Chapter 12

Vector boson character of the static electric field

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Abstract

The field equations of Einstein Cartan Evans (ECE) are used to develop the concept of the static electric field as a vector boson with spin indices -1, 0, $+1$, which occur in addition to the vector character of the electric field. The existence of the electric vector boson in physics is inferred directly from Cartan geometry, using the concept of a spinning space-time that defines the electromagnetic field. When the electromagnetic field is independent of the gravitational field the spin connection is dual to the tetrad, producing a set of equations with which to define the electric vector boson. Angular momentum theory is used to develop the basic concept.

Keywords: Einstein Cartan Evans (ECE) field theory, generally covariant unified field theory, electric vector boson, spinning space-time, spin connection, tetrad, angular momentum theory.

12.1 Introduction

Recently, the concept of electromagnetic field as spinning space-time in general relativity has been developed using standard Cartan geometry [1]– [20]. This produces a generally covariant unified field theory in which the electromagnetic field is the well known Cartan torsion within a factor $A^{(0)}$, where $cA^{(0)}$ has the units of volts. The theory is known as Einstein Cartan Evans (ECE) field theory because it is based on the work of Einstein and Cartan. The ECE theory has been tested extensively for technical correctness and also against a range of experimental data. It has therefore been accepted [21] as a valid theory of physics.

In Section 12.2, the duality equations between spin connection and tetrad are developed in a set of six equations, which are valid when the electromagnetic field is independent of the gravitational field. The basic ECE Ansatz [1]– [19] is used to define the scalar potential and from this the electric field is defined in terms of the scalar potential and the spin connection. It is shown that in ECE theory, the static electric field develops a vector boson character. In Section 12.3, angular momentum theory is used to develop the concept of electric vector boson and the complex circular basis [1]– [19] introduced for the internal indices of the electric vector boson.

12.2 Scalar potential and electric field

In the absence of gravitation the spin connection and tetrad forms [1]– [20] are dual in the tangent space-time, so:

$$\omega^a{}_{\mu b} = -\frac{\kappa}{2}\epsilon^a{}_{bc} q^c_\mu \tag{12.1}$$

Here $\omega^a{}_{\mu b}$ is the spin connection form of Cartan geometry, κ has the units of wave-number, $\eta^a{}_{bc}$ is the Levi-Civita tensor in the tangent space-time, and q^c_μ is the tetrad form of Cartan geometry. Eq.(12.1) may be expanded in six equations as follows:

$$\omega^0{}_{\mu 1} = -\frac{\kappa}{2}(q^2_\mu + q^3_\mu) \tag{12.2}$$

$$\omega^0{}_{\mu 2} = -\frac{\kappa}{2}(q^3_\mu + q^1_\mu) \tag{12.3}$$

$$\omega^0{}_{\mu 3} = -\frac{\kappa}{2}(-q^1_\mu - q^2_\mu) \tag{12.4}$$

$$\omega^1{}_{\mu 2} = \frac{\kappa}{2}(q^0_\mu + q^3_\mu) \tag{12.5}$$

$$\omega^1{}_{\mu 3} = -\frac{\kappa}{2}(-q^2_\mu + q^0_\mu) \tag{12.6}$$

$$\omega^2{}_{\mu 3} = -\frac{\kappa}{2}(q^1_\mu + q^0_\mu) \tag{12.7}$$

The basic ECE Ansatz is:

$$A^a_\mu = A^{(0)} q^a_\mu \tag{12.8}$$

where A^a_μ is the electromagnetic potential form. The scalar potential may be defined in two ways:

$$A^a_\mu \rightarrow (A^a_0, \mathbf{0})\text{or}(A^0_\mu, \mathbf{0}) \tag{12.9}$$

However, these two definitions are equivalent as shown as follows.

