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The Continuity Equation in ECE Theory

by

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Abstract

In Einstein Cartan Evans (ECE) field theory the charge on the electron, -e, is a fundamental constant, and therefore, electric charge/current density is conserved in a fundamental continuity equation. In this paper the latter is derived in a space-time with torsion and curvature within the context of Cartan geometry, thus proving that the continuity equation is valid in a generally covariant unified field theory.

Keywords: ECE theory, unified field theory, continuity equation, conservation of charge/current density.

23.1 Introduction

The ECE theory has been accepted as being a valid unified field theory within the context of general relativity. It is therefore a generally covariant unified field theory of physics, or natural philosophy. The equations of dynamics and electrodynamics are expressed in a space-time with torsion and curvature present in general. In both subject areas the vector equations have the same format, and this is also the same as the Maxwell Heaviside and gravitomagnetic equations of standard physics, but expressed in ECE self consistently in a generally covariant mathematical framework based on Cartan geometry [1–12]. In ECE theory the charge on the electron, -e, is a universal constant, which implies that charge/current density must be conserved in a continuity equation that must be developed in a space-time with curvature and torsion both present. In standard physics the continuity equation [13, 14] is part of Noether's Theorem expressed in a Minkowski space-time with no torsion and no curvature. In Section 23.2 the ECE equations of classical electrodynamics and dynamics are reviewed and the electric charge and current density defined. In Section 23.3 the ECE continuity equation is derived self - consistently with the generally covariant Proca equation of ECE theory [1–12] by making the charge current density proportional to the vector potential and using an identically non-zero photon mass. In so doing the ECE continuity equation is identified as an example of the tetrad postulate of Cartan geometry and the continuity equation is derived from geometry as required in the philosophy of relativity.

23.2 The ECE Equations of Classical Dynamics and Electrodynamics

The ECE equations of dynamics are found from the following equation of geometry:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{23.1}$$

where $T^{\kappa\mu\nu}$ is the Cartan torsion tensor and $R^{\kappa}{}_{\mu}{}^{\mu\nu}$ is the Cartan curvature tensor. This is a tensor equation in a space-time with curvature and torsion [1–12] and is a tensorial expression of the Hodge dual of the Bianchi identity. The covariant derivative of the torsion is the curvature. In vector format this tensor equation becomes two generally covariant gravitomagnetic equations valid for all field strengths. The first one is the covariant generalization of the Newton inverse square law:

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = 4\pi G \rho_m . \tag{23.2}$$

Here G is Newton's gravitational constant. The acceleration due to gravity is:

$$\boldsymbol{g} = c^2 (T^{010}\boldsymbol{i} + T^{020}\boldsymbol{j} + T^{030}\boldsymbol{k}) \tag{23.3}$$

and the mass density is:

$$\rho = J^{0}{}_{1}{}^{10} + J^{0}{}_{2}{}^{20} + J^{0}{}_{3}{}^{30} \tag{23.4}$$

where c is the vacuum speed of light, a universal constant in the theory of relativity, of which ECE is the most developed form to date. The current terms defining the mass density are made up of curvature, torsion and spin connection elements as in the following general formula [1–12]:

$$J^{\kappa}{}_{\mu}{}^{\mu\nu} = \frac{c^2}{4\pi G} \left(R^{\kappa}{}_{\mu}{}^{\mu\nu} - \omega^{\kappa}{}_{\mu\lambda} T^{\lambda\mu\nu} \right) \tag{23.5}$$

where $\omega^{\kappa}{}_{\mu\lambda}$ denotes the spin connection. The second vector equation is the gravitomagnetic analogue of the Ampère Maxwell law of electrodynamics written in a space-time with torsion and curvature both present in general, and is:

$$\boldsymbol{\nabla} \times \boldsymbol{h} - \frac{1}{c}\frac{\partial \boldsymbol{g}}{\partial t} = 4\pi G \boldsymbol{J}_m \tag{23.6}$$

where $\boldsymbol{h}$ is the gravitomagnetic analogue of the magnetic field strength $\boldsymbol{H}$ of electrodynamics, $\boldsymbol{g}$ being the gravitomagnetic analogue of the electric displacement $\boldsymbol{D}$. In Eq. (23.6):

$$\boldsymbol{h} = c^2 (T^{332}\boldsymbol{i} + T^{113}\boldsymbol{j} + T^{221}\boldsymbol{k}) \tag{23.7}$$

and

$$\boldsymbol{g} = c^2 (T^{110}\boldsymbol{i} + T^{220}\boldsymbol{j} + T^{330}\boldsymbol{k}). \tag{23.8}$$

