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Spin Connection Resonance in the Bedini Machine

by

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Abstract

Spin connection resonance (SCR) is used to explain theoretically why devices in electrical engineering can use the properties of space-time to induce voltage. Einstein Cartan Evans (ECE) theory has shown why classical electrodynamics is a theory of general relativity in which covariant derivatives are used with the spin connection playing a central role. These concepts are applied to a device known as the Bedini machine.

Keywords: Spin connection resonance, electrodynamics in general relativity, Einstein Cartan Evans theory, Bedini machine.

1.1 Introduction

Recently [1–10] the Einstein Cartan Evans (ECE) field theory has been generally accepted as the first successful unified field theory on the classical and quantum levels. It shows that classical electrodynamics is a theory of general relativity, not of special relativity. In ECE theory the spin connection plays a central role in the structure of the laws of electrodynamics and in the way the electric and magnetic fields are related to the scalar and vector potentials. The ECE equations of classical electrodynamics allow the existence of resonances in potential which can be used to extract electric power from the structure of space-time. This structure is not the vacuum, the latter in relativity theory is a universe devoid of all curvature and torsion. The resonance phenomenon induced by these equations is known as spin connection resonance (SCR). In this paper it is applied to a device known as the Bedini machine [11], which has been patented and which has been shown to be experimentally reproducible and repeatable. In section 1.2 the equations of classical electrodynamics are given in ECE theory. These are given in the vector notation used by engineers, and the reduction of the original differential form equations of ECE theory to the vector equations is given in technical appendices. In section 3 models of the Bedini device are developed, in section 1.4 the occurrence of resonances is identified and graphed using computer algebra to check the derivations.

1.2 The Equations of Classical Electrodynamics in General Relativity

All electromagnetic devices of engineering are governed by these equations, which are the generally covariant form of classical electrodynamics. Each device must be considered separately, and the general equations applied systematically to each device. The electric field in ECE theory is defined in general by the scalar and vector potentials and by the scalar and vector components of the spin connection:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - c\boldsymbol{\nabla}\phi - c\omega^0\mathbf{A} + c\phi\boldsymbol{\omega}. \tag{1.1}$$

Here ϕ is the scalar potential, $\mathbf{A}$ is the vector potential, ω^0 is the scalar part of the spin connection and $\boldsymbol{\omega}$ is the vector part of the spin connection (see technical appendices). The Coulomb law in ECE theory [1–10] is

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} := c\mu_0 J^0 \tag{1.2}$$

where ϵ_0 is the vacuum permittivity and ρ is the scalar part of the inhomogeneous charge current density of ECE theory. The magnetic field in ECE theory is defined by:

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A} \tag{1.3}$$

and the Gauss law of magnetism is:

$$\boldsymbol{\nabla} \cdot \mathbf{B} = \mu_0 j^0 \tag{1.4}$$

where j^0 is the scalar part of the homogeneous charge current density. The Faraday law of induction in ECE theory is:

$$\boldsymbol{\nabla} \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = c\mu_0 \mathbf{j} \tag{1.5}$$

where $\mathbf{j}$ is the vector part of the homogeneous charge current density and the Ampère Maxwell law is:

$$\boldsymbol{\nabla} \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \tag{1.6}$$

where $\mathbf{J}$ is the vector part of the inhomogeneous charge current density.

The explanation of various devices that are reproducible and repeatable depends on the systematic application of these general equations. It has been shown [1–10] that they are resonance equations in general, so that a small driving term can produce a very large amplification of space-time effects through the inter-mediacy of the spin connection. Devices which find no explanation in the standard model can be explained in this way. For example, we consider the Bedini device [11] as one in which an electric pulse produced by the rate of change of a magnetic field is induced in a generator. The electric field pulse produces a pulse of electrons in a battery [11] as controlled by Eqs. (1.1) and (1.2), from which:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla}\phi + \frac{1}{c}\frac{\partial}{\partial t}(\boldsymbol{\nabla} \cdot \mathbf{A}) + \boldsymbol{\nabla} \cdot \left(\omega^0 \mathbf{A}\right) - \boldsymbol{\nabla} \cdot (\phi \boldsymbol{\omega}) = -\mu_0 J^0. \tag{1.7}$$

This equation produces resonances in two ways, each of which gives a resonance equation.

1. If it is assumed that the origin of $\mathbf{E}$ is purely due to ϕ, we obtain the basic resonance equations of paper 63 and 92 of the ECE series [1–10].

2. If it is assumed that the origin of $\mathbf{E}$ is purely magnetic, and that the scalar potential is zero, we have:

$$\frac{1}{c}\frac{\partial}{\partial t}(\boldsymbol{\nabla} \cdot \mathbf{A}) + \boldsymbol{\nabla} \cdot \left(\omega^0 \mathbf{A}\right) = -\mu_0 J^0. \tag{1.8}$$

i.e.

$$\boldsymbol{\nabla} \cdot \left(\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \omega^0 \mathbf{A}\right) = -\mu_0 J^0. \tag{1.9}$$

which can be integrated to give a resonance equation. It is also possible to produce a time dependent resonance equation from Eqs. (1.1) and (1.6). The Ampère Maxwell law (1.6) is considered to produce a driving term:

$$\frac{\partial \mathbf{E}}{\partial t} = c^2 \left(\boldsymbol{\nabla} \times \mathbf{B} - \mu_0 \mathbf{J} \right)_{driving} = -\boldsymbol{\nabla} \frac{\partial \phi}{\partial t} - \frac{\partial^2 \mathbf{A}}{\partial t^2} - \frac{\partial}{\partial t} \left(c\omega^0 \mathbf{A} \right) + \frac{\partial}{\partial t} \left(c\phi \boldsymbol{\omega} \right) \tag{1.10}$$

so that the most general resonance equation of time-dependent type is:

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} + c\frac{\partial \omega^0}{\partial t} \mathbf{A} + c\omega^0 \frac{\partial \mathbf{A}}{\partial t} = c\frac{\partial \phi}{\partial t} \boldsymbol{\omega} + c\phi \frac{\partial \boldsymbol{\omega}}{\partial t} + c^2 \mu_0 \mathbf{J} - \boldsymbol{\nabla} \frac{\partial \phi}{\partial t} - c^2 \boldsymbol{\nabla} \times \mathbf{B}. \tag{1.11}$$

If there is no charge and current density this equation reduces to:

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} + c\omega^0 \frac{\partial \mathbf{A}}{\partial t} + c\frac{\partial \omega^0}{\partial t} \mathbf{A} = -c^2 \left(\boldsymbol{\nabla} \times \mathbf{B} \right)_{driving}. \tag{1.12}$$

There is resonance in **A** under the following conditions:

1. the scalar part, ω^0, of the spin connection is non-zero,
2. the time derivative, $\frac{\partial \omega^0}{\partial t}$, is non-zero,
3. the curl $\boldsymbol{\nabla} \times \mathbf{B}$ is non-zero and also time dependent.

When investigating various claims such as the Bedini machine it is necessary to use equations such as this, which show for example that the magnetic field in the design must be both space and time dependent, and produced by a device that satisfies these requirements. That is an example of a design prediction of ECE theory in engineering.

