \pm 1$ denotes the spincoordinate.

If the nuclear Coulomb potential resulting from all atoms of the system is denoted by $V_{ext}(\boldsymbol{r})$ the original Hamiltonian has the form (a.u.: energy: Hartree, length: a_B)

$$\widehat{H}_{interact} = \sum_{k=1}^{N} \left[-\frac{1}{2}\nabla_k^2 + V_{ext}(\boldsymbol{r}_k) \right] + \frac{1}{2} \sum_{k=1, l \neq k}^{N} \frac{1}{\mid \boldsymbol{r}_k - \boldsymbol{r}_l \mid} \tag{9.99}$$

In the intermediate state when the electron-electron interaction is not yet completely switched off, the Hamiltonian attains the form

$$\sum_{k=1}^{N} \left[-\frac{1}{2}\nabla_k^2 + V_{ext}(\boldsymbol{r}_k) + \widehat{V}_{ext}(\lambda, \boldsymbol{r}_k) \right] + \frac{\lambda}{2} \sum_{k, l \neq k} \frac{1}{\mid \boldsymbol{r}_k - \boldsymbol{r}_l \mid} \tag{9.100}$$

where λ controls the strength of the electron-electron interaction, i. e. $0 \leq \lambda \leq 1$. Furthermore, $\widehat{V}_{ext}(\lambda, \boldsymbol{r})$ denotes the additional potential that has to be switched on at coupling strength λ to ensure the conservation of the original interacting density $\rho(\boldsymbol{r})$. If this potential exists, it is unique, as has been shown by Hohenberg and Kohn (1964). However, the actual existence can only be shown for densities that are defined on a lattice of points in real-space [28]. The Hamiltonian of the non-interacting substitute system derives from Eq.(9.100) for $\lambda = 0$:

$$\widehat{H}_{substitute} = \sum_{k=1}^{N} \left[-\frac{1}{2}\nabla_k^2 + V_{ext}(\boldsymbol{r}_k) + \widehat{V}_{ext}(\boldsymbol{r}_k) \right] \tag{9.101}$$

where $\widehat{V}_{ext}(\boldsymbol{r}) \equiv \widehat{V}_{ext}(\lambda = 0, \boldsymbol{r})$.

2. The Schrödinger equation associated with the substitute Hamiltonian (9.101) can be decomposed into N one-particle equations

$$\left[-\frac{1}{2}\nabla^2 + V_{ext}(\boldsymbol{r}) + \widehat{V}_{ext}(\boldsymbol{r})\right]\psi_i(\boldsymbol{r}) = \epsilon_i\psi_i(\boldsymbol{r}) \tag{9.102}$$

which represent the so-called "Kohn-Sham equations". It can be shown that the non-interacting wavefunction $\Phi_0(x_1, x_2, \cdots, x_N)$ which can be cast as a Slater determinant has to be formed from the N energetically lowest lying solutions $\psi_i(\boldsymbol{r})$. Hence one has from Eq.(9.98)

$$\rho(\boldsymbol{r}) = \sum_{i=1}^{N} \mid \psi_i(\boldsymbol{r}) \mid^2 \tag{9.103}$$

3. By using the Hellmann-Feynman theorem it is easy to show that the total energy E_0 of the actual **interacting** system can be cast as

$$E_0 = \underbrace{\langle\phi_0 \mid \sum_{k=1}^{N}\left[-\frac{1}{2}\nabla_k^2\right] \mid \phi_0\rangle}_{\equiv T_0} + \int \rho(\boldsymbol{r})V_{ext}(\boldsymbol{r})d^3r + \langle\overline{V_{e-e}}\rangle \tag{9.104}$$

where

$$T_0 = \sum_{i=1}^{N}\int \psi_i^*(\boldsymbol{r})\left[-\frac{1}{2}\nabla^2\right]\psi_i(\boldsymbol{r})d^3r\ ,\ \rho(\boldsymbol{r}) = \sum_{i=1}^{N} \mid \psi_i(\boldsymbol{r}) \mid^2 \tag{9.105}$$

and

$$\langle\overline{V_{e-e}}\rangle = \iint \frac{\overline{\rho_2(\boldsymbol{r}', \boldsymbol{r})}}{\mid \boldsymbol{r}' - \boldsymbol{r} \mid} d^3r'd^3r \tag{9.106}$$

with $\overline{\rho}_2$ denoting the electronic pair density averaged over the coupling strength:

$$\overline{\rho_2(\boldsymbol{r}', \boldsymbol{r})} = \int_0^1 \rho_2(\lambda, \boldsymbol{r}', \boldsymbol{r})d\lambda \tag{9.107}$$

where

$$\begin{aligned}&\rho_2(\lambda, \boldsymbol{r}', \boldsymbol{r}) = \\ &N(N-1)\sum_{\sigma', \sigma}\int \mid \widehat{\Psi}_0(\lambda, \boldsymbol{x}', \boldsymbol{x}, \boldsymbol{x}_3, \cdots, \boldsymbol{x}_N) \mid^2 d^4x_3 \cdots d^4x_N\end{aligned} \tag{9.108}$$

and $\widehat{\Psi}_0(\lambda, \boldsymbol{x}', \boldsymbol{x}, \boldsymbol{x}^3, \cdots, \boldsymbol{x}^N)$ solves the Schrödinger equation associated with the Hamiltonian (9.99) and ground-state energy E_0.

4. Eqs.(9.102) and (9.104) define the framework of density functional theory. At this stage it is still equivalent to a rigorous (non-relativistic) N-electron theory since no approximations have been made so far. But in practice it is useless in this form, because the density conserving extra potential $\widehat{V}_{ext}(\boldsymbol{r})$ and $\overline{\rho}_2$ are unknown. However, exploiting universal properties of

the wavefunction $\widehat{\Psi}_0(\boldsymbol{x}, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N)$, in particular its antisymmetry giving rise to "Fermi-holes" in the pair-density $\rho_2(\boldsymbol{r}', \boldsymbol{r})$, one arrives at a surprisingly simple approximation to $\langle\overline{V_{e-e}}\rangle$:

$$\langle\overline{V_{e-e}}\rangle = \frac{1}{2}\iint \frac{\rho(\boldsymbol{r}')\rho(\boldsymbol{r})}{\mid \boldsymbol{r}' - \boldsymbol{r} \mid} d^3r' d^3r - \int \frac{3}{4}\left(\frac{3}{\pi}\right)^{1/3} [\rho(\boldsymbol{r})]^{4/3}\, d^3r \quad (9.109)$$

For details see [29]. More advanced expressions account for contributions of the gradient of $\rho(\boldsymbol{r})$ ("Generalized Gradient Approximation", "GGA") see [30].

