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On the Symmetry of the Connection in Relativity and ECE Theory

by

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Abstract

It is shown that the connection in relativity theory must always be anti-symmetric and that relativity theory must be based on a non-zero space-time torsion as in the Einstein Cartan Evans (ECE) field theory. These results follow straightforwardly from the action of the commutator of covariant derivatives on any tensor in any space-time. The commutator is anti-symmetric by construction and generates the torsion and curvature tensors with anti-symmetric connection. The assumption of a symmetric connection as used in the standard model is irretrievably incorrect, meaning that the Einsteinian era in gravitational theory and cosmology is over. The ECE equations are geometrically correct and provide new cosmologies and technologies.

Keywords: Anti-symmetric connection, ECE theory, space-time torsion.

29.1 Introduction

The philosophy of relativity is that of objectivity, without which there is no natural or life science. The central thesis of relativity is that physics, or natural philosophy, is geometry. Physics is also causal in nature. These ideas go back to ancient times, but the most well known example of relativity is the theory developed from the late eighteen eighties to 1915 by many scientists. General relativity is the term used to describe the type of relativity suggested by Einstein from about 1906 onwards, when he developed the metric diag

$(-1, 1, 1, 1)$ of the Minkowski space-time into one which in general is space-time dependent. Tensor analysis had been proposed mainly by Ricci and Levi-Civita and published in about 1900. The idea of a space-time connection goes back via Christoffel and others to Riemann in the early nineteenth century, and the connection appears in the definition of the covariant derivative. An anti-symmetric commutator made up of covariant derivatives may act on any tensor [1] to produce torsion and curvature tensors. In ECE theory [2–10] the torsion has assumed central importance and its role in the natural and life sciences recognized. This is not the case in the Einsteinian era of about 1915 to 2003, when ECE theory first began to be developed. The reason is that the connection in the Einsteinian era was incorrectly assumed to be symmetric. In Section 2 it is shown straightforwardly that the connection must be anti-symmetric in both the torsion and curvature tensors, and that the torsion must always be non-zero in any theory of relativity. It follows that no physical inference may be drawn from the Einsteinian era in gravitation and cosmology. The proof of the anti-symmetry of the connection is simple, and it is not clear why a symmetric connection has been incorrectly used for over a century. The situation can be rescued by adopting the well known ECE equations of dynamics, electrodynamics and quantum mechanics [2–10]. In these equations a symmetric connection is not assumed, and the torsion is the centrally important concept.

Before proceeding to Section 2 a few historical remarks are given for ease of reference. It appears that the assumption of a symmetric connection was first made in about 1900 by Ricci and Levi-Civita for ease of calculation. This assumption should be researched further by historians of science, and it may emerge that Christoffel or Riemann assumed such a symmetry. It is well known that Einstein relied on advice by Grossman and that Einstein frequently read the 1900 paper, corresponding with Levi-Civita. The latter corrected some errors in Einstein's use of tensor analysis. In attempting to develop his 1915 field equation, Einstein made several errors and false turns before finally making a type of covariant Noether Theorem proportional to an identity of geometry known in the Einsteinian era as "the second Bianchi identity". The proportionality constant is the Einstein constant k as is well known. In the ECE era it has been recognized that this is not an identity because it omits space-time torsion. Similarly the "first Bianchi identity" of the Einsteinian era also omits torsion and is not correct. There is only one true identity, and it was first given by Cartan in the early twenties. In short hand notation (with indices left out for clarity), the Cartan Bianchi identity is:

$$D \wedge T := R \wedge q. \tag{29.1}$$

Here $D\wedge$ represents the covariant exterior derivative, defined by:

$$D\wedge \ := d\wedge \ + \omega\wedge \tag{29.2}$$

where $d\wedge$ is the exterior derivative and ω is the spin connection form of differential geometry. The symbol T denotes the torsion form, R denotes the curvature form, and q denotes the Cartan tetrad. In the ECE era it has also been recognized that there exists the Cartan Evans dual identity:

$$D \wedge \widetilde{T} := \widetilde{R} \wedge q \tag{29.3}$$

where $\widetilde{T}$ is the Hodge dual of the torsion form, and $\widetilde{R}$ is the Hodge dual of the curvature form. Neglect of torsion leads to a violation of the dual identity [2–10], thus ending the Einsteinian era.