Use the fundamental definitions:

$$U^a = q^a_0 U^0 \tag{12.10}$$

$$U^0 = q^0_\mu U^\mu \tag{12.11}$$

and assume that:

$$U^0 U_0 = 1 \tag{12.12}$$

for simplicity of argument. Multiply both sides of Eqs.(12.10) and (12.11) by U_0 to obtain:

$$U^a U_0 = q_0^a \tag{12.13}$$

and

$$q_\mu^0 U^\mu U_0 = 1 \tag{12.14}$$

The fundamental normalization property of the tetrad [20] is:

$$q_\mu^a q_a^\mu = 1 \tag{12.15}$$

so

$$q_\mu^0 q_0^\mu = 1 \tag{12.16}$$

and:

$$q_0^a q_a^0 = 1 \tag{12.17}$$

Using Eqs.(12.13), (12.14) and (12.17):

$$q_a^0 U^a = q_\mu^0 U^\mu \tag{12.18}$$

The a and μ indices are therefore equivalent and interchangeable if one index of the tetrad is fixed at zero. Therefore the scalar potential is either:

$$A_0^a = A^{(0)} q_0^a \tag{12.19}$$

or:

$$A_\mu^0 = A^{(0)} q_\mu^0 \tag{12.20}$$

where

$$\phi^{(0)} = cA^{(0)} \tag{12.21}$$

The scalar component is $\phi^{(0)}$, but q_0^a and q_μ^0 are mixed index rank two tensors. The equivalence of (12.19) and (12.20) means that one frame spinning with respect to a second is indistinguishable from a second spinning with respect to the first.

In special relativity (Maxwell Heaviside field theory) the index a is missing and the scalar potential is the time-like part of A_μ:

$$A_\mu = \left(\frac{\phi}{c}, \mathbf{0}\right) \tag{12.22}$$

In general relativity (ECE theory):

$$A_\mu^a = \left(\frac{\phi_0^a}{c}, \mathbf{0}\right) \tag{12.23}$$

where A_0^a has the four components A_0^0, A_0^1, A_0^2, and A_0^3. Each of these four components is a generally covariant quantity. In special relativity there is only one quantity (12.21). That quantity is not generally covariant, it is Lorentz covariant. From the basic ECE Ansatz (12.8):

$$A_0^0 = A^{(0)} q_0^0, \cdots, A_0^3 = A^{(0)} q_0^3 \tag{12.24}$$

where $A^{(0)}$ is frame invariant and a scalar. The four tetrad elements in Eq.(12.24) are generally covariant, i.e. the tetrad q_0^a retains its tensorial character under any type of coordinate transformation. This property represents objectivity in science. All four elements A_0^0, A_0^1, A_0^2, A_0^3 exist in general and are not the same in general. Thus ECE theory contains more information than Maxwell Heaviside (MH) theory and is ECE theory is rigorously objective. In general, all four A_0^a $(a = 0, ..., 3)$ may be observed experimentally. At present, nothing is known about them experimentally, the only thing that is known is the Coulomb Law's potential:

$$\phi = -\frac{e}{4\pi\epsilon_0 r}\epsilon_0 \tag{12.25}$$

in spherical polar coordinates. Here η_0 is the vacuum permittivity, $-e$ is the charge on the electron, and r is the radial coordinate. In special relativity, ϕ is the time-like part of A_μ in electro-statics. As such ϕ is Lorentz covariant, part of the Lorentz covariant A_μ. In MH theory the electric field is well known to be represented by:

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \frac{\partial A}{\partial t} \tag{12.26}$$

where

$$A_\mu = \left(\frac{\phi}{c}, -\boldsymbol{A}\right) \tag{12.27}$$

In MH the electric field is therefore a 3-D vector and part of the 4-D electromagnetic tensor $F_{\mu\nu}$. In electro-statics [22]:

$$\frac{\partial A}{\partial t} = \boldsymbol{0} \tag{12.28}$$

so:

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi = \frac{e}{4\pi\epsilon_0 r^2}\boldsymbol{e}_r \tag{12.29}$$

where $\boldsymbol{e}_r$ is the radial unit vector of the spherical polar coordinate system. In ECE theory the electric field is obtained from the first Cartan structure equation:

$$T^a = d \wedge q^a + \omega^a{}_b \wedge q^b \tag{12.30}$$

This is transformed into:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{12.31}$$

by the ECE Ansatz:

$$A^a = A^{(0)} q^a \tag{12.32}$$

$$F^a = A^{(0)} T^a \tag{12.33}$$

Here F^a is the electromagnetic field form and T^a is the Cartan torsion form. These are vector valued two-forms carrying an index a. It is this index that gives the electric field its vector boson character. From Eq.(12.31) [1]– [19]:

$$\boldsymbol{E}^a = -\boldsymbol{\nabla}\phi^a - \frac{\partial \boldsymbol{A}^a}{\partial t} - c\omega^{0a}{}_b \boldsymbol{A}^b + \boldsymbol{\omega}^a{}_b \phi^b \tag{12.34}$$

and it is seen that the electric field is a vector with $a = 0, \ldots, 3$ and can be considered in general as four vectors, $\boldsymbol{E}^0$, $\boldsymbol{E}^1$, $\boldsymbol{E}^2$ and $\boldsymbol{E}^3$. This is the correctly objective description of the electric field, but again, almost nothing is known experimentally about. In special relativity the entity known as the static electric field is the 3-D vector $\boldsymbol{E}$, which is Lorentz covariant. This has no internal label a.

In special relativity the work done on a charge against the action of a static electric field is:

$$W = e(\phi_A - \phi_B) \tag{12.35}$$

in transporting a charge from A to B. In general relativity the amount of work done depends on the index a:

$$W_0^a = e(\phi_{0A}^a - \phi_{0B}^a) \tag{12.36}$$

and this reflects the structure of ECE space-time. The Minkowski space-time of MH theory has no structure. However, the work may still be expressed in ECE theory as the difference of two potentials. Electro-statics may therefore be developed in general relativity using the potential. It may be assumed that:

$$\boldsymbol{A}^a = \boldsymbol{0} \tag{12.37}$$

and therefore the electric field is defined as:

$$\boldsymbol{E}^a = -\boldsymbol{\nabla}\phi^a + \boldsymbol{\omega}^a{}_b\phi^b \tag{12.38}$$

From this point onwards it is necessary to rely on the only information available experimentally about ϕ, and this is the Coulomb Law, Eq.(12.25). Although this is known to be very accurate [22], it is objectively only a special case of general relativity. In Eq.(12.25), ϕ is the time-like component of A_μ, which is Lorentz covariant. Objectivity demands that there must be $\phi_0^a(a = 0, ..., 3)^\mu$ in physics. In the absence of any experimental data on ϕ_0^a it is assumed for simplicity of argument only that:

$$\phi := \phi_0^0 = \phi_0^1 = \phi_0^2 = \phi_0^3 \tag{12.39}$$

Thus:

$$\phi = \phi^{(0)} q_0^0 \tag{12.40}$$

where

$$q_0^0 = \phi_0^1 = \phi_0^2 = \phi_0^3 \tag{12.41}$$

has been assumed for simplicity. From units analysis:

$$q_0^0 = 1 \tag{12.42}$$

Note carefully however that q_0^0 takes this value only in a given frame. In any other frame it is still unit-less but may become different from unity in general. This is the result of general covariance in physics, the basic ansatz of general relativity. The spin connection elements responsible for the scalar potential in ECE may now be defined in terms of q_0^0 using Eqs.(12.2)–(12.7), as follows:

$$\omega_{02}^1 = -\omega_{01}^2 = \omega_{03}^1 = -\omega_{01}^3 = \omega_{03}^2 = -\omega_{02}^3 = \frac{\kappa}{2} q_0^0 \tag{12.43}$$

There is only one independent element, and its value is $\kappa/2$. This greatly simplifies the treatment of the scalar potential in terms of the spin connection.