If one inserts the expression (9.109) in Eq.(9.104) and uses Eq.(9.103), E_0 becomes a functional of the orbital set $\{\psi_i(\boldsymbol{r})\}$. Requiring E_0 to be stationary against variations of these orbitals one arrives at

$$\begin{aligned} \delta E_0 = \sum_{i=1}^{N_\sigma} \int \delta\psi_i^*(\boldsymbol{r}) \Big[-\frac{1}{2}\nabla^2 + \\ V_{ext}(\boldsymbol{r}) + V_H(\boldsymbol{r}) + V_{XC}(\boldsymbol{r}, \sigma) \Big] \psi_i(\boldsymbol{r}) d^3r + \text{c.c.} = 0 \end{aligned} \quad (9.110)$$

Here $V_H(\boldsymbol{r})$ and $V_{xc}(\boldsymbol{r})$ denote, respectively, the so-called Hartree- and exchange-correlation potential which derive from the respective two contributions on the right-hand side of Eq.(9.109). They have the form

$$\begin{aligned} V_H(\boldsymbol{r}) &= \int \frac{\rho(\boldsymbol{r}')}{\mid \boldsymbol{r}' - \boldsymbol{r} \mid} d^3r' \\ &\text{and} \\ V_{XC}(\boldsymbol{r}) &= -\frac{3}{\pi}^{1/3} [\rho(\boldsymbol{r})]^{1/3} \end{aligned} \quad (9.111)$$

Eq.(9.110) is obviously fulfilled if

$$\widehat{V}_{ext}(\boldsymbol{r}) = V_H(\boldsymbol{r}) + V_{XC}(\boldsymbol{r}) \quad (9.112)$$

The DFT-scheme is now fully defined and easily accessible to applications. In case of ECE resonances the potentials $V_H(\boldsymbol{r})$ and $V_{XC}(\boldsymbol{r})$ have to incorporate the spin connection effects via the modified charge density $\rho(\boldsymbol{r})$. This in turn is determined from Eqs.(9.103) and (9.102) where the spin connection terms of Eq.(9.92) have to be respected.

It should be observed, however, that the rigorous version requires the pair density $\rho_2(\lambda, \boldsymbol{r}', \boldsymbol{r})$, averaged over the coupling strength to be known. This, in turn, implies the availability of $\widehat{\Psi}_0(\lambda, \boldsymbol{x}', \boldsymbol{x}, \boldsymbol{x}_3, \cdots, \boldsymbol{x}_N)$ for all values of λ, which means one has to solve the N-electron Schrödinger equation associated with the Hamiltonian (9.100). Hence, if the DFT-scheme is defined by the property to solve the N-electron problem without determining the N-electron wavefunction, it constitutes always an approximate theory.

Finally we give some hints how the resonant Coulomb law can be used in electrical engineering. At the end of Section 9.2 it was already stated that the circuit equivalent of Eq.(9.35) or (9.38) is an undamped electrical oscillator. This is a connection in series of an induction coil and a capacitor (see Fig. 9.14).

The differential equation for the oscillating charge q in this circuit is (according to Eq.(9.96)

$$\ddot{q} + \omega_0^2 q = U_{emf} \tag{9.113}$$

with

$$\omega_0 = 2\pi\nu_0 = \frac{1}{\sqrt{LC}} \tag{9.114}$$

being the resonance frequency (ν_0) and resonance angular frequency (ω_0). U_{emf} is the driving voltage representing the electromotive force. According to conventional Maxwell Heaviside theory, resonance occurs if U_{emf} is a harmonic A.C. voltage of the form

$$U_{emf} = U_0 \cos(\omega_0 t) \tag{9.115}$$

with an amplitude U_0. In order to activate a spin connection resonance we have to apply the driving force given at the right hand side of Eq.(9.38) which has been graphed in Fig. 9.4. In the equivalent circuit the length coordinate has to be replaced by time and the spatial resonance frequency κ_r (wave number) by the time frequency ω_0. In the same way the varying wave number of the driving force κ is to be changed to a frequency ω. From Fig. 9.5 we know that there are only three significant frequency contributions in the Fourier spectrum of the driving term. We can compose it by adding three single voltages for a given frequency ω:

$$U_{emf} = U_0(0.997\cos(2\omega t) - 0.503\cos(4\omega t) + 0.040\cos(6\omega t)) \tag{9.116}$$

This Fourier synthesis is graphed in Fig. 9.17. There is no visible difference to the exact form using all Fourier coefficients. Spin connection resonance should occur when ω is chosen in the ratio to ω_0 w.r.t. the resonance frequencies found from Fig. 9.2:

$$\begin{aligned} \omega_1 &= \frac{1}{2}\omega_0 \\ \omega_2 &= \frac{1}{4}\omega_0 \\ \omega_3 &= 0.166\omega_0 \end{aligned} \tag{9.117}$$

In particular the first and strongest resonance is obtained if

$$U_{emf}^{(1)} = U_0\,(0.997\cos(\omega_0 t) - 0.503\cos(2\omega_0 t) + 0.040\cos(3\omega_0 t)) \tag{9.118}$$

The voltage enhanced by resonance should occur then at the components of the equivalent circuit and can be tapped at the positions denoted by U_{res1} and U_{res2} in Fig. 9.14. The question remains how the mechanism for spin connection resonance in the circuit works. Some of the circuit material has to be brought to Coulomb resonance so that the material is ionized and electrons are emitted making up the additional current and voltage. How this is achieved has to be found experimentally. The material of the capacitor plates (or foils) could be this as well as the dielectric in-between. Alternatively a suitable material may be incorporated in the induction coil. Then the free electrons would appear in this material and could be extracted by a small voltage (see Fig. 9.15). In addition a feedback loop could be established by connecting the ends of the coil core with U_{emf} (dashed lines in Fig. 9.15). Since the voltage at the coil has a phase shift compared to the exciter emf, some phase shifting elements have to be

added (denoted symbolically by P in Fig. 9.15). After having been started by a small initial voltage, such a machine would run by its own. From Eq.(9.118) it is seen that U_{emf} consists of harmonics of the conventional resonance frequency ω_0. This means that the driving force is "compatible" with the circuit and could be enhanced by resonance without impairing its function.

As a last example we present an advanced circuit design suggested by Douglas Mann [31], see Fig. 9.16. The capacitor has been replaced by a secondary coil made by a bifilar wire. This special construction suppresses its own magnetic field nearly completely. Interaction with the inner induction coil is by electric fields of the wires, building a capacity. Thus both elements of the equivalent circuit are present and the resonant medium consists of the wires exclusively. The emf can be applied magnetically by a further induction coil with few windings which surrounds the two other coils. It is expected that devices similar like this will be able to deliver energy from spacetime in the near future.

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Appendix 1: Analytical solutions of the undamped oscillator

Consider the basic undamped oscillator equation (see text):

$$\frac{d\phi^2}{dR^2} + \kappa^2\phi = \frac{\rho\,(0)}{\epsilon_0} f\,(\kappa R) \tag{A-1}$$

where:

$$f\,(\kappa R) = e^{2i\kappa R}\cos\left(e^{i\kappa R}\right) \tag{A-2}$$

If $f\,(\kappa R)$ satisfies the Dirichlet condition, i.e. is single valued and continuous in an interval such as $\pi < f\,(\kappa R) \leq \pi$ it can be expanded in a Fourier series:

$$f\,(\kappa R) = \frac{a_0}{2} + \sum_{d=1}^{\infty}\left(a_\alpha \cos\left(\alpha\kappa R\right) + b_\alpha \sin\left(\alpha\kappa R\right)\right) \tag{A-3}$$

where:

$$\left.\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f\,(\kappa R)\,d\,(\kappa R) \\ a_\alpha &= \frac{1}{\pi}\int_{-\pi}^{\pi} f\,(\kappa R)\cos\left(\alpha\kappa R\right)d\,(\kappa R) \\ b_\alpha &= \frac{1}{\pi}\int_{-\pi}^{\pi} f\,(\kappa R)\sin\left(\alpha\kappa R\right)d\,(\kappa R)\,. \end{aligned}\right\} \tag{A-4}$$