In addition to Eq. (1.6) there exists the Coulomb law (1.2), which is the resonance equation [1–10]: [1–10]

$$\boldsymbol{\nabla} \cdot \left(c\phi \boldsymbol{\omega} - \boldsymbol{\nabla} \phi - \frac{\partial \mathbf{A}}{\partial t} - c\omega^0 \mathbf{A} \right) = \frac{\rho}{\epsilon_0}. \tag{1.13}$$

In the absence of charge this equation reduces to:

$$\boldsymbol{\nabla} \cdot \left(\frac{\partial \mathbf{A}}{\partial t} + c\omega^0 \mathbf{A} \right) = 0 \tag{1.14}$$

so ω^0 may be eliminated between equations (1.12) and (1.14). Eq. (1.14) is:

$$\boldsymbol{\nabla} \cdot \frac{\partial \mathbf{A}}{\partial t} = -c\left(\mathbf{A} \cdot \boldsymbol{\nabla}\omega^0 + \omega^0 \boldsymbol{\nabla} \cdot \mathbf{A}\right). \tag{1.15}$$

Therefore ω^0 is governed by Eqs. (1.12) and (1.15) which must be solved simultaneously. The latter equation can be integrated with the divergence theorem [12]. For any well behaved vector field $\mathbf{V}(\mathbf{r})$ defined with a volume surrounded by a closed surface S:

$$\oint_S \mathbf{V} \cdot \mathbf{n}\, da = \int_V \boldsymbol{\nabla} \cdot \mathbf{V}\, d^3r. \tag{1.16}$$

Thus for the Coulomb law 1.12:

$$\int_V \left(\boldsymbol{\nabla} \cdot \mathbf{E} - \frac{\rho}{\epsilon_0}\right) d^3r = 0 \tag{1.17}$$

i.e.

$$\oint_S \mathbf{E} \cdot \mathbf{n}\, da = \frac{1}{\epsilon_0} \int_V \rho(r)\, d^3r. \tag{1.18}$$

So the integration of Eq. (1.14) is:

$$\oint_S \left(\frac{\partial \mathbf{A}}{\partial t} + c\omega^0 \mathbf{A}\right) \cdot \mathbf{n}\, da = 0 \tag{1.19}$$

i.e.

$$\oint_S \frac{\partial \mathbf{A}}{\partial t} \cdot \mathbf{n}\, da = -c \oint_S \omega^0 \mathbf{A} \cdot \mathbf{n}\, da. \tag{1.20}$$

Eq. (1.20) is a relation between ω^0 and $\mathbf{A}$. The correct way of solving (1.12) is simultaneously with (1.18). This can be carried out numerically for various models of $\boldsymbol{\nabla} \times \mathbf{B}$ produced by various devices. It can be seen that ω^0 can be eliminated and that Eq. (1.12) reduces to an undamped oscillator [1–10] because $\frac{\partial \mathbf{A}}{\partial t}$ is eliminated in favour of $\mathbf{A}$. So in this example $\mathbf{A}$ can be amplified to INFINITY for various models of $\boldsymbol{\nabla} \cdot \mathbf{B}$ acting as a driving force. There is no need to model ω^0 because it can be expressed in terms of $\mathbf{A}$.

1.3 Systematic Evaluation of Equations for the Bedini Machine

If no scalar potential is present, the ECE field equations (1.1–1.6) in the base manifold take the simple form:

$$\boldsymbol{\nabla} \times \mathbf{E} + \dot{\mathbf{B}} = 0 \tag{1.21}$$

$$\boldsymbol{\nabla} \times \mathbf{B} - \frac{1}{c^2}\dot{\mathbf{E}} = 0 \tag{1.22}$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{1.23}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 0 \tag{1.24}$$

with the definition equations

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A} \tag{1.25}$$

$$\mathbf{E} = -\dot{\mathbf{A}} - c\,\omega^0 \mathbf{A}. \tag{1.26}$$

Here the dot denotes the time derivative, $\mathbf{A}$ is the vector potential, $\boldsymbol{\omega}$ the vector spin connection and ω^0 the scalar spin connection, both in units of 1/m. It is more convenient to transform the scalar spin connection to a time frequency:

$$\omega_0 := c\,\omega^0. \tag{1.27}$$

Eqs. (1.21-1.24) represent a system of eight equations and by the right-hand side of Eqs. (1.25-1.26) seven variables are defined. In the most general case the scalar potential Φ is the eights variable so that (1.21)–(1.24) can be solved uniquely. Here we restrict consideration to the case without charges and therefore without a scalar potential.

In classical electrodynamics we have the same equations, but without the spin connection. This leads to an inconsistency for solving the equations. Sometimes solely the fields E and B are considered, then only the equations (1.21)–(1.22) can be used. The Gauss and Coulomb law are tried to be handled as "constraints", but this leads to an over-determined equation system. In other cases (when charges and currents are present) the potentials $\mathbf{A}$ and

Φ are taken as variables. Then only the Eqs. (1.22) and (1.24) can be used, the other two are homogeneous and lead to the trivial solution $\mathbf{A} = 0$. In contrast, ECE theory presents a perfectly well-defined situation with eight equations and eight variables.

There are basically two methods to combine these equations to obtain resonances for particular cases:

1. use (1.21) and (1.22) completely to define driving terms, use (1.25) and (1.26) as basis for resonance solutions,
2. use the terms $\dot{\mathbf{B}}, \dot{\mathbf{E}}$ in (1.21), (1.22) as driving terms, insert curl of (1.25) and (1.26) into (1.21) and (1.22) and use these equations for resonance solutions.

We will see that both methods are not applicable in all possible cases.

In addition to both methods, we have to use one of the equations (1.3), (1.4). The actual choice depends on the case if $\boldsymbol{\omega}$ or ω_0 occurs in the equations (1.21) and (1.22). In the following we work out the distinguished cases 1 and 2 each for Eq. (1.21) (called sub-case a) and Eq. (1.22) (called sub-case b).

1a: Faraday Law as driving term, B field resonance

By definition we have

$$(\boldsymbol{\nabla} \times \mathbf{E})_{driving} = -(\dot{\mathbf{B}})_{driving} \tag{1.28}$$

Inserting the time derivative of (1.25) into (1.28):

$$\boldsymbol{\nabla} \times \dot{\mathbf{A}} - \dot{\boldsymbol{\omega}} \times \mathbf{A} - \boldsymbol{\omega} \times \dot{\mathbf{A}} = (\dot{\mathbf{B}})_{driving} = -(\boldsymbol{\nabla} \times \mathbf{E})_{driving} \tag{1.29}$$

In order to obtain resonance a differential equation of second order in time is required, therefore we take a further time derivative:

$$\boldsymbol{\nabla} \times \ddot{\mathbf{A}} - \ddot{\boldsymbol{\omega}} \times \mathbf{A} - 2\dot{\boldsymbol{\omega}} \times \dot{\mathbf{A}} - \boldsymbol{\omega} \times \ddot{\mathbf{A}} = (\ddot{\mathbf{B}})_{driving} \tag{1.30}$$

This is a resonance equation in $\mathbf{A}$ (for constant $\boldsymbol{\omega}$) as well as in $\boldsymbol{\omega}$ (for constant $\mathbf{A}$). The spin connection can be obtained from simultaneously solving Eq. (1.23). This could be sufficient, if not all components of $\mathbf{A}$ or $\boldsymbol{\omega}$ are different from zero. In the most general case further equations have to be added.

1b: Ampère-Maxwell Law as driving term, E field resonance

In analogy to case 1a we obtain from (1.22):

$$(\boldsymbol{\nabla} \times \mathbf{B})_{driving} = \frac{1}{c^2}(\dot{\mathbf{E}})_{driving} \tag{1.31}$$

and by applying (1.26):

$$\ddot{\mathbf{A}} + \dot{\omega_{\mathbf{0}}}\mathbf{A} + \omega_{\mathbf{0}}\dot{\mathbf{A}} = -(\dot{\mathbf{E}})_{driving} \tag{1.32}$$

This is an equation for a damped resonance for $\omega_0 > 0$. The spin connection can be determined by combining (1.32) with (1.24).