The current density term of the gravitomagnetic equation (23.6) is defined by:

$$\boldsymbol{J}_m = J_X \boldsymbol{i} + J_Y \boldsymbol{j} + J_Z \boldsymbol{k} \tag{23.9}$$

where:

$$J_X = J^1{}_0{}^{01} + J^1{}_2{}^{21} + J^1{}_3{}^{31}, \tag{23.10}$$
$$J_Y = J^2{}_0{}^{02} + J^2{}_1{}^{12} + J^2{}_3{}^{32}, \tag{23.11}$$
$$J_Z = J^3{}_0{}^{03} + J^3{}_1{}^{13} + J^3{}_2{}^{23}. \tag{23.12}$$

Eq. (23.6) is a spin equation whereas Eq. (23.2) is an orbital equation. Note carefully that the torsion components defining $\boldsymbol{g}$ are in general different in the two equations.

There are two more gravitomagnetic field equations which are found by Hodge dual transformation of Eqs. (23.2) and (23.6). The Hodge dual of the orbital equation (23.2) is:

$$\boldsymbol{\nabla} \cdot \boldsymbol{h} = 4\pi G \widetilde{\rho}_m \tag{23.13}$$

where:

$$\boldsymbol{h} = c^2 (\widetilde{T}^{010}\boldsymbol{i} + \widetilde{T}^{020}\boldsymbol{j} + \widetilde{T}^{030}\boldsymbol{k}) \tag{23.14}$$

and

$$\widetilde{\rho} = \widetilde{J}^{0}{}_{1}{}^{01} + \widetilde{J}^{0}{}_{2}{}^{02} + \widetilde{J}^{0}{}_{3}{}^{03}. \tag{23.15}$$

The tilde denoting Hodge dual transformation. The Hodge dual of the spin equation (23.6) is:

$$\boldsymbol{\nabla} \times \boldsymbol{g} + \frac{1}{c}\frac{\partial \boldsymbol{h}}{\partial t} = 4\pi G \widetilde{\boldsymbol{J}}_m \tag{23.16}$$

where:

$$\boldsymbol{g} = c^2(\widetilde{T}^{332}\boldsymbol{i} + \widetilde{T}^{113}\boldsymbol{j} + \widetilde{T}^{221}\boldsymbol{k}) \tag{23.17}$$

and

$$\boldsymbol{h} = c^2(\widetilde{T}^{101}\boldsymbol{i} + \widetilde{T}^{202}\boldsymbol{j} + \widetilde{T}^{303}\boldsymbol{k}) \tag{23.18}$$

and where the Hodge dual current is:

$$\widetilde{\boldsymbol{J}}_m = \widetilde{J}_X\boldsymbol{i} + \widetilde{J}_Y\boldsymbol{j} + \widetilde{J}_Z\boldsymbol{k}, \tag{23.19}$$

$$\widetilde{J}_X = \widetilde{J}^{1}{}_{0}{}^{01} + \widetilde{J}^{1}{}_{2}{}^{21} + \widetilde{J}^{1}{}_{3}{}^{31} \tag{23.20}$$

$$\widetilde{J}_Y = \widetilde{J}^{2}{}_{0}{}^{02} + \widetilde{J}^{2}{}_{1}{}^{12} + \widetilde{J}^{2}{}_{3}{}^{32} \tag{23.21}$$

$$\widetilde{J}_Z = \widetilde{J}^{3}{}_{0}{}^{03} + \widetilde{J}^{3}{}_{1}{}^{13} + \widetilde{J}^{3}{}_{2}{}^{23}. \tag{23.22}$$