These integrals can be computed straightforwardly to any required precision in any interval, the latter is not necessarily constrained to $\pi < f\,(\kappa R) \leq \pi$, the latter is used for illustration. Therefore Eq.(A-1) becomes:

$$\begin{aligned} \frac{d^2\phi}{dR^2} + \kappa^2\phi = \frac{\rho\,(0)}{\epsilon_0}\Big(&\frac{a_0}{2} + a_1\cos\left(\kappa R\right) + a_2\cos\left(2\kappa R\right) \\ &+ \cdots + b_1\sin\left(\kappa R\right) + b_2\sin\left(2\kappa R\right) \\ &+ \cdots\Big) \end{aligned} \tag{A-5}$$

Assume a solution of the type:

$$\begin{aligned} \phi = \frac{\rho\,(0)}{\epsilon_0}\Big(&A_0\frac{a_0}{2} + A_1a_1\cos\left(\kappa R\right) + A_2a_2\cos\left(2\kappa R\right) + \\ &\cdots + B_1b_1\sin\left(\kappa R\right) + B_2b_2\sin\left(2\kappa R\right) + \cdots\Big) \end{aligned} \tag{A-6}$$

Substituting Eq.(A-6) in Eq.(A-5) and comparing terms by term:

$$\left.\begin{aligned} \kappa^2 A_0\frac{a_0}{2} &= \frac{a_0}{2} \\ A_1\kappa^2\left(1 - a_1\right)\cos\left(\kappa R\right) &= \cos\left(\kappa R\right) \\ A_1 2\kappa^2\left(1 - 4a_1\right)\cos\left(\kappa R\right) &= \cos\left(\kappa R\right) \\ &\vdots \\ B_n\kappa^2\left(1 - n^2b_n\right)\sin\left(n\kappa R\right) &= \sin\left(n\kappa R\right)\,. \end{aligned}\right\} \tag{A-7}$$

Thus:

$$\phi = \frac{\rho(0)}{\epsilon_0 \kappa^2}\left(\frac{a_0}{2} + \frac{\cos(\kappa R)}{(1-a_1)} + \frac{\cos(2\kappa R)}{(1-4a_2)} + \cdots + \frac{\sin(\kappa R)}{(1-b_1)} + \frac{\sin(2\kappa R)}{(1-4b_2)} + \cdots\right) \tag{A-8}$$

Infinite resonances occur at:

$$\left.\begin{aligned} a_n &= 1/n^2, \quad n = 1, \cdots, m, \\ b_n &= 1/n^2, \quad n = 1, \cdots, m \end{aligned}\right\} \tag{A-9}$$

In general these resonances occur at:

$$\text{Real}\left(\frac{1}{\pi}\int_{-\pi}^{\pi} e^{2i\kappa R}\cos\left(e^{i\kappa R}\right)\cos(n\kappa R)\,d(\kappa R)\right) = \frac{1}{n^2} \tag{A-10}$$

and:

$$\text{Real}\left(\frac{1}{\pi}\int_{-\pi}^{\pi} e^{2i\kappa R}\cos\left(e^{i\kappa R}\right)\sin(n\kappa R)\,d(\kappa R)\right) = \frac{1}{n^2} \tag{A-11}$$

This analysis can be repeated straightforwardly for any driving term:

$$f(\kappa R) = e^{2i\kappa R} f_1\left(e^{i\kappa R}\right) \tag{A-12}$$

A constrained particular integral of Eq.(A-1) can be obtained for any driving function $f(\kappa R)$. In this case the undamped oscillator is:

$$\frac{d^2\phi}{dR^2} + \kappa^2\phi = \frac{\rho(0)}{\epsilon_0} e^{2i\kappa R} f\left(e^{i\kappa R}\right) \tag{A-13}$$

Assume a solution of the type:

$$\phi = \frac{A\rho(0)}{\epsilon_0} e^{2i\kappa R} f\left(e^{i\kappa R}\right) \tag{A-14}$$

subject to the constraint:

$$\frac{dA}{dR} = 0 \tag{A-15}$$

Then:

$$\frac{d\phi}{dR} = i\kappa\frac{A\rho(0)}{\epsilon_0}\left(2e^{2i\kappa R} f + e^{3i\kappa R} f'\right) \tag{A-16}$$

and:

$$\frac{d^2\phi}{dR^2} = -\kappa^2\frac{A\rho(0)}{\epsilon_0}\left(4e^{2i\kappa R} f + 5e^{3i\kappa R} f' e^{4i\kappa R} f''\right) \tag{A-17}$$

So the particular integral is:

$$\phi = -\frac{\phi(0)}{\epsilon_0}\left(\frac{r^2 f^2}{3f + 5\kappa r f' + \kappa^2 r^2 f''}\right) \tag{A-18}$$

subject to the constraint:

$$\frac{d\phi}{dr} = 0 \tag{A-19}$$

A solution of Eq.(A-19) is the general resonance condition:

$$3f + 5\kappa r f' + \kappa^2 r^2 f'' = 0 \tag{A-20}$$

To explain the notation in Eq.(A-20) consider for example a cosine driving term:

$$f = \cos x, \quad x = \kappa r \tag{A-21}$$

Then the notation means:

$$f' = -\sin x, \quad f'' = -\cos x \tag{A-22}$$

The resonance condition (A-20) then becomes:

$$\tan x = \frac{3 - x^2}{5x} \tag{A-23}$$

to which there is an infinite number of solutions. For a driving term:

$$f = e^{-x}, \quad f' = -e^{-x}, \quad f'' = e^{-x} \tag{A-24}$$

the resonance condition is:

$$x^2 - 5x + 3 = 0 \tag{A-25}$$

and there are two solutions at

$$x = 4.3028, \quad 0.6972 \tag{A-26}$$

For a driving term:

$$\left.\begin{aligned} f &= e^{-x}\cos x \\ f' &= e^{-x}(\cos x - \sin x) \\ f'' &= -2e^{-x}\sin x \end{aligned}\right\} \tag{A-27}$$

the resonance equation is:

$$\tan x = \frac{3 + 5x}{x(5 - 6x)} \tag{A-28}$$

and there are again an infinite number of solutions.

Appendix 2 : Simultaneous equations for ϕ and $\boldsymbol{\omega}$

The general resonance equation for the Coulomb Law is:

$$\nabla^2\phi + \boldsymbol{\nabla}\phi \cdot \boldsymbol{\omega} + (\boldsymbol{\nabla} \cdot \boldsymbol{\omega})\,\phi = -\frac{\rho}{\epsilon_0} \tag{B-1}$$

The limit of the standard model is reached when:

$$\boldsymbol{\nabla}\phi = \boldsymbol{\omega}\phi \tag{B-2}$$

i.e.

$$\nabla^2\phi = \boldsymbol{\nabla}\phi \cdot \boldsymbol{\omega} + (\boldsymbol{\nabla} \cdot \boldsymbol{\omega})\,\phi \tag{B-3}$$

and

$$\phi = \frac{-e}{4\pi\epsilon_0 r} \tag{B-4}$$

Therefore:

$$\boldsymbol{\omega} = -\frac{1}{r}\mathbf{e}_r \tag{B-5}$$

where $\mathbf{e}_r$ is the radial unit vector of the spherical polar coordinate system. So:

$$\omega_r = -\frac{1}{r} \tag{B-6}$$

Eq.(B-3) is a limiting case or boundary value of the general resonance equation (B-1). There is not sufficient information in Eq.(B-1) alone to completely determine ϕ and $\boldsymbol{\omega}$ under all conditions, because there are two variables and only one equation. In the text of the paper it has been assumed in order to proceed that Eq.(B-6) holds under all conditions, so Eq.(B-1) becomes (in spherical polar coordinates):

$$\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{1}{r^2}\phi = -\frac{\rho\,(0)}{\epsilon_0}\cos(\kappa r) \tag{B-7}$$

Eq.(B-7) has been developed in the text into the undamped oscillator:

$$\frac{d^2\phi}{dR^2} + \kappa^2 R = \frac{\rho\,(0)}{\epsilon_0} e^{2i\kappa R}\cos\left(e^{i\kappa R}\right) \tag{B-8}$$

and Eq.(B-8) has been solved numerically and analytically to show the presence in general of an infinite number of resonant voltage peaks of theoretically infinite amplitude at which the voltage becomes infinite.