2a: B field definition as driving term, Faraday Law as resonance equation

Taking the magnetic field in (1.21) as driving term gives

$$\boldsymbol{\nabla} \times \mathbf{E} = -(\dot{\mathbf{B}})_{driving}. \tag{1.33}$$

Inserting (1.26) into (1.33):

$$\boldsymbol{\nabla} \times \dot{\mathbf{A}} + \boldsymbol{\nabla} \times (\omega_0 \mathbf{A}) = (\dot{\mathbf{B}})_{driving} \tag{1.34}$$

or after taking a further time derivative:

$$\boldsymbol{\nabla} \times \ddot{\mathbf{A}} + \boldsymbol{\nabla} \times (\dot{\omega_0} \mathbf{A}) + \boldsymbol{\nabla} \times (\omega_0 \dot{\mathbf{A}}) = (\ddot{\mathbf{B}})_{driving} \tag{1.35}$$

which is the equivalent of (1.30) with the other type of spin connection.

2b: E field definition as driving term, Ampère-Maxwell Law as resonance equation

Starting with Eq. (1.22) we obtain

$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{1}{c^2} (\dot{\mathbf{E}})_{driving} \tag{1.36}$$

and with (1.25):

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{A} - \boldsymbol{\nabla} \times \boldsymbol{\omega} \times \mathbf{A} = \frac{1}{c^2} (\dot{\mathbf{E}})_{driving} \tag{1.37}$$

or

$$\begin{aligned} \boldsymbol{\nabla} (\boldsymbol{\nabla} \cdot \mathbf{A}) - \boldsymbol{\nabla}^2 \mathbf{A} - \boldsymbol{\omega} (\boldsymbol{\nabla} \cdot \mathbf{A}) + \mathbf{A} (\boldsymbol{\nabla} \cdot \boldsymbol{\omega}) \\ - (\mathbf{A} \cdot \boldsymbol{\nabla}) \boldsymbol{\omega} + (\boldsymbol{\omega} \cdot \boldsymbol{\nabla}) \mathbf{A} = \frac{1}{c^2} (\dot{\mathbf{E}})_{driving}. \end{aligned} \tag{1.38}$$

This is a resonance equation for the space coordinates of $\mathbf{A}$. Investigating time-dependent resonances requires a twofold additional time derivation which makes this equation impractible.

1.4 Detailed Investigation of the Bedini Machine

1.4.1 Description of the Bedini machine

In the book "Free Energy Generation" [13], see also [14], Bedini explains his battery charging device of 1984. He presents some variants of the machine constructed within 20 years. The basic design has remained the same. The patented Bedini machine inventor claims that his machines are able to extract energy from the surrounding space in the form of radiant energy. The authors here attempt to show that the energy produced by these machines is the result of disturbing the local space-time unit volume, creating a resonance effect, which allows energy to flow out of the local unit volume, in the form of asymmetric electromagnetic wave forms into a rectifier circuit, where it can be then sent to a storage device. The mathematical expressions developed are based on ECE field theory. The resulting expressions will allow electrical designers to produce productive circuits, based on this math, since the inventor has not furnished an adequate explanation of the machines' operation. One of the authors has replicated two of the Bedini machines successfully, and others have had success building and operating the machines.

The Bedini machine has several distinct elements (see Fig. 1.1). The input power supply, which can be a battery or a rectified power supply from an external supply, provides the transducer coil and trigger circuitry with energy to pulse the unit volume through the trigger winding. The magnet induces an asymmetric pulse into the transducer core, which induces an e-m pulse in

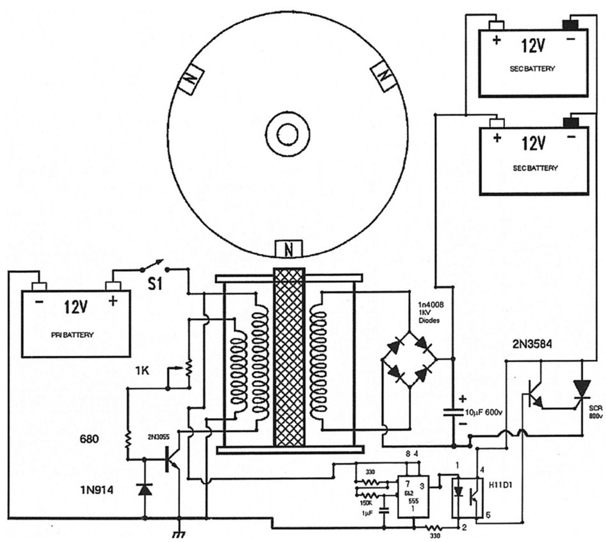

Fig. 1.1. Bedini machine (from [13], p. 47).

the trigger winding, power winding, and generator winding. The trigger pulse causes power to flow into the power winding, giving a boot to the magnet as it goes on by, thereby powering the rotor to the next magnet. The transducer pulse from the coils flows into the unit volume, upsetting the local field, and the resulting return energy is rectified after flowing through the generator winding.

All of these windings of the transducer are separate coils, wound concentrically on a spool, which has a core consisting of mild iron rods, typically 1/16" in diameter. Once the rotor is spun manually, and the power source and storage device are connected, the rotor will accelerate to a select speed determined by a tuning rheostat, and the machine will maintain that speed indefinitely, charging the storage device, using less energy to run than it stores, thereby achieving over unity in its operation. One of the authors has determined that the machine operates more efficiently at 24 Volts DC, than at 12 Volts DC, and the machine operates at almost twice the rpm as compared to 12 Volt operation.

Mr. Bedini has built several demonstrator machines in the kilowatt size, however, one of the authors' machines is only capable of 10-15 watts of output, but this size is adequate to provide meaningful test results. One of the authors is presently building a larger machine to replicate Mr. Bedini's claims of higher power outputs. In addition to the rotor style machines, the inventor has shown solid state designs, which the authors have not replicated yet, but others have, with limited output success. A company using Mr. Bedini's designs is presently marketing a line of battery chargers claiming to use radiant energy to enhance battery life and longevity.

1.4.2 Models of the Bedini machine

Charging of a battery means a flow of ions in the electrolyte in direction reverse to the discharging current. According to the explanations of Bedini, the battery charging process is evoked by high frequency pulses. This type of charging is completely different from the conventional DC charging process where the ion transport is effected by applying a DC voltage. Bedini points out that the high frequency / high voltage oscillations initiate a coupling to spacetime so that the ions resonate and move in the direction opposite to the discharge current. No significant conventional recharging energy has to be expended in this process.

Key of understanding the process is the mechanism of tapping the vacuum background energy, i.e. to evoke a resonant coupling to the spacetime background. As has been shown by ECE field theory [papers 63, 92], a coupling to spacetime background can be achieved by a resonance circuit. Such an original circuit from Bedini is shown in Fig. 1.1.

The key component of the Bedini machine is the trifilar wound coil which acts as a combined transmitter-receiver transducer. In the following we use the working hypothesis that the spacetime coupling takes place by means of this coil. Therefore we need not consider the complex electro-chemical processes in the battery, and an electrical potential Φ can be omittet as already done in Equations (1.21–1.26).