This treatment relies on the Coulomb potential as the only experimentally determined scalar potential to date for the static electric field. In general, the latter is defined from Eq.(12.38) as:

$$\boldsymbol{E}^a = -\boldsymbol{\nabla}\phi^a + \boldsymbol{\omega}^a{}_0\phi^0 + \cdots + \boldsymbol{\omega}^a{}_3\phi^3 \tag{12.44}$$

$$\boldsymbol{E}^a_1 = -\frac{\partial\phi^a}{\partial x_1} + \boldsymbol{\omega}^a{}_{10}\phi^0 + \cdots + \boldsymbol{\omega}^a{}_{13}\phi^3 \tag{12.45}$$

and so on. From Eq.(12.1), it is seen that the spin connection elements that contribute to Eq.(12.45) are of the type:

$$\mu = 1, 2, 3 \tag{12.46}$$

Furthermore, only tetrad elements of the type q^0_μ may contribute in Eqs.(12.2)–(12.7). These are defined, as we have seen at the start of this Section, by:

$$q^\mu_0 q^0_\mu = 1 \tag{12.47}$$

and

$$U^0 = q^0_\mu U^\mu \tag{12.48}$$

The tetrad elements of type q^0_μ in Eqs.(12.2)–(12.7), with $\mu = 1, 2, 3$, define:

$$\omega^1_{\mu 2} = \omega^1_{\mu 3} = \omega^2_{\mu 3} = \frac{\kappa}{2} q^0_\mu \tag{12.49}$$

with:

$$\begin{gathered} \omega^2_{\mu 3} = -\omega^1_{\mu 2} \\ \text{etc.} \end{gathered} \tag{12.50}$$

This means that in Eq.(12.45):

$$a = 1, 2, 3 \tag{12.51}$$

and the electric field takes on a vector boson character in general relativity. Thus:

$$\boldsymbol{E}^1 = -\boldsymbol{\nabla}\phi + \boldsymbol{\omega}^1\phi \tag{12.52}$$

$$\boldsymbol{E}^2 = -\boldsymbol{\nabla}\phi + \boldsymbol{\omega}^2\phi \tag{12.53}$$

$$\boldsymbol{E}^3 = -\boldsymbol{\nabla}\phi + \boldsymbol{\omega}^3\phi \tag{12.54}$$

where the three vector boson spin connections are defined by:

$$\omega^1{}_1 = \omega^1{}_{10} + \omega^1{}_{11} + \omega^1{}_{12} + \omega^1{}_{13} \tag{12.55}$$

$$\omega^1{}_2 = \omega^1{}_{20} + \omega^1{}_{21} + \omega^1{}_{22} + \omega^1{}_{23} \tag{12.56}$$

$$\omega^1{}_3 = \omega^1{}_{30} + \omega^1{}_{31} + \omega^1{}_{32} + \omega^1{}_{33} \tag{12.57}$$

$$\boldsymbol{\omega}^1 = \omega^1{}_1\boldsymbol{i} + \omega^1{}_2\boldsymbol{j} + \omega^1{}_3\boldsymbol{k} \tag{12.58}$$

and so on for $\boldsymbol{\omega}^2$ and $\boldsymbol{\omega}^2$. From Eq.(12.49):

$$\omega^1{}_1 = \omega^1{}_{12} + \omega^1{}_{13} = \kappa q^0_1 = \kappa \tag{12.59}$$

$$\omega^1{}_2 = \omega^1{}_{21} + \omega^1{}_{23} = \kappa q^0_2 = \kappa \tag{12.60}$$

$$\omega^1{}_3 = \omega^1{}_{32} + \omega^1{}_{33} = \kappa \tag{12.61}$$

$$\omega^2{}_1 = \omega^2{}_2 = \omega^2{}_3 = 0 \tag{12.62}$$

and:

$$E_1^1 = -\frac{\partial \phi}{\partial x_1} + (\omega^1{}_{10} + \omega^1{}_{11} + \omega^1{}_{12} + \omega^1{}_{13})\phi = -\frac{\partial \phi}{\partial x_1} + \kappa\phi \quad \text{etc} \tag{12.63}$$

Thus:

$$\boldsymbol{E}^1 = -\boldsymbol{\nabla}\phi + \boldsymbol{\kappa}\phi \tag{12.64}$$