More information can be obtained about ϕ and $\boldsymbol{\omega}$ by using the ECE Faraday Law of induction:

$$\boldsymbol{\nabla} \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mu_0 \mathbf{j}^a \tag{B-9}$$

where $\mathbf{j}^a$ is the homogeneous current density of ECE theory. When there is no magnetic field present (as in the Coulomb Law) Eq.(B-9) becomes the electrostatic law:

$$\boldsymbol{\nabla} \times \mathbf{E}^a = \mu_0 \mathbf{j}^a \tag{B-10}$$

Since Eq.(B-10) holds for all a it can be written simply as:

$$\boldsymbol{\nabla} \times \mathbf{E} = \mu_0 \mathbf{j} \tag{B-11}$$

where:

$$\mathbf{E} = -\left(\boldsymbol{\nabla} + \boldsymbol{\omega}\right)\phi \tag{B-12}$$

From Eqs.(B-11) and (B-12):

$$\boldsymbol{\nabla} \times \left(\boldsymbol{\nabla}\phi + \boldsymbol{\omega}\phi\right) = -\mu_0 \mathbf{j} \tag{B-13}$$

Using the vector relations:

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla}\phi = \mathbf{0} \tag{B-14}$$

$$\boldsymbol{\nabla} \times \left(\phi\boldsymbol{\omega}\right) = \phi\boldsymbol{\nabla} \times \boldsymbol{\omega} + \boldsymbol{\nabla}\phi \times \boldsymbol{\omega} \tag{B-15}$$

Eq.(B-13) becomes:

$$\phi\boldsymbol{\nabla} \times \boldsymbol{\omega} + \boldsymbol{\nabla}\phi \times \boldsymbol{\omega} = -\mu_0 \mathbf{j} \tag{B-16}$$

Thus Eqs.(B-1 and B-16) are the two simultaneous equations needed to solve for ϕ and $\boldsymbol{\omega}$ under all conditions in general.

If it is assumed that gravitation has no effect on electromagnetism the homogeneous current disappears:

$$\mathbf{j} = \mathbf{0} \tag{B-17}$$

so that Eq.(B-16) simplifies to:

$$\phi\boldsymbol{\nabla} \times \boldsymbol{\omega} + \boldsymbol{\nabla}\phi \times \boldsymbol{\omega} = \mathbf{0} \tag{B-18}$$

Eq.(B-1) and (B-18) must be solved simultaneously by computer in general to find the class of solutions for the spin connection that gives resonance. In the far off resonance condition they reduce to the Poisson equation:

$$\nabla^2\phi = -\frac{\rho}{\epsilon_0} \tag{B-19}$$

and thus to the Coulomb potential (B-4) and spin connection (B-5). The latter is a valid solution of Eqs. (B-1) and (B-19) because from Eq.(B-5):

$$\boldsymbol{\nabla} \times \boldsymbol{\omega} = \mathbf{0} \tag{B-20}$$

so Eq.(B-18) reduces to:

$$\boldsymbol{\nabla}\phi \times \boldsymbol{\omega} = \mathbf{0} \tag{B-21}$$

If:

$$\boldsymbol{\nabla}\phi = \boldsymbol{\omega}\phi \tag{B-22}$$

Eq.(B-21) is true identically. Also, if consideration is restricted to the radial component:

$$\boldsymbol{\nabla} = \frac{\partial}{\partial r}\mathbf{e}_r \tag{B-23}$$

in the spherical polar coordinate system, then Eq.(B-21) is valid for any radial spin connection of the type:

$$\boldsymbol{\omega} = \omega_r \mathbf{e}_r \tag{B-24}$$

because

$$\mathbf{e}_r \times \mathbf{e}_r = \mathbf{0} \tag{B-25}$$

So Eq.(B-18) is true for any radially directed spin connection. The one that gives the standard model as a limit is Eq.(B-5), Q.E.D.

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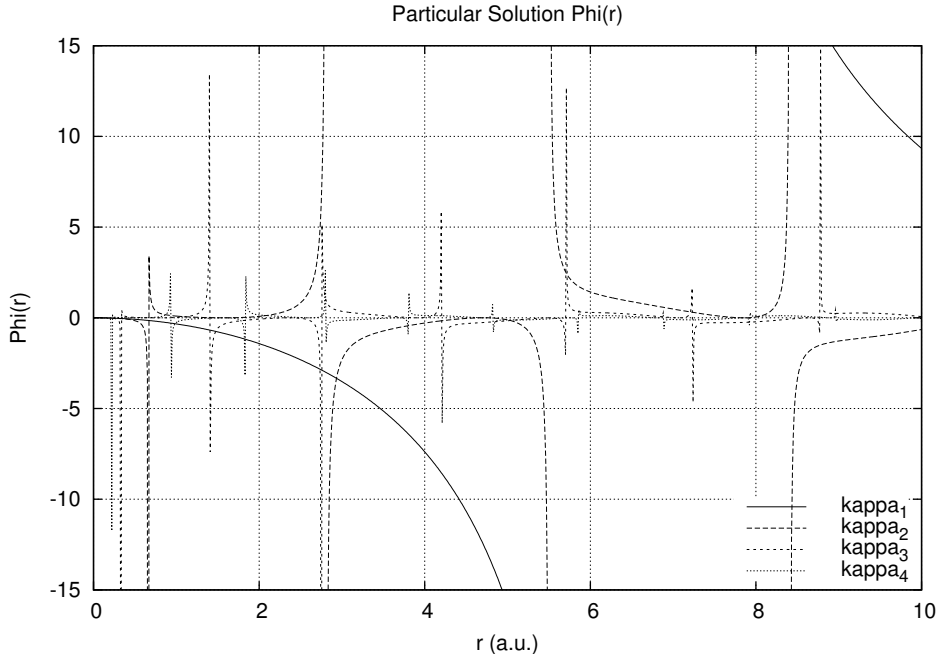

Figure 9.1: Graph of particular solution (9.46) for four κ different values (0.1,1,2,3)