These four equations of gravitomagnetism are valid for any field strength and are generally covariant. They describe the interaction of field and matter. If the field is propagating infinitely distant from its source the four equations reduce to the free field or vacuum gravitomagnetic equations:

$$\boldsymbol{\nabla}\cdot\boldsymbol{h}_0 = 0, \tag{23.23}$$

$$\boldsymbol{\nabla}\times\boldsymbol{g}_0 + \frac{1}{c}\frac{\partial \boldsymbol{h}_0}{\partial t} = \boldsymbol{0}, \tag{23.24}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{g}_0 = 0, \tag{23.25}$$

$$\boldsymbol{\nabla}\times\boldsymbol{h}_0 - \frac{1}{c}\frac{\partial \boldsymbol{g}_0}{\partial t} = \boldsymbol{0}, \tag{23.26}$$

with plane wave solutions indicating gravitational radiation. In the laboratory the gravitational radiation is about twenty one orders of magnitude weaker than electromagnetic radiation. The gravitomagnetic field/potential relations are:

$$\boldsymbol{g}_0 = -\boldsymbol{\nabla}\Phi - c\frac{\partial \boldsymbol{q}}{\partial t} + \Phi\boldsymbol{\omega} - c\omega^0\boldsymbol{q}, \tag{23.27}$$

$$\boldsymbol{h}_0 = c^2(\boldsymbol{\nabla} \times \boldsymbol{q} - \boldsymbol{\omega} \times \boldsymbol{q}), \tag{23.28}$$

where $\boldsymbol{q}$ is the vector potential of the gravitomagnetic equations and where Φ is the scalar potential of the gravitomagnetic equations. Here ω° is the spin connection scalar and $\boldsymbol{\omega}$ is the spin connection vector. In the Newtonian limit:

$$\boldsymbol{g}_0 \to -\boldsymbol{\nabla}\Phi. \tag{23.29}$$

The generally covariant equations of classical electromagnetism in ECE theory have the same vector structure as the standard Maxwell Heaviside equations but are written as follows in a space-time with torsion and curvature. The Coulomb law of ECE theory is an orbital torsion law:

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho \tag{23.30}$$

where the electric displacement is:

$$\boldsymbol{D} = D^{010}\boldsymbol{i} + D^{020}\boldsymbol{j} + D^{030}\boldsymbol{k} \tag{23.31}$$

and where the electric charge density is:

$$\rho = J^0{}_1{}^{10} + J^0{}_2{}^{20} + J^0{}_3{}^{30}. \tag{23.32}$$

The Ampère Maxwell law is a spin torsion law:

$$\boldsymbol{\nabla} \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{J} \tag{23.33}$$

in which the magnetic field strength is:

$$\boldsymbol{H} = H^{332}\boldsymbol{i} + H^{113}\boldsymbol{j} + H^{221}\boldsymbol{k} \tag{23.34}$$

and the electric displacement is:

$$\boldsymbol{D} = D^{110}\boldsymbol{i} + D^{220}\boldsymbol{j} + D^{330}\boldsymbol{k}. \tag{23.35}$$

The electric current density in Eq. (23.33) has the same form as in Eq. (23.19)–(23.22):

$$\boldsymbol{J} = J_X \boldsymbol{i} + J_Y \boldsymbol{j} + J_Z \boldsymbol{k}. \tag{23.36}$$

The other two laws of ECE classical electrodynamics are the Gauss law of magnetism, an orbital law:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \tag{23.37}$$

in which the magnetic flux density is:

$$\boldsymbol{B} = B^{010} \boldsymbol{i} + B^{020} \boldsymbol{j} + B^{030} \boldsymbol{k} \tag{23.38}$$

and the Faraday law of induction, a spin law where:

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 \tag{23.39}$$

and:

$$\boldsymbol{E} = E^{332} \boldsymbol{i} + E^{113} \boldsymbol{j} + E^{221} \boldsymbol{k}, \tag{23.40}$$

$$\boldsymbol{B} = B^{101} \boldsymbol{i} + B^{202} \boldsymbol{j} + B^{303} \boldsymbol{k}, \tag{23.41}$$

The field potential relations are:

$$\boldsymbol{E} = -\boldsymbol{\nabla} \phi - \frac{\partial \boldsymbol{A}}{\partial t} + \phi \boldsymbol{\omega} - \omega^0 \boldsymbol{A} \tag{23.42}$$

and

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} - \boldsymbol{\omega} \times \boldsymbol{A} \tag{23.43}$$

in which ϕ is the scalar potential and $\boldsymbol{A}$ is the vector potential.