Since we have to model the fields of a cylindrical coil, we choose cylinder coordinates (r, φ, z) for convenience with unit vectors $\mathbf{e}_r, \mathbf{e}_\varphi, \mathbf{e}_Z$ as shown in Fig. 1.2. Inside a conventional coil the magnetic field is parallel to the z direction and the vector potential is tangential to circles around **B**. We assume that the magnetic field maintains its direction in case of resonance. Then the vector spin connection has to lie in the r-φ plane as well as **A**. In the simplest case it is perpendicular to **A**.

Whether type a or b of setcion 3.1 should be chosen for modeling the device, depends on the type of excitation mechanism. Inside the transducer we have

$$(\boldsymbol{\nabla} \times \mathbf{B})_{driving} \approx 0, \tag{1.39}$$

during the pulsing phase. In the preceding phase when a rotor-mounted magnet approaches the transducer, the moving magnet induces a non-symmetric magnetic field within the iron core of the transducer. Therefore condition (1.39) is not always valid. To obtain a viable model, we make the following additional simplifying assumptions. The B field is in z direction:

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ B_Z \end{pmatrix}. \tag{1.40}$$

The vector potential in classical electrodynamics then has only a φ and r component:

$$\mathbf{A} = \begin{pmatrix} A_r \\ A_\varphi \\ 0 \end{pmatrix}. \tag{1.41}$$

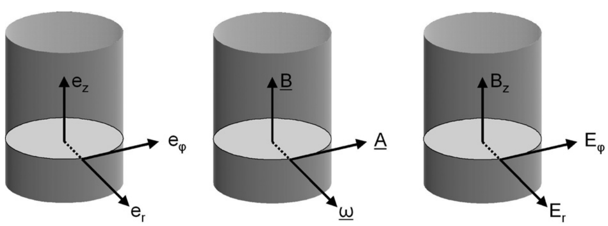

Fig. 1.2. Cylindrical coordinate system and fields in a coil.

Since the spin connection $\boldsymbol{\omega}$ cannot be in parallel to **A** and **B** according to Eq. (1.25), we choose

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_r \\ \omega_\varphi \\ 0 \end{pmatrix} . \tag{1.42}$$

Due to the rotational symmetry of the device, there cannot be a φ dependence of the fields. In total we have the functional dependencies

$$\begin{aligned} B_Z &= B_Z(r,t) \\ A_r &= A_r(r,t) \\ A_\varphi &= A_\varphi(r,t) \\ \omega_r &= \omega_r(r,t) \\ \omega_\varphi &= \omega_\varphi(r,t) \\ \omega_0 &= \omega_0(r,t) \end{aligned} \tag{1.43}$$

With (1.40–1.42) we have (using the differential operators in cylinder coordinates)

$$\boldsymbol{\nabla} \times \mathbf{A} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi) - \frac{\partial A_r}{\partial \varphi} \end{pmatrix} , \tag{1.44}$$

$$\boldsymbol{\omega} \times \mathbf{A} = \begin{pmatrix} 0 \\ 0 \\ \omega_r A_\varphi - \omega_\varphi A_r \end{pmatrix} . \tag{1.45}$$

The divergence of a vector **V** is in cylindric coordinates:

$$\boldsymbol{\nabla} \cdot \mathbf{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial}{\partial \varphi}(V_\varphi) + \frac{\partial}{\partial z}(V_Z) . \tag{1.46}$$

We are now ready to apply the methods 1a, 1b, 2a. Starting with 1a, we obtain from Eq. (1.30) with the special form of **A** and $\boldsymbol{\omega}$ (1.40-1.45):

$$\begin{aligned} &\frac{1}{r}\frac{\partial}{\partial r}(r\ddot{A}_\varphi) - \frac{\partial \ddot{A}_r}{\partial \varphi} - \ddot{\omega}_r A_\varphi + \ddot{\omega}_\varphi A_r - 2(\dot{\omega}_r \dot{A}_\varphi - \dot{\omega}_\varphi \dot{A}_r) \\ &\quad - \omega_r \ddot{A}_\varphi + \omega_\varphi \ddot{A}_r = (\ddot{B}_Z)_{driving} \end{aligned} \tag{1.47}$$

From (1.23) follows

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A}) = 0 \tag{1.48}$$

or

$$\frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi) - \omega_r A_\varphi + \omega_\varphi A_r\right) = 0. \tag{1.49}$$

This equation is trivially fulfilled. Even if we additionally assume $A_r = \omega_\varphi = 0$ we have one equation with two unknowns A_φ and ω_r so that no unique solution is obtained.

Considering the alternative case 2a we get from Eq. (1.35):

$$\begin{aligned}&\frac{1}{r}\frac{\partial}{\partial r}\left(r\ddot{A}_\varphi + r\dot{\omega}_0 A_\varphi + r\omega_0 \dot{A}_\varphi\right)\\ &\quad - \frac{\partial}{\partial \varphi}\left(\ddot{A}_r + \dot{\omega}_0 A_r + \omega_0 \dot{A}_r\right) = (\ddot{B}_Z)_{driving}.\end{aligned} \tag{1.50}$$

From Eq. (1.24) follows

$$\boldsymbol{\nabla}\cdot\left(-\dot{\mathbf{A}} - \omega_0 \mathbf{A}\right) = 0 \tag{1.51}$$

or

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\dot{A}_r + r\omega_0 A_r\right) + \frac{1}{r}\frac{\partial}{\partial \varphi}\left(\dot{A}_\varphi + \omega_0 A_\varphi\right) = 0 \tag{1.52}$$

According to (1.43) Eqs. (1.50) and (1.52) can be simplified to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\ddot{A}_\varphi + r\dot{\omega}_0 A_\varphi + r\omega_0 \dot{A}_\varphi\right) = (\ddot{B}_Z)_{driving} \tag{1.53}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\dot{A}_r + r\omega_0 A_r\right) = 0 \tag{1.54}$$

These are two equations for three unknowns and not unique as before.

Finally we apply case 1b. This is different from the previous ones since the electrical field is considered to be the driving term. From Eq. (1.32) we obtain the two equations

$$\ddot{A}_r + \dot{\omega}_0 A_r + \omega_0 \dot{A}_r = -(\dot{E}_r)_{driving} \tag{1.55}$$

$$\ddot{A}_\varphi + \dot{\omega}_0 A_\varphi + \omega_0 \dot{A}_\varphi = -(\dot{E}_\varphi)_{driving} \tag{1.56}$$

and from Eq. (1.24):

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\dot{A}_r + r\omega_0 A_r\right) = 0 \tag{1.57}$$

or

$$\dot{A}_r + \left(\omega_0 + r\frac{\partial \omega_0}{\partial r}\right) A_r + r\frac{\partial \dot{A}_r}{\partial r} + r\omega_0\frac{\partial A_r}{\partial r} = 0. \tag{1.58}$$

We see that the spin connection is coupled to the radial part of the vector potential. This indicates that the unit volume interacting with spacetime may be somewhat extended beyond the transducer. The occurrence of $\dot{\mathbf{E}}$ implies a non-vanishing curl of $\mathbf{B}$ according to (1.31).