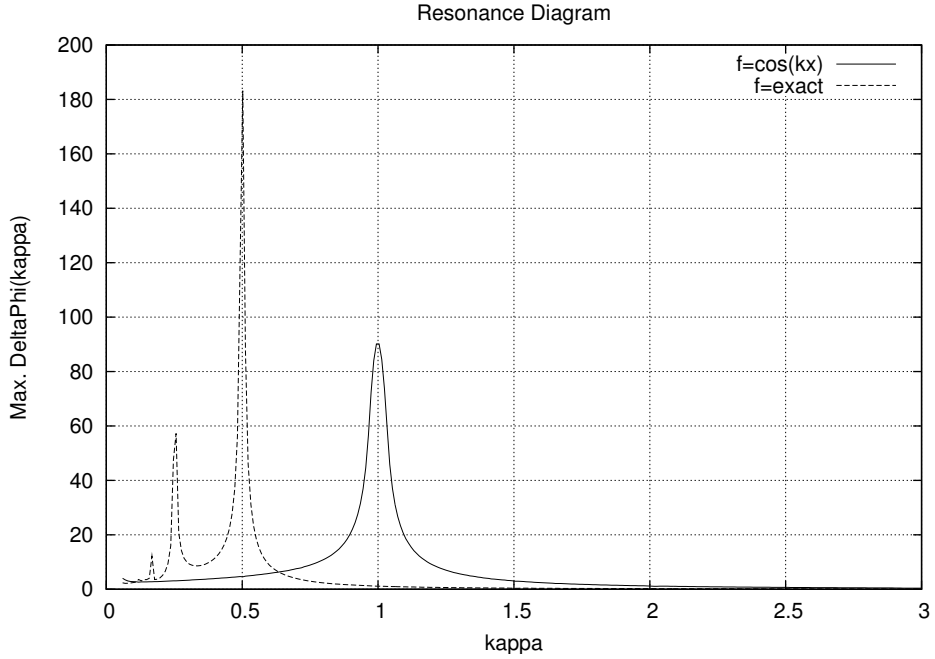

Figure 9.2: Resonance diagram, max. amplitude difference after 15 wavelengths $\lambda = 2\pi/\kappa$

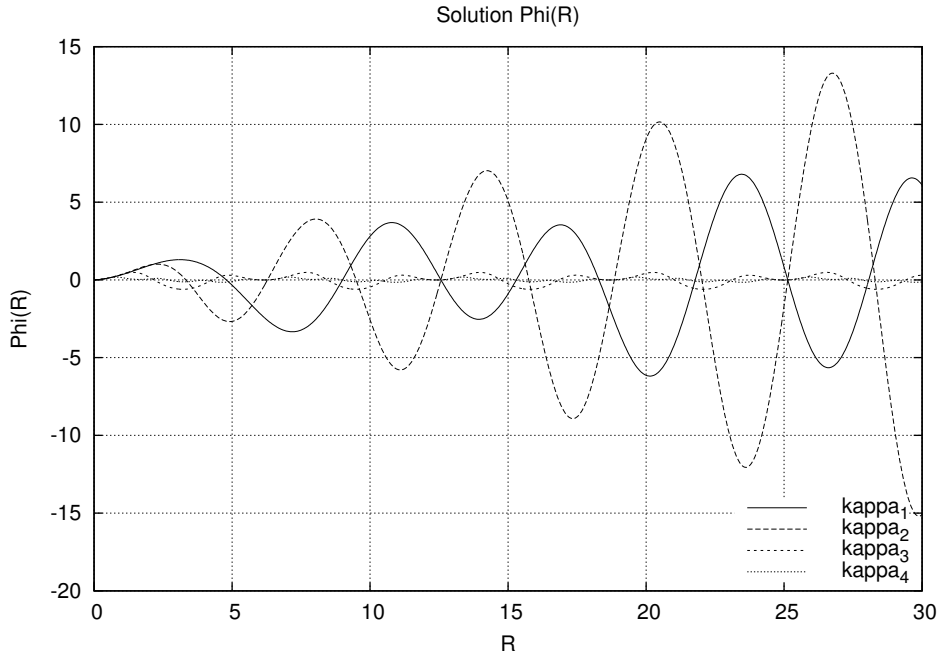

Figure 9.3: Numerical solution of Eq.(9.38) for four different κ values (0.25, 0.5, 1, 2)

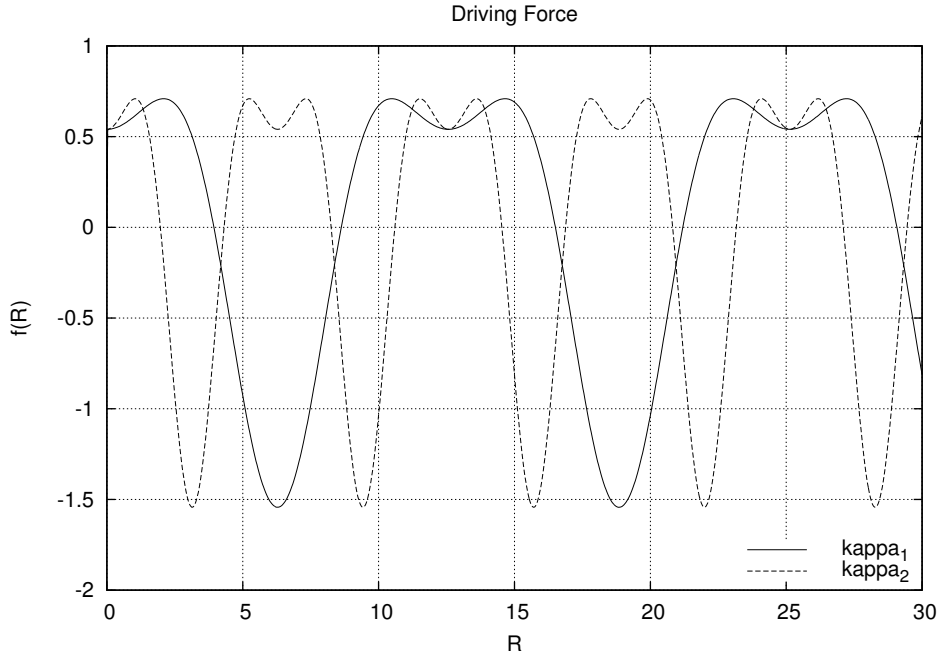

Figure 9.4: Driving force of Eq.(9.38) for $\kappa = 0.25$ and $\kappa = 0.5$

Figure 9.5: Fourier spectrum of driving force (Fig. 9.4) for four different κ values (0.25, 0.5, 1, 2)