23.3 Derivation of the Generally Covariant Continuity Equation

In this section it is shown that the continuity equation is:

$$D_\mu j^a_\nu = 0 \tag{23.44}$$

where j^a_ν is a vector valued differential one-form defined by:

$$j^a_\nu = -\epsilon_0 k T A^a_\nu. \tag{23.45}$$

Here, the generally covariant Proca equation of ECE theory is [1–12]:

$$(\Box + kT) A^a_\nu = 0. \tag{23.46}$$

where k is the Einstein equation, T is the index reduced canonical energy-momentum density, and where the potential is defined by the fundamental ECE postulate:

$$A^a_\nu = A^{(0)} q^a_\nu \tag{23.47}$$

where q^a_ν is the Cartan tetrad. In Eq. (23.45) ϵ_0 is the vacuum permittivity. In the limit of Minkowski space-time:

$$kT \to \left(\frac{mc}{\hbar}\right)^2 = \frac{1}{\lambda_c^2} \tag{23.48}$$

where m is the photon mass, $\hbar$ is the reduced Planck constant and where λ_c is the Comton wavelength.

Therefore Eq. (23.44) is an example of the tetrad postulate [1–12]:

$$D_\nu q^a_\mu = 0. \tag{23.49}$$

The structure of the inhomogeneous ECE field equation is:

$$\partial_\mu F^{\kappa\mu\nu} = j^{\kappa}{}_{\mu}{}^{\mu\nu} / \epsilon_0 \tag{23.50}$$

therefore by index contraction the charge-current density is a rank two tensor in ECE theory:

$$j^{\kappa\nu} = j^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{23.51}$$

Lowering an index:

$$j^{\kappa}{}_{\nu} = j^{\kappa}{}_{\mu}{}^{\mu}{}_{\nu} \tag{23.52}$$

and by definition [1–12]:

$$j^a_\nu = q^a_\kappa j^\kappa_\nu. \tag{23.53}$$

Therefore in general, Eq. (23.50) is:

$$\partial_\mu F^{a\mu}{}_{\nu} = j^{a}{}_{\nu}/\epsilon_0 \tag{23.54}$$

Proceed now with reference to the Proca equation in standard physics. The Proca equation in standard physics is defined in a Minkowski space-time by:

$$\partial_\mu F^{\mu\nu} = j^\nu/\epsilon_0 = -\left(\frac{mc}{\hbar}\right)^2 A^\nu \tag{23.55}$$

where the field tensor is:

$$F^{\mu\nu} = \partial^\nu A^\nu - \partial^\nu A^\mu. \tag{23.56}$$

Using Eq. (23.56) in Eq. (23.55) we obtain the Lorentz covariant Proca equation of standard physics:

$$\left(\Box + \left(\frac{mc}{\hbar}\right)^2\right) A^\nu = 0 \tag{23.57}$$

provided that:

$$\partial_\mu A^\mu = 0. \tag{23.58}$$

The latter "Lorenz gauge" result follows from the continuity equation of standard physics:

$$\partial_\mu j^\mu = 0 \tag{23.59}$$

and Eq. (23.55). It is well known that if the photon mass is not zero:

$$m \neq 0 \tag{23.60}$$

the Proca equation is not gauge invariant, and the "Lorenz gauge" is not arbitrary. This leads to the collapse of gauge theory if the photon mass is not identically zero. In general relativity on the other hand the photon mass is identically non-zero, as seen in the bending of light by gravity for example. Therefore in ECE theory the gauge principle is rejected and the potential is considered to be physically meaningful, as observed in such phenomena as ESR and NMR, and in the Aharonov Bohm effects [1–12].