Similarly:

$$E_1^2 = -\frac{\partial \phi}{\partial x_2} + (\omega_{10}^2 + \omega_{11}^2 + \omega_{12}^2 + \omega_{13}^2)\phi = -\frac{\partial \phi}{\partial x_2} \tag{12.65}$$

$$\boldsymbol{E}^2 = -\boldsymbol{\nabla}\phi \tag{12.66}$$

and:

$$E_1^3 = -\frac{\partial \phi}{\partial x_3} + (\omega_{10}^3 + \omega_{11}^3 + \omega_{12}^3 + \omega_{13}^3)\phi = -\frac{\partial \phi}{\partial x_3} - \kappa\phi \tag{12.67}$$

$$\boldsymbol{E}^3 = -\boldsymbol{\nabla}\phi - \boldsymbol{\kappa}\phi \tag{12.68}$$

It is seen that the spin connections and tetrads are always space-time properties but that ϕ is defined by the scalar amplitude $\phi^{(0)}$. The spin connection plays the role of an "additional $\bar{\boldsymbol{V}}$". Using experimental measurements to date the three vector boson components $\boldsymbol{E}^1$,$\boldsymbol{E}^2$,$\boldsymbol{E}^3$ have been observed only as the static field $\boldsymbol{E}$ without realizing that it has three labels. However, using the concept of spin connection resonance [1]– [19] they become distinguishable, because $\boldsymbol{E}^1$ and $\boldsymbol{E}^3$ cause resonance but $\boldsymbol{E}^3$ does not. For convenience of development in angular momentum theory we re-label the three components as:

$$\boldsymbol{E}^1 = \boldsymbol{E}_1 \tag{12.69}$$

$$\boldsymbol{E}^2 = \boldsymbol{E}_0 \tag{12.70}$$

$$\boldsymbol{E}^3 = \boldsymbol{E}_{-1} \tag{12.71}$$

12.3 Development of the electric vector boson with angular momentum theory

Angular momentum theory is developed in ref. [18], volume one, chapter five. In ECE theory, the electric field is a vector boson defined by:

$$\boldsymbol{E}_1 = -\boldsymbol{\nabla}\phi + \boldsymbol{\omega}\phi \tag{12.72}$$

$$\boldsymbol{E}_0 = -\boldsymbol{\nabla}\phi \tag{12.73}$$

$$\boldsymbol{E}_{-1} = -\boldsymbol{\nabla}\phi - \boldsymbol{\omega}\phi \tag{12.74}$$

The components are written as $(-1, 0, +1)$ to emphasize the angular momentum character of the electric vector boson. In the simplest instance, angular momentum theory can be developed by considering $O(3)$ vector relations. In the Cartesian basis:

$$\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k} \tag{12.75}$$

$$\boldsymbol{k} \times \boldsymbol{i} = \boldsymbol{j} \tag{12.76}$$

$$\boldsymbol{j} \times \boldsymbol{k} = \boldsymbol{i} \tag{12.77}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are unit vectors. Defining the operator:

$$\hat{J}_Z := \boldsymbol{k}\times \tag{12.78}$$

then:

$$\hat{J}_Z\boldsymbol{i} = 1\boldsymbol{j} \tag{12.79}$$

$$\hat{J}_Z\boldsymbol{j} = -1\boldsymbol{i} \tag{12.80}$$

$$\hat{J}_Z\boldsymbol{k} = 0\boldsymbol{k} \tag{12.81}$$

so the eigenvalues of $\hat{J}_Z$ are -1, 0 and $+1$. In the complex circular basis [1]– [19]:

$$\boldsymbol{e}^{(1)} = \boldsymbol{e}^{(2)*} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}) \tag{12.82}$$

$$\boldsymbol{e}^{(2)} = \boldsymbol{e}^{(1)*} = \frac{1}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j}) \tag{12.83}$$

$$\boldsymbol{e}^{(3)} = \boldsymbol{e}^{(3)*} = \boldsymbol{k} \tag{12.84}$$

Thus, $O(3)$ symmetry is represented by:

$$\boldsymbol{e}^{(1)} \times \boldsymbol{e}^{(2)} = i\boldsymbol{e}^{(3)*} \tag{12.85}$$

$$\boldsymbol{e}^{(2)} \times \boldsymbol{e}^{(3)} = i\boldsymbol{e}^{(1)*} \tag{12.86}$$

$$\boldsymbol{e}^{(3)} \times \boldsymbol{e}^{(1)} = i\boldsymbol{e}^{(2)*} \tag{12.87}$$