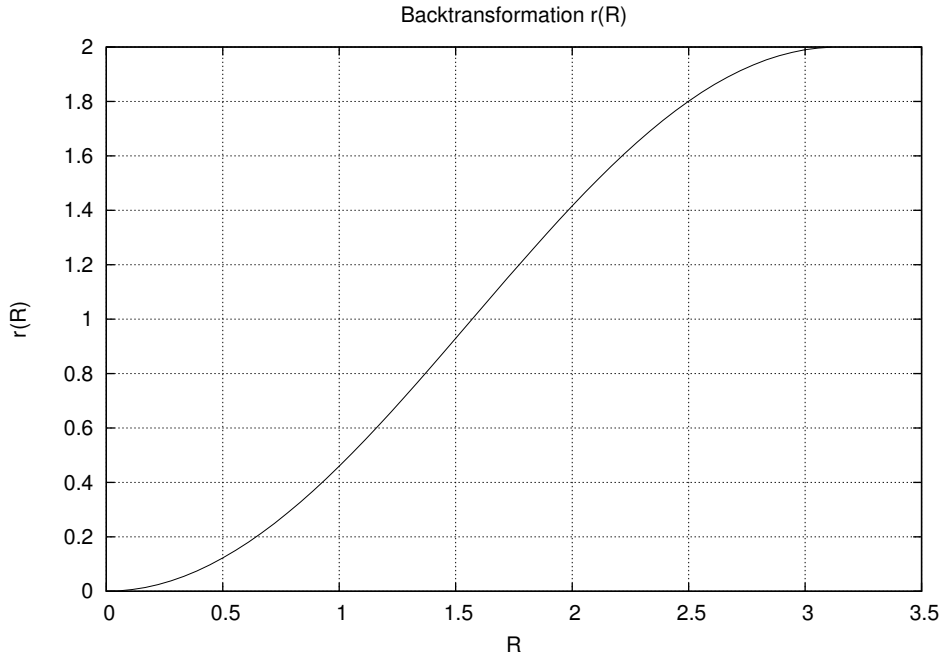

Figure 9.6: Backtransformation $R \rightarrow r$ for $\kappa_r = 1$

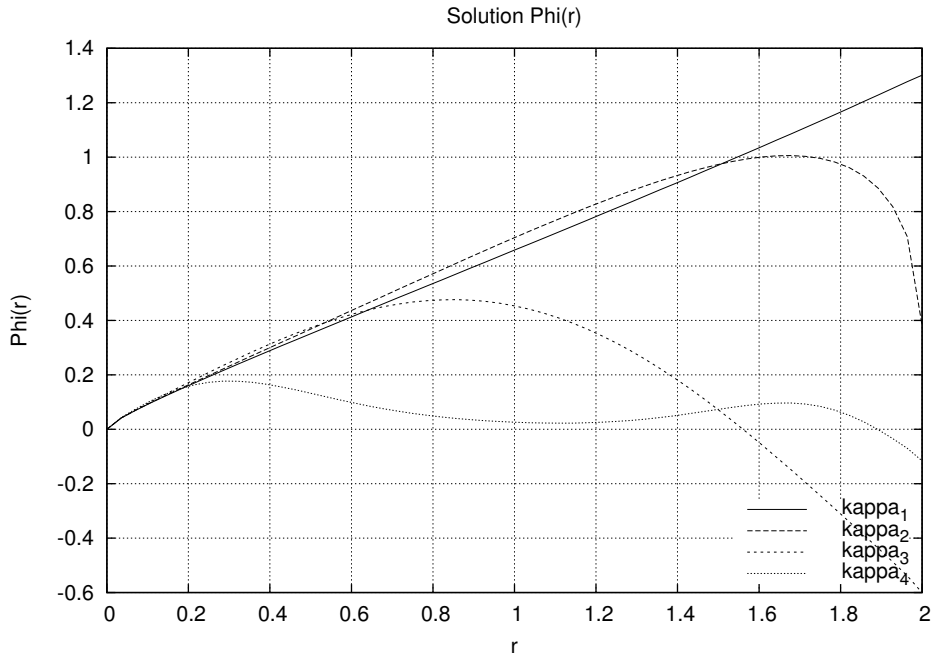

Figure 9.7: Back-transformed $\Phi(r)$ according to Fig. 9.6

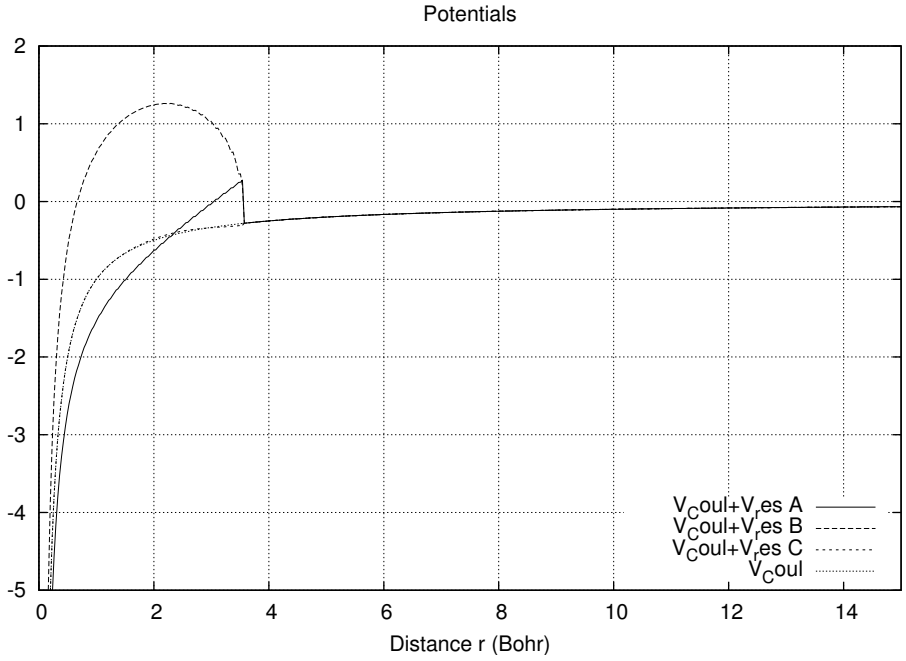

Figure 9.8: Coulomb and resonance potential of H. A: $\kappa = 0.175$, B: $\kappa = 0.225$, C: $\kappa = 1.0$

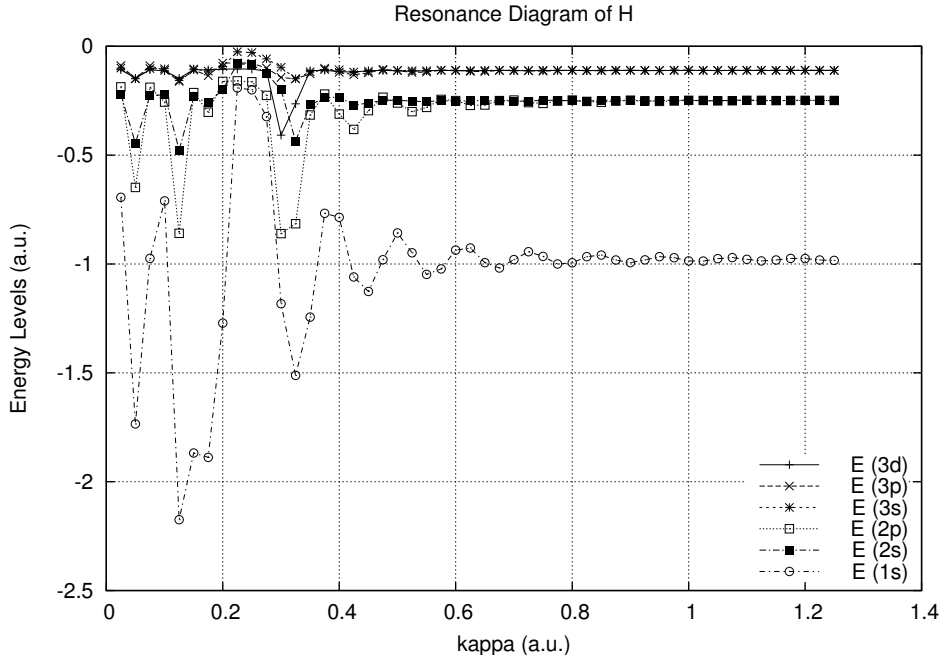

Figure 9.9: Resonance diagram of atomic Hydrogen ($\kappa_r = 0.5$)