The generally covariant Proca equation (23.46) is based on

$$D_\nu A^a_\mu = 0 \tag{23.61}$$

from which:

$$\partial^\mu \left(\partial_\mu A^a_\nu + \omega^a_{\mu b} A^b_\nu - \Gamma^\lambda_{\mu\nu} A^a_\lambda \right) = 0 \tag{23.62}$$

i.e. :

$$\Box A^a_\mu = \frac{j\nu^a}{\epsilon_0} = \partial^\mu (\Gamma^\lambda_{\mu b} A^a_\lambda - \omega^a_{\mu b} A^b_\nu). \tag{23.63}$$

This equation is the correctly covariant form of the standard physics equation:

$$\Box A^\mu = \frac{j^\mu}{\epsilon_0} \tag{23.64}$$

whose solutions are the Lienard Wiechert potentials [13]. In ECE theory:

$$F^{a\mu\nu} = \partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + \omega^{a\mu}_b A^{b\nu} - \omega^{a\nu}_b A^{b\mu} \tag{23.65}$$

Using this equation with Eq. (23.54) it is found that:

$$(\Box + kT) A^{a\nu} = \partial_\mu (\partial^\nu A^{a\mu} - \omega^{a\mu}_b A^{b\nu} + \omega^{a\nu}_b A^{b\mu}) = 0 \tag{23.66}$$

which is the generally covariant "Lorenz gauge" condition, but is now a rigorous geometrical requirement and not an arbitrary choice of gauge. If Eq. (23.66) is compared with the tetrad postulate:

$$D^\nu A^{a\mu} = \partial^\mu A^{a\nu} + \omega^{a\nu}_b A^{b\mu} - \Gamma^{\lambda\nu\mu} A^a_\lambda = 0 \tag{23.67}$$

it is found that Eq. (23.66) is true if:

$$\omega^{a\mu}_b A^{b\nu} = \Gamma^{\lambda\nu\mu} A^a_\lambda \tag{23.68}$$

which may be taken as the condition for Eq. (23.45).

The charge current density from Eq. (23.45) may be written as:

$$j^a_\mu = j^{(0)} q^a_\mu \tag{23.69}$$

where:

$$j^{(0)} = -\epsilon_0 k T A^{(0)}. \tag{23.70}$$

An example of Eq. (23.69) is:

$$j^{\kappa\nu} = j^{(0)} q^{\kappa\nu} \tag{23.71}$$

with:

$$D_\mu j^{\kappa\nu} = 0. \tag{23.72}$$

This is the continuity equation associated with Eq. (23.50). The covariant derivative in Eq. (23.44) may be replaced by the ordinary derivative if:

$$\omega^a{}_{\mu b} j^b_\nu = \Gamma^\lambda_{\mu\nu} j^a_\lambda \tag{23.73}$$

and under this condition, also that of Eq. (23.68), the continuity equation is:

$$\partial_\mu j^a_\nu = 0. \tag{23.74}$$

A special case of Eq. (23.74) is:

$$\partial_\mu j^{\kappa\mu} = 0 \tag{23.75}$$

and in ECE theory [1–12]:

$$\rho = \frac{1}{c} j^{00}, \boldsymbol{J} = j^{11}\boldsymbol{i} + j^{22}\boldsymbol{j} + j^{33}\boldsymbol{k} \tag{23.76}$$

so:

$$\frac{1}{c}\frac{\partial j^{00}}{\partial t} + \frac{\partial j^{11}}{\partial X} + \frac{\partial j^{22}}{\partial Y} + \frac{\partial j^{33}}{\partial Z} = 0 \tag{23.77}$$

which in vector notation is the continuity equation:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{J} = 0 \tag{23.78}$$

but now written in a space-time with torsion and curvature as required, and not in the flat or Minkowski equation of standard electrodynamics, which is Lorentz covariant but not generally covariant as required.

Acknowledgments

The British Government is thanked for a Civil List Pension and the staffs of AIAS and TGA for many interesting discussions.

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