The result (1.57) can further be simplified by applying the divergence theorem as explained at the end of section 1.2. The surface integral of Eq. (1.19) is to be taken over the cylinder surface of the model. The parts over the circular areas cancel out due to the assumed symmetry in z direction. For the cylindrical part the φ component of the vector potential is perpedicular to the surface normal and does not contribute anything. The only contributing part is the radial component:

$$\int_V \boldsymbol{\nabla}\cdot\mathbf{A} d^3r = \int_S (\dot{A}_r + \omega_0 A_r) da = 0 \tag{1.59}$$

Since A_r and ω_0 are independent on the individual surface points, the integral can be evaluated trivially and results in

$$\omega_0 = -\frac{\dot{A}_r}{A_r}. \tag{1.60}$$

The equations (1.55, 1.56, 1.60) are three equations for three unknowns A_r, A_φ, ω_0. This set of equations has to be solved numerically to provide guidance to designers in sizing the transducer, designing the trigger and power circuits, and predicting power outputs. Since the unit volume is surrounded by the large number of unit volumes in a spherical configuration(the rest of space), the theoretical power input to the machine transducer is limited by its conductor size and impedance seen looking into the transducer from the space side.

This paper discusses the Bedini machine in particular, but the concept of a transducer acting as a transmitter-receiver for power extraction from the surrounding space should be applicable to other machine designs also.

The inventor has put forth a hypothesis as to how his machines operate, which is non-conventional in its premise. The authors here suggest that the latest ECE theory will provide a rational explanation to the machines' operation, using conventional mathematical notation, and recognized physical theory.

1.4.3 Resonance behaviour of the vector potential

Without doing any numerical calculations, we can demonstrate that resonance solutions for Eqs. (1.55, 1.56) exist. We assume a harmonic time dependence

$$A_r = A_1(r)\sin(\omega t) \tag{1.61}$$

$$\omega_0 = \omega_1(r)\sin(\omega t) \tag{1.62}$$

with a frequency ω (not to be confused with the spin connection ω_0) and radius dependent functions A_1 and ω_1. Let's further denote the right-hand side of (1.55) by f_1, then this equation can be written:

$$2A_1\omega_1\omega\cos(\omega t)\sin(\omega t) - A_1\omega^2\sin(\omega t) = -f_1. \tag{1.63}$$

For $\omega t = \pi/4$ we have

$$\sin(\omega t) = \cos(\omega t) = \frac{1}{\sqrt{2}} \tag{1.64}$$

and (1.63) simplifies to

$$A_1\left(\omega_1\omega - \frac{\omega^2}{\sqrt{2}}\right) = -f_1 \tag{1.65}$$

which gives the solution for A_r:

$$A_1 = \frac{f_1}{\frac{\omega^2}{\sqrt{2}} - \omega_1\omega}. \tag{1.66}$$

There is resonance when the denominator approaches zero, i.e.

$$\omega_1 = \frac{\omega}{\sqrt{2}}. \tag{1.67}$$

If we had defined (1.61, 1.62) by the cosine function, we had got the same value for ω_0 with a negative sign. From this simple model we learn that the spin connection can assume both signs (in contrast to a real frequency) and show up sharp resonances for certain phases of the time period. This is in accordance with the experimental findings. From the original Eqs. (1.55, 1.56) we would expect a damped oscillation, but these equations are non-linear and therefore some unexpected results can occur, in this case an undamped oscillation.

1.4.4 Computation of the energy balance

The theory should provide a method to estimate the energy balance of the Bedini machine. According to the previous section it is assumed that the excess energy comes from the spacetime processes in the extended unit volume, where they are evoked by the transducer. So a calculation has to compare the energy density of the input fields $(\mathbf{E})_{driving}$ or $(\mathbf{B})_{driving}$ to the energy of the total fields being present in the resonance case. The result may depend on whether we consider the energy of the force fields only or whether we include the effects on the spacetime potential $\mathbf{A}$. In the first case we can define the energy densities for input and output:

$$u_{in} = \frac{\epsilon_0}{2}(\mathbf{E}^2)_{driving} + \frac{1}{2\mu_0}(\mathbf{B}^2)_{driving}, \tag{1.68}$$

$$u_{out} = \frac{\epsilon_0}{2}\mathbf{E}^2 + \frac{1}{2\mu_0}\mathbf{B}^2. \tag{1.69}$$

The resulting total energies then are obtained by integrating over the unit volume and time:

$$E_{in} = \int u_{in}\, d^3r\, dt \tag{1.70}$$

$$E_{out} = \int u_{out}\, d^3r\, dt \tag{1.71}$$

and the "coefficient of performance" is

$$COP = \frac{E_{out}}{E_{in}}. \tag{1.72}$$

Alternatively, the output energy can be related to the spacetime potential. From the minimal prescription of momentum density p

$$p \rightarrow p + eA \tag{1.73}$$

we can define the kinetic energy density of the field by

$$u = \frac{e^2 A^2}{2m} \tag{1.74}$$

where m is the "mass" of the field volume. According to the de Broglie equation

$$m = \frac{\hbar\omega}{c^2} \tag{1.75}$$

the mass corresponds to a frequency ω. This leads to the expression

$$u = u_{out} = \frac{e^2 c^2}{2\hbar\omega} A^2. \tag{1.76}$$

1.4.5 Analytical and numerical solutions

The equations to be solved for the model we have developed (Eqs. 1.55, 1.56, 1.60) read

$$\ddot{A}_r + \dot{\omega}_0 A_r + \omega_0 \dot{A}_r = -f_1 \tag{1.77}$$
$$\ddot{A}_\varphi + \dot{\omega}_0 A_\varphi + \omega_0 \dot{A}_\varphi = -f_2 \tag{1.78}$$

$$\omega_0 = -\frac{\dot{A}_r}{A_r}. \tag{1.79}$$

with driving terms $f_1(r)$ and $f_2(r)$. Instead of Eq. (1.79) we can alternatively use its original form (1.58) without application of the divergence theorem:

$$\dot{A}_r + \left(\omega_0 + r\frac{\partial \omega_0}{\partial r}\right) A_r + r\frac{\partial \dot{A}_r}{\partial r} + r\omega_0 \frac{\partial A_r}{\partial r} = 0. \tag{1.80}$$

The difference is that the original form represents a differential equation in r while the r differentiation has vanished in the other form. Thus Eqs. (1.77–1.79) are only to be solved in the time domain which is a great alleviation. In this case Eq. (1.79) can be inserted into (1.77). Then all terms on the left cancel out, leading to the condition

$$f_1 = 0. \tag{1.81}$$

Obviously this is a compatibility condition, indicating that a driving force f_1 cannot be applied. The second Equation (1.78) can be solved analytically by computer algebra and gives the particular solution

$$A_\varphi = \omega_0 f_2 \left(e^{-\omega_0 t} \int \frac{e^{\omega_0 t}}{\omega_0 \dot{\omega}_0 t - \dot{\omega}_0 + \omega_0^2} dt - \int \frac{1}{\omega_0 \dot{\omega}_0 t - \dot{\omega}_0 + \omega_0^2} dt \right) \tag{1.82}$$

As already made plausible in section (1.4.3), this is a resonance equation if the denominator goes to zero. This means that resonances occur at solutions of the differential equation

$$\omega_0 \dot{\omega}_0 t - \dot{\omega}_0 + \omega_0^2 = 0. \tag{1.83}$$

Computer algebra gives for this equation the general solution

$$\omega_0\, t - log\,(\omega_0) = c \tag{1.84}$$

with a constant c. This is a transcendent equation for ω_0. Since c is arbitrary, there is an infinite number of resonances in the whole interval of real numbers for ω_0.