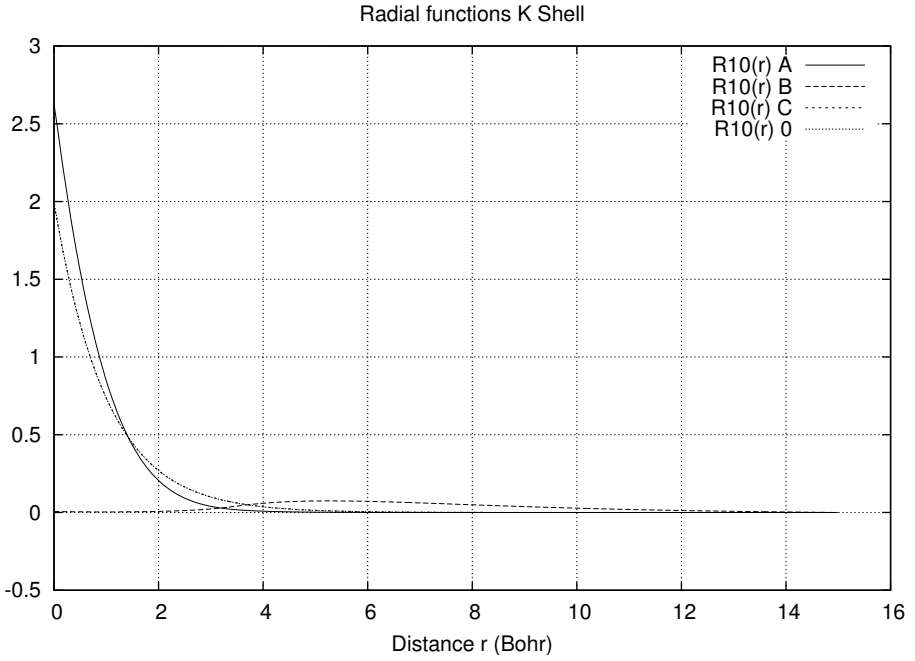

Figure 9.10: Radial 1s wavefunction of H. A: $\kappa = 0.175, B : \kappa = 0.225, C : \kappa = 1.0$

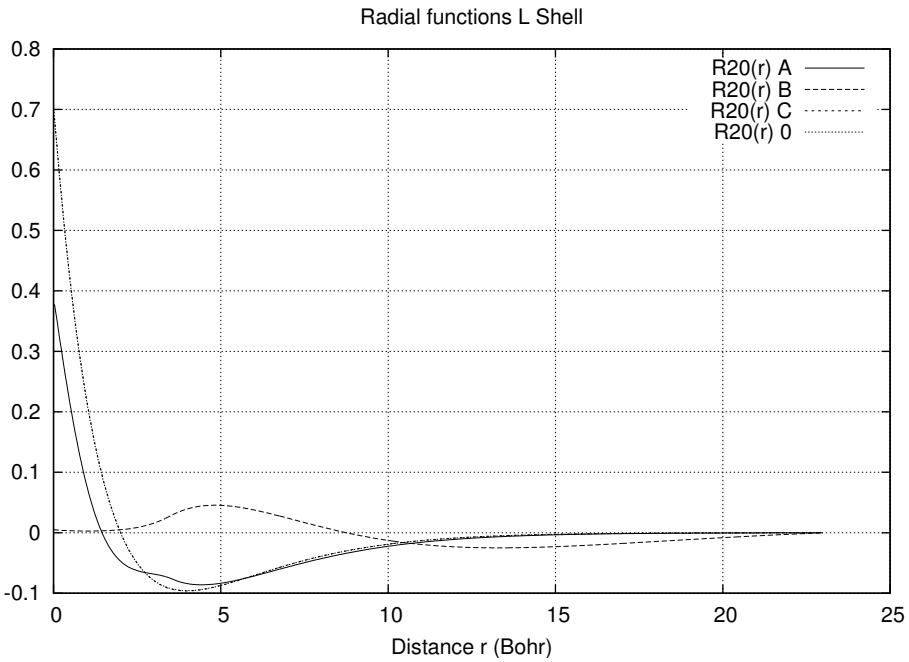

Figure 9.11: Radial 2s wavefunction of H. A: $\kappa = 0.175, B : \kappa = 0.225, C : \kappa = 1.0$

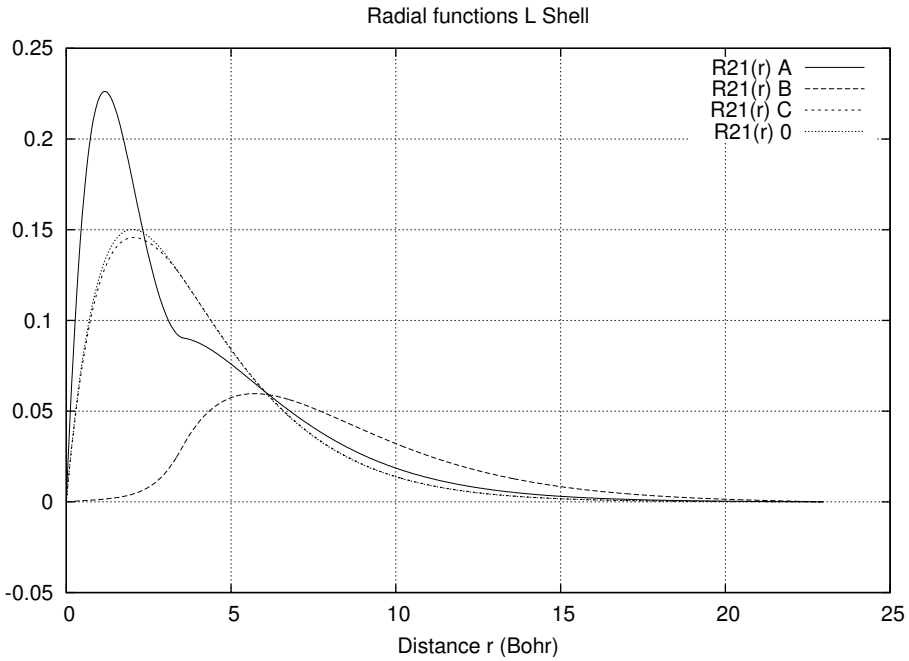

Figure 9.12: Radial 2p wavefunction of H. A: $\kappa = 0.175, B : \kappa = 0.225, C : \kappa = 1.0$