If we now define the operator:

$$\hat{J}^{(3)} := i\boldsymbol{e}^{(3)}\times \tag{12.88}$$

then:

$$\hat{J}^{(3)}\boldsymbol{e}^{(1)} = -1\boldsymbol{e}^{(1)} \tag{12.89}$$

$$\hat{J}^{(3)}\boldsymbol{e}^{(2)} = 1\boldsymbol{e}^{(2)} \tag{12.90}$$

$$\hat{J}^{(3)}\boldsymbol{e}^{(3)} = 0\boldsymbol{e}^{(3)} \tag{12.91}$$

Thus $\hat{J}^{(3)}$ is a type of angular momentum operator with eigenvalues -1, 0 and 1. These eigenvalues signify the existence of $O(3)$ symmetry. These eigenvalues, -1, 0 and 1, appear again in the matrix equivalents of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$:

$$\boldsymbol{i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, \boldsymbol{j} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \boldsymbol{k} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tag{12.92}$$

In the theory of irreducible tensorial sets, these rotation operators in Euclidean space are first rank $\hat{T}$ operators, which are irreducible tensor operators and under rotations transform into linear combinations of each other. The $\hat{T}$ operators are directly proportional to the scalar spherical harmonic operators. The rotation operators of the full rotation group are related to the $\hat{T}$ operators as follows:

$$\hat{T}^1_{-1} = i\hat{J}^{(1)} \, , \, \hat{T}^1_1 = i\hat{J}^{(2)} \, , \, \hat{T}^1_0 = i\hat{J}^{(3)} \tag{12.93}$$

and are also related to the spherical harmonics and vector spherical harmonics [18].

The complex circular basis (1), (2) and (3) is also a convenient basis with which to represent the electric vector boson as follows:

$$\boldsymbol{E}^{(1)} = -\boldsymbol{\nabla}\phi - \boldsymbol{\omega}^{(1)}\phi \tag{12.94}$$

$$\boldsymbol{E}^{(2)} = -\boldsymbol{\nabla}\phi + \boldsymbol{\omega}^{(1)}\phi \tag{12.95}$$

$$\boldsymbol{E}^{(3)} = -\boldsymbol{\nabla}\phi \tag{12.96}$$

and the vector boson character of the static electric field may be represented as:

$$\hat{J}^{(3)}\boldsymbol{E}^{(1)} = -\phi\boldsymbol{\omega}^{(1)} \tag{12.97}$$

$$\hat{J}^{(3)}\boldsymbol{E}^{(2)} = \phi\boldsymbol{\omega}^{(2)} \tag{12.98}$$

$$\hat{J}^{(3)}\boldsymbol{E}^{(3)} = \phi\boldsymbol{0} \tag{12.99}$$

For plane waves the circularly polarized electric field is:

$$\boldsymbol{E}^{(1)} = \frac{E^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i(\omega t - \kappa Z)} = \boldsymbol{E}^{(2)*} \tag{12.100}$$

and its static limit may be defined as the limit:

$$\omega t - \kappa Z \to 0 \tag{12.101}$$

in which case:

$$\text{Real}\,(\boldsymbol{E}^{(1)}) = \text{Real}\,(\boldsymbol{E}^{(2)}) = \frac{E^{(0)}}{\sqrt{2}}\boldsymbol{i} \tag{12.102}$$

$$\text{Im}\,(\boldsymbol{E}^{(1)}) = -\text{Im}\,(\boldsymbol{E}^{(2)}) = -\frac{E^{(0)}}{\sqrt{2}}\boldsymbol{j} \tag{12.103}$$

Therefore the spin connection for plane waves is:

$$\boldsymbol{\omega}^{(1)} = \boldsymbol{\omega}^{(2)*} = \frac{\omega^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i(\omega t - \kappa Z)} \tag{12.104}$$

This means that the frame itself is both spinning and translating. This is an example of the more general spin connection of Cartan geometry.

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