All further investigations are made by a numerical model. As we have seen by analysing Eq. (1.77) the vector potential A_r can be chosen freely. Considering the Bedini machine, such a radial component can only be created by an asymmetric disturbance of the field potential of the transducer coil. This is achieved by the magnets of the wheel passing the transducer. We model these pulses by a sinoidal function:

$$A_r(t) = A_1\, sin^6(\omega t) \tag{1.85}$$

with an arbitrary amplitude A_1 and a time frequency ω. This function and its time derivative are shown in Figs. 1.3 and 1.4 for three frequencies. With this ansatz, Eq. (1.79) takes the form

$$\omega_0(t) = -6\,\omega\, cot(\omega t). \tag{1.86}$$

This function has vertical tangents where the values approach infinity, see Fig. 1.5 for a plot of $|\omega_0|$ for three frequencies in a logarithmic scale. Consequently, the derivative shows also this behaviour (Fig. 1.6).

Eq. (1.78) has been solved numerically for A_φ. The driving force f_2 was assumed to be in proportion to the "symmetry breaking" potential A_r. With ω_0 having the singular behaviour, the solution spans a remarkable order of magnitude and is vulnerable to numerical instabilities. Therefore the solution was checked by inserting it back into Eq. (1.78) and checking for equality with f_2. In all cases the equality was maintained within sufficient precision. The result (Fig. 1.7) shows giant resonance peaks over 15 orders of magnitude which occur in coincidence with the structure of ω_0. Obviously these peaks correspond to the peak signals in the Bedini machine. The time frequency is to be identified with the passing rate of the magnets over the transducer. To make comparison even more appropriate, in Fig. (1.8) the derivative of A_φ is shown which should correspond to the induced voltage

$$U_{ind} = -\dot{A}_\varphi. \tag{1.87}$$

The structure is very similar to that of A_φ itself.

Next we have tested the dependence of the solution on the driving force f_2. It results that A_φ is practically insensitive to the form of f_2, provided the value is different from zero where ω_0 has its poles. It is even sufficient to take

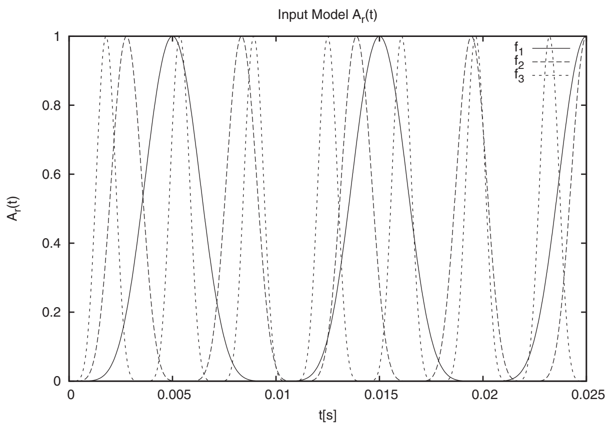

Fig. 1.3. Radial component of vector potential A_r for three frequencies f=50 Hz, 90 Hz, 140 Hz.

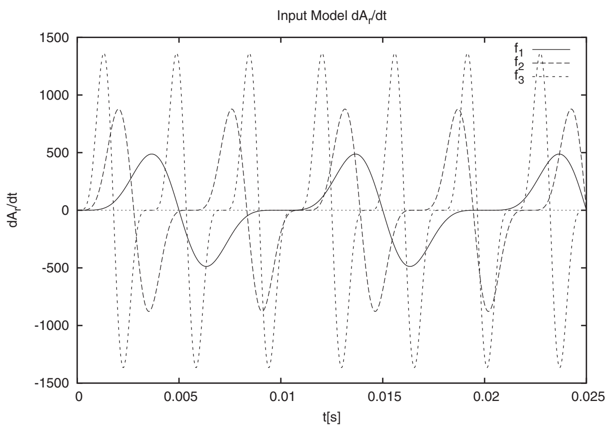

Fig. 1.4. Time derivative $\dot{A}_r$ for three frequencies f=50 Hz, 90 Hz, 140 Hz.

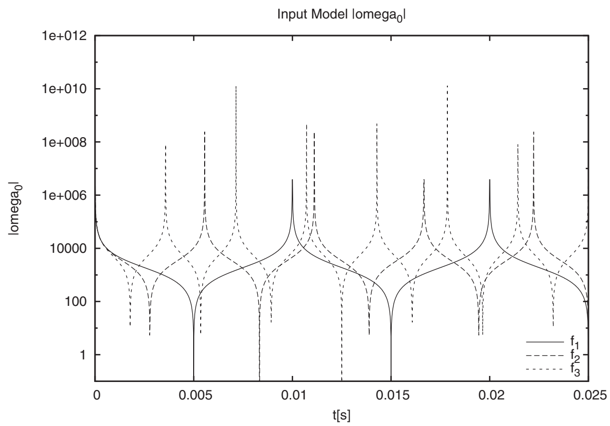

Fig. 1.5. $|\omega_0|$ for three frequencies f=50 Hz, 90 Hz, 140 Hz.

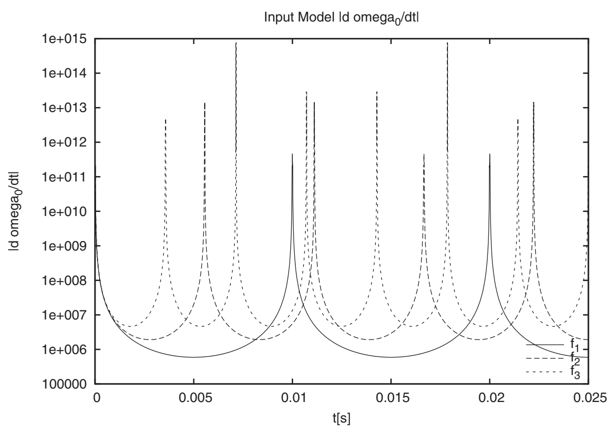

Fig. 1.6. Time derivative $|\dot{\omega}_0|$ for three frequencies f=50 Hz, 90 Hz, 140 Hz.

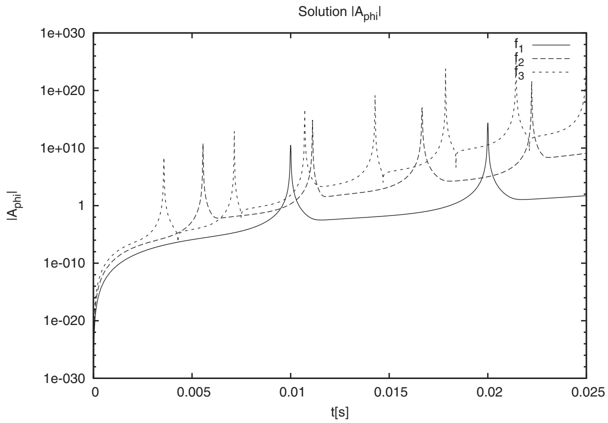

Fig. 1.7. Solution $|A_\varphi(t)|$ for three frequencies f=50 Hz, 90 Hz, 140 Hz.

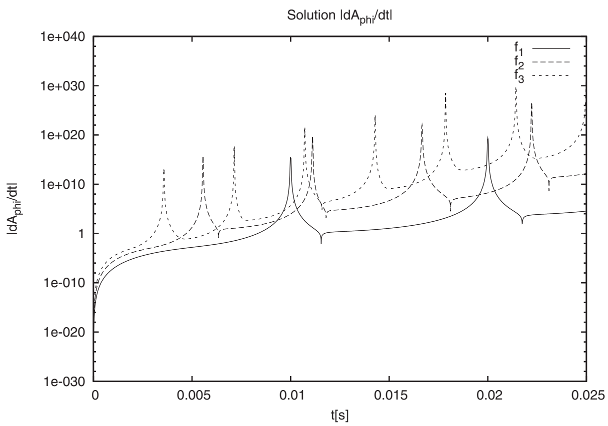

Fig. 1.8. Time derivative $|\dot{A}_\varphi(t)|$ for three frequencies f=50 Hz, 90 Hz, 140 Hz.

a spike pulse of one percent of the time period. Fig. 1.9 shows the result for a constant value of f_2.