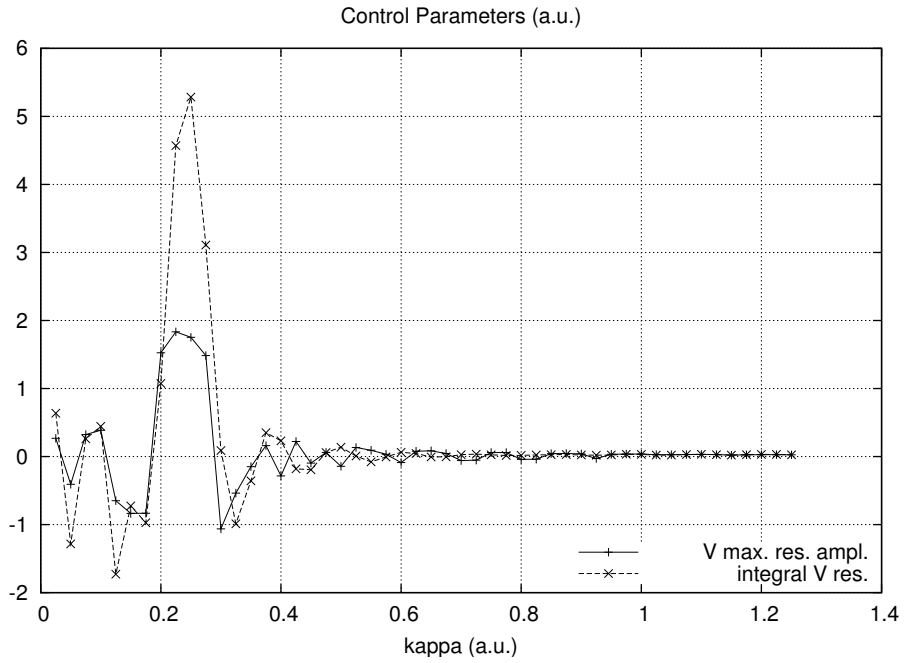

Figure 9.13: Control parameters

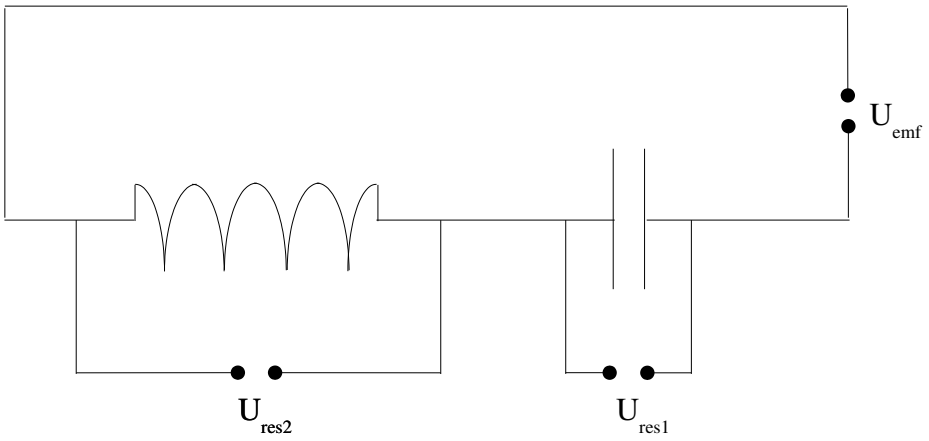

Figure 9.14: Equivalent circuit for the resonant Coulomb law

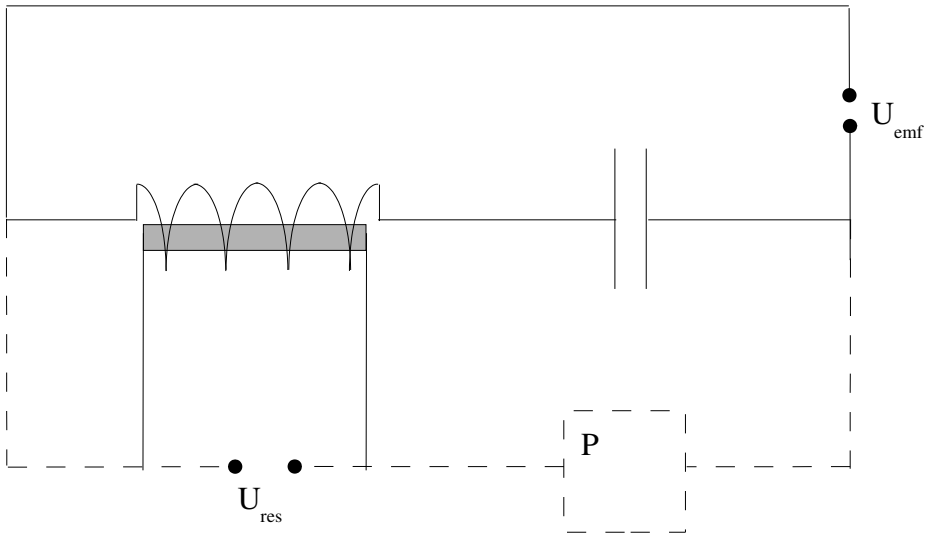

Figure 9.15: Alternative tapping of energy from equivalent circuit

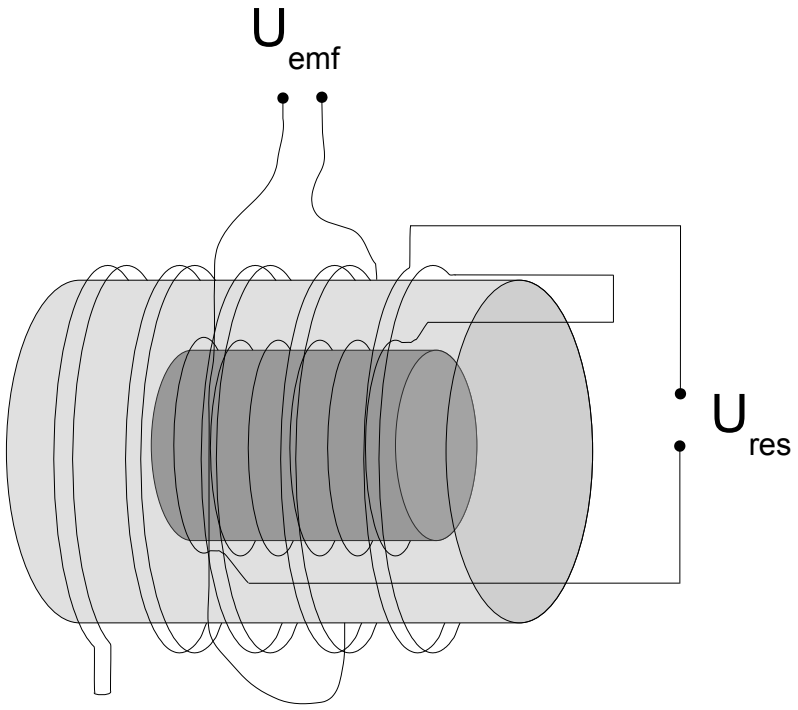

Figure 9.16: Alternative tapping of energy from equivalent circuit

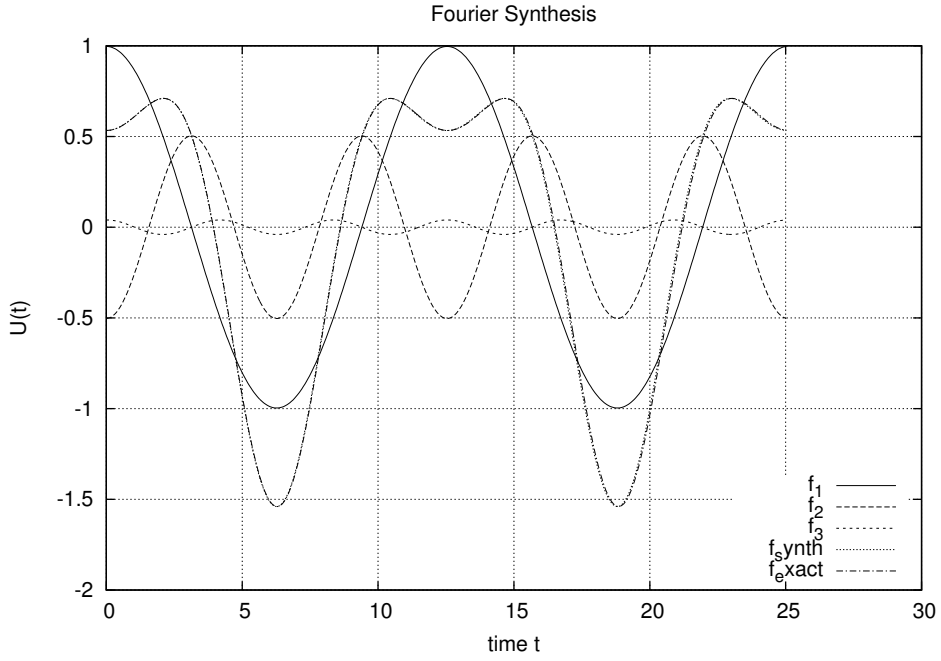

Figure 9.17: Fourier synthesis of the driving force U_{emf}