The fundamental reason for this error is the assumption by Einstein that the connection is symmetric. It was known as early as 1918, in independent criticisms by Bauer and Schroedinger, [11], that there was something amiss with the Einstein field equation of 1915. However, the concept of torsion was not fully developed until about 1922, when Cartan and Maurer gave the first structure equation:

$$T = D \wedge q = d \wedge q + \omega \wedge q. \tag{29.4}$$

In tensor notation this is equivalent to [1–10]:

$$T^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\mu\nu} - \Gamma^{\kappa}_{\nu\mu} \tag{29.5}$$

where $T^{\kappa}_{\mu\nu}$ is the connection. If the latter is arbitrarily forced to be symmetric:

$$\Gamma^{\kappa}_{\mu\nu} \; = ? \; \Gamma^{\kappa}_{\nu\mu} \tag{29.6}$$

the torsion vanishes. In the Einsteinian era therefore the torsion was seen as a complication which was arbitrarily removed. It was shown in paper 93 ff. on www.aias.us that this removal of torsion leads to a violation of geometry, i.e. a violation of the Cartan Evans dual identity. In Section 29.2 this conclusion is reinforced using a straightforward demonstration that the connection must be anti-symmetric, both in the torsion and curvature tensors. It appears that the erroneous use of a symmetric connection was perpetrated uncritically throughout the Einsteinian era, either by lack of scholarship, by peer pressure, or by ignoring new developments. The result is that pseudo-scientific concepts such as big bang, black hole theory, dark matter theory and associated paraphenalia have proliferated, and are being taught as if they were mathematically correct. The intense international interest in ECE theory however has brought this era to an end and ECE theory has been adapted in the industrial sector. Its equations are able to provide cosmologies and technologies based on correct geometry. The use of correct geometry is of course a fundamental requirement of relativity theory.

29.2 Anti-Symmetry of the Connection

Define the covariant derivative of a vector V^ν of any dimension in any space-time as

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda} V^\lambda. \tag{29.7}$$

The commutator of the covariant derivative is anti-symmetric by construction:

$$[D_\mu,\ D_\nu] = -[D_\nu,\ D_\mu] \tag{29.8}$$

and may operate on any tensor in any space-time of any dimension. Let the commutator (29.8) operate on the vector V^ρ . The result is well known [1–10] to be:

$$[D_\mu,\ D_\nu]V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda_{\mu\nu} D_\lambda V^\rho \tag{29.9}$$

where $T^\lambda_{\mu\nu}$ is the torsion tensor:

$$T^\lambda_{\mu\nu} := \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \tag{29.10}$$

and

$$R^\rho{}_{\sigma\mu\nu} := \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \tag{29.11}$$

is the curvature tensor. These tensors are anti-symmetric in their last two indices by construction:

$$T^\lambda_{\mu\nu} = -T^\lambda_{\nu\mu} \tag{29.12}$$

$$R^\rho{}_{\sigma\mu\nu} = -R^\rho{}_{\sigma\nu\mu}. \tag{29.13}$$

If it is assumed that:

$$\mu = \nu \tag{29.14}$$

the commutator operator becomes the null operator, and the curvature and torsion tensors BOTH vanish. It is incorrect to assume that the torsion vanishes and that the curvature does not vanish when the assumption (29.14) is made. This is an error that was perpetrated throughout the Einsteinian era for over a century. The torsion is defined as:

$$[D_\mu,\ D_\nu]V^\rho = -T^\lambda_{\mu\nu} D_\lambda V^\rho + \ldots := -\left(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}\right) D_\lambda V^\rho + \ldots \tag{29.15}$$

and is anti-symmetric, and it follows that:

$$\Gamma^{\lambda}_{\mu\nu} = -\Gamma^{\lambda}_{\nu\mu}. \tag{29.16}$$

The connection in the theory of relativity must be anti-symmetric whenever it occurs, Q.E.D.