Since the zero crossing of A_r is essential for the resonances, we have modified Eq. (1.85) by adding a constant value of 0.001, thus displacing the curve of Fig. 1.3 by this value from zero. The result (Fig. 1.10) shows a far smaller resonance structure indicating that resonances are very sensitive to the form of A_r via ω_0.

Next we inspect the development of the maximum amplitude. In Fig. 1.11 the maximum difference over the first six time periods is plotted in dependence of the time frequency. Obviously the resonance is most dramatic for low frequencies. In the next figure (Fig. 1.12) the maximum amplitude difference was recorded over a constant simulated time of 0.1 sec. To avoid numerical instabilities inferred by the calculation we used a modified A_r input value as discussed for Fig. 1.10 (shifted by 0.1 upwards, no zero crossing). Solutions are stable in the low frequency range but there are windows of unstability for higher frequencies. We argue that the differential equation (1.78) can show chaotic behaviour and must be carefully evaluated.

Finally we present the amount of transferred energy integrated over time. According to Eq. (1.76) this is proportional to

$$u(t) = \int_0^t \frac{A_\varphi^2(t')}{\omega} dt'. \tag{1.88}$$

This term is represented in Fig. 1.13. Since A_φ crosses zero at the resonances (remember that the modulus is shown in the figures), a considerable amount of energy is pushed back to the vacuum after having been transferred to the system, but there is enough energy left after each resonance peak so that the energy in the system rises considerably.

As a last item in this section let us consider the radius dependence of the fields which can not be determined from Eqs. (1.77–1.79) as discussed above. Therefore let's start from Eqs. (1.77) and (1.80):

$$\ddot{A}_r + \dot{\omega}_0 A_r + \omega_0 \dot{A}_r = -f_1 \tag{1.89}$$

$$\dot{A}_r + \left(\omega_0 + r\frac{\partial \omega_0}{\partial r}\right) A_r + r\frac{\partial \dot{A}_r}{\partial r} + r\omega_0 \frac{\partial A_r}{\partial r} = 0 \tag{1.90}$$

We will make an ansatz for A_r and compute the solution for ω_0 which is compatible with this. We choose

$$A_r = C\, e^{-\alpha r - i\beta t} \tag{1.91}$$

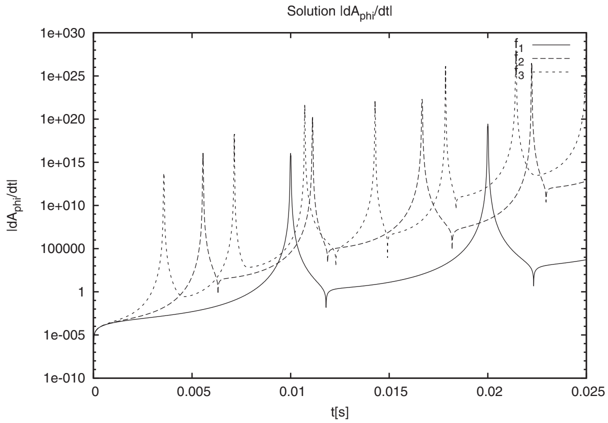

Fig. 1.9. Time derivative $|\dot{A}_\varphi(t)|$ with $f_2 = 1$ for three frequencies f=50 Hz, 90 Hz, 140 Hz.

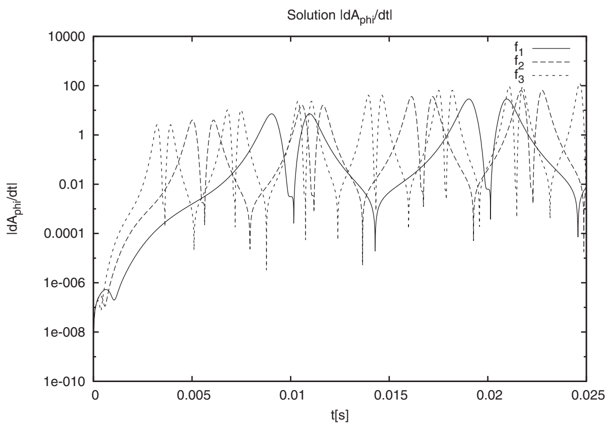

Fig. 1.10. Time derivative $|\dot{A}_\varphi(t)|$ with A_r shifted by 0.001 for three frequencies f=50 Hz, 90 Hz, 140 Hz.

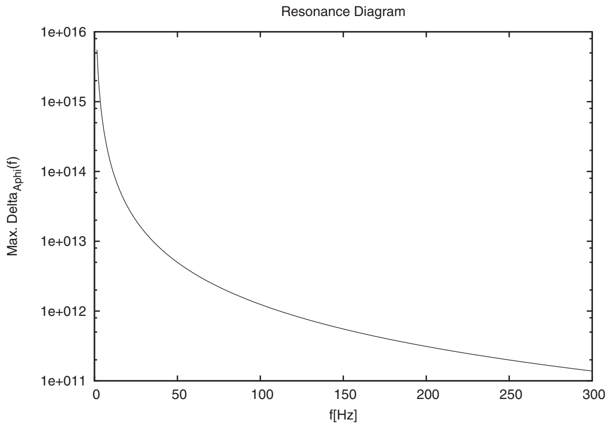

Fig. 1.11. Resonance behaviour: amplitude of A_φ after 6 periods of frequency f.

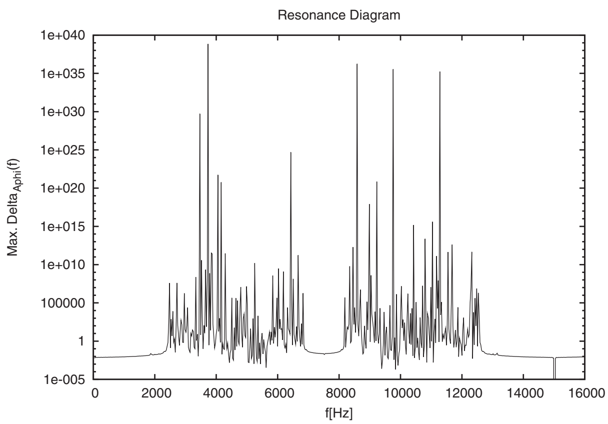

Fig. 1.12. Resonance behaviour: maximum amplitude of A_φ within 0.1 sec runtime, A_r shifted by 0.1.