It is important to note that the connection is also anti-symmetric in the curvature tensor. There is only one connection symmetry, which is generated directly by the commutator acting on a vector or tensor. The commutator of covariant derivatives arises by the well known fact [1] that the covariant derivative of any tensor in any space-time in a given direction measures how much the tensor changes relative to what it would have been if it had been parallel transported. The covariant derivative of a tensor in the direction along which it is parallel transported is zero [1]. The commutator of covariant derivatives measures the difference between parallel transporting the tensor clockwise and anti-clockwise. In a flat space-time with no connection such a commutator is zero, i.e. :

$$[\partial_\mu,\ \partial_\nu] = -[\partial_\nu, \partial_\mu] = 0. \tag{29.17}$$

For the arbitrary tensor in any space-time, the commutator operates on the tensor to produce the torsion and curvatures as follows:

$$\begin{aligned}[D_\rho, D_\sigma]X^{\mu_1\ldots\mu_k}_{\nu_1\ldots\nu_l} &= -T^{\lambda}_{\rho\sigma} D_\lambda X^{\mu_1\ldots\mu_k}_{\nu_1\ldots\nu_l} + R^{\mu_1}{}_{\lambda\rho\sigma} X^{\lambda\mu_2\ldots\mu_k}_{\nu_1\ldots\nu_l}\\ &\quad + R^{\mu_2}_{\lambda\rho\sigma} X^{\mu_1\lambda\ldots\mu_k}_{\nu_1\ldots\nu_l} + \ldots - R^{\lambda}_{\nu_1\rho\sigma} X^{\mu_1\ldots\mu_k}_{\lambda\nu_2\ldots\nu_l}\\ &\quad - R^{\lambda}_{\nu_2\rho\sigma} X^{\mu_1\ldots\mu_k}_{\nu_1\lambda\ldots\nu_l} - \ldots \end{aligned} \tag{29.18}$$

So it is always incorrect to use a null torsion, because that means a null commutator and a null curvature (i.e. removing the torsion removes all the information needed for relativity theory).

For example, let:

$$\mu = 0,\ \nu = 1,\ \rho = 2 \tag{29.19}$$

in Eq. (29.9). Then:

$$[D_0, D_1]V^2 = R^2{}_{\sigma 01} V^\sigma - T^\lambda{}_{01}\, D_\lambda V^2. \tag{29.20}$$

It follows that:

$$T^\lambda{}_{01} = \Gamma^\lambda{}_{01} - \Gamma^\lambda{}_{10} = -T^\lambda{}_{10} \tag{29.21}$$

i.e.

$$\Gamma^{\lambda}{}_{01} = -\Gamma^{\lambda}{}_{10}. \tag{29.22}$$

The curvature tensor is:

$$R^{2}{}_{\sigma 01} = -R^{2}{}_{\sigma 10} \tag{29.23}$$

For example, when:

$$\sigma = 0 \tag{29.24}$$

the curvature tensor is:

$$R^{2}{}_{001} = \partial_0 \Gamma^2_{10} - \partial_1 \Gamma^2_{00} + \Gamma^2_{0\lambda}\Gamma^{\lambda}_{10} - \Gamma^2_{1\lambda}\Gamma^{\lambda}_{00} \tag{29.25}$$

where there is summation over repeated λ indices. By anti-symmetry in μ and ν:

$$\partial_0 \Gamma^2_{10} = -\partial_1 \Gamma^2_{00} = -\partial_0 \Gamma^2_{01} = \partial_1 \Gamma^2_{00} \tag{29.26}$$

i.e.

$$\Gamma^2_{00} = -\Gamma^2_{00} = 0, \tag{29.27}$$
$$\Gamma^2_{01} = -\Gamma^2_{10} \neq 0. \tag{29.28}$$

The symmetric connections are zero, and the other connections are anti-symmetric Q.E.D.

Similarly we may systematically consider all other μ and ν indices:

$$\left.\begin{array}{l} \mu = 0,\ \nu \quad = 2;\ \mu = 1,\ \nu \quad = 3; \\ \mu = 0,\ \nu \quad = 3;\ \mu = 0,\ \nu \quad = 2; \end{array}\right\} \tag{29.29}$$

to find that:

$$\Gamma^{\lambda}_{00} = \Gamma^{\lambda}_{11} = \Gamma^{\lambda}_{22} = \Gamma^{\lambda}_{33} = 0 \tag{29.30}$$

for all λ. It is seen immediately that any metric of the Einsteinian era that does not obey the symmetry of Eq. (29.30) is incorrect geometrically, and cannot produce any meaningful results in physics. It is important to realize that all the commonly taught metrics of the Einsteinian era are incorrectly based on a symmetric connection, for example the mis-named [2–10] Schwarzschild metric, The Robertson Walker metric of big bang, all black hole metrics, and so on. So the Einsteinian era must be discarded as a matter of urgency and replaced by the ECE era.

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