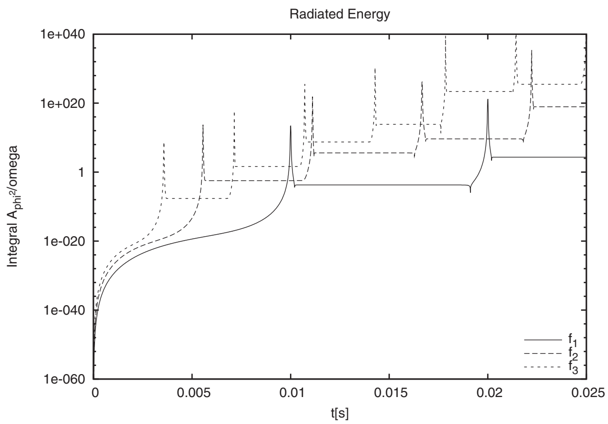

Fig. 1.13. Integral over radiated energy for three frequencies f=50 Hz, 90 Hz, 140 Hz.

which is a conventional approach for a radially decreasing vector potential which oscillates in time with frequency β. Inserting this into Eq. (1.89) results in a differential equation for ω_0 in the variable t:

$$-\left(\beta^2 + i\,\omega_0\,\beta - \dot{\omega}_0\right)\,e^{-\alpha\,r - i\,\beta}\,C = -f_1 \tag{1.92}$$

The solution of this equation is

$$\omega_0(t) = c(r)\,e^{i\beta t} + i\,\beta - \frac{f_1 t}{C} e^{\alpha r + i\,\beta t} \tag{1.93}$$

with a constant $c(r)$ which is dependent on r in general. Inserting (1.93) into (1.90) yields

$$(i\,\alpha\,\beta\,r - \omega_0\,\alpha\,r + \dot{\omega}_0\,r - i\,\beta + \omega_0)\;e^{-\alpha\,r - i\,\beta\,t}\,C = 0 \tag{1.94}$$

which has the solution

$$\omega_0(r) = \frac{c(t)}{r}\,e^{\alpha\,r} + i\,\beta. \tag{1.95}$$

Since both solutions (1.93) and (1.95) must be compatible, we have to assume

$$c(r) = 0. \tag{1.96}$$

By comparison of both equations for ω_0 (1.93 and 1.95) we find

$$c(t) = -\frac{f_1\, t}{C} e^{i\beta\, t} \tag{1.97}$$

and finally

$$\omega_0(r) = -\frac{f_1\, t}{C} e^{\alpha\, r + i\beta\, t} + i\beta. \tag{1.98}$$

We see that the spin connection has a diverging behaviour in space as well as in time which is consistent with the results of the numerical model.

1.4.6 Summary and discussion

The Bedini device has been analysed by analytical and numerical methods. Based on a model of cylindrical symmetry of the transducer, which is considered to be the essential part for spacetime coupling, the following mechanism of spacetime interaction could be identified:

Under undisturbed conditions, the magnetic field in the transducer is cylindrically symmetric. The radial part of the vector potential must vanish due to the Gauss law. The passing magnets of the wheel distort the symmetry of the magnetic field in the transducer by inducing an asymmetric signal. This leads to a radial component of the vector potential which was not present before. The vector potential changes in time and therefore induces an electric field. Consequently, the Coulomb law has to be fulfilled as an additional condition, in this case for a vanishing charge density (the electric field is completely a radiated field). The ECE field equations show that for the Coulomb law the radial component of the vector potential has either to be zero, or the scalar spin connection must exist to compensate a non-vanishing radial component of the vector potential. The latter case is fulfilled in the Bedini device and leads to the observed resonant behaviour.

A model has been developed which takes a timely varying radial vector potential A_r as a given input. By means of the Coulomb law, a spin connection ω_0 is produced. The zero crossings of A_r lead to a singular value of the spin connection, leading in turn to very high values of the φ component of the vector potential. These are the giant resonances which are strong enough in practice to transfer significant amounts of energy from spacetime to the machine.

For some spacetime resonance experiments it is reported that there is a glowing or fluorescent light effect around the apparatus when it is at resonance. At the same time the measured current of the driving mechanism takes a minimum. This can qualitatively be explained by analyzing the contributions of the ECE electric charge current density. It is given in general by

$$J = (\tilde{R} \wedge A - \omega \wedge \tilde{F}) \tag{1.99}$$

(in short hand notation) for the Hodge duals of curvature $\tilde{R}$ and the electromagnetic field $\tilde{F}$ as well as the potential A. At spin connection resonance, it may happen that the term $\omega \wedge \tilde{F}$ outweights the curvature term. Then the charge current can become significantly smaller while the region of space has a high energy density due to the large spin connection term. Obviously no "negative energy" is required to explain the effect.

The only experimental feature which cannot be directly related to our model is the required behaviour of the driving current. According to Bedini, the motor pulse acts as driving force for the spacetime resonance and must be very short and sharp without oscillations. The model calculations showed that the form of the driving force is not important as long as it is different from zero at the diverging time positions of the spin connection.

Based on the results of this paper we can give some recommendations for further investigations and improvements of the Bedini design, under the prerequisite that our model is correct:

1. The vector potential A_r has to be provided in a way to have zero crossings. This could be enforced by positioning magnets with alternating polarity on the wheel.

2. Since mechanical parts limit the lifetime of a device, a design without moving parts is desirable. The principles of the design can be retained by replacing the wheel by a rotating electromagnetic field (for example based on a three-phase AC voltage). Then arbitrary rotation frequencies can be applied without mechanical restrictions.

3. Effects of the asymmetry of the signal inducing the resonance should be investigated. For example it could be tested if a linear motion of a magnet perpendicular to the transducer would evoke resonance effects too.

A

Appendix 1: Reduction of Form Notation to Vector Notation

In differential form notation the electromagnetic field in ECE theory is:

$$F^a = d \wedge A^a + \omega^a_b \wedge A^b \tag{A.1}$$

which in tensor notation is 1–10:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \omega^a_{\mu b} A^b_\nu - \omega^a_{\nu b} A^b_\mu. \tag{A.2}$$

The electromagnetic potential is:

$$A^a_\mu = A^{(0)} q^a_\mu \tag{A.3}$$

where q^a_μ is a rank two mixed index tensor defined by:

$$V^a = q^a_\mu V^\mu. \tag{A.4}$$

Here V^a and V^μ are four vectors in different frames of reference labeled a and μ in four dimensional space-time. Consider a particular example of Eq. (A.2):

$$F^1_{23} = \partial_2 A^1_3 - \partial_3 A^1_2 + \omega^1_{2b} A^b_3 - \omega^1_{3b} A^b_2. \tag{A.5}$$

Either side of the equation there are rank three tensors whose components must correspond to each other on both sides. Thus:

$$F^1_{23} = \left(\partial_2 A_3 - \partial_3 A_2\right)^1 + \left(\omega_{2b} A^b_3 - \omega_{3b} A^b_2\right)^1. \tag{A.6}$$

Inside the brackets on the right hand side are anti-symmetric tensor components which correspond to the components of an axial vector (magnetic field) or polar vector (electric field). The magnetic vector components are defied by:

$$B^1_i = \frac{1}{2}\epsilon_{ijk}F^1_{jk} \tag{A.7}$$

thus:

$$B^1_1 = \frac{1}{2}\left(\epsilon_{123}F_{23} + \epsilon_{132}F_{32}\right)^1 = F^1_{23}. \tag{A.8}$$

This is recognized as the X component:

$$B_X = B^1_1 \tag{A.9}$$

of the magnetic field:

$$\mathbf{B} = B_X\mathbf{i} + B_Y\mathbf{j} + B_Z\mathbf{k}. \tag{A.10}$$

Similarly:

$$B_Y = B^2_2 = F^2_{31}, \tag{A.11}$$

$$B_Z = B^3_3 = F^3_{12}. \tag{A.12}$$

These results were checked by computer in paper 93 of the ECE series [1–10]. So Eq. (A.6) becomes:

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} - \boldsymbol{\omega}_b \times \mathbf{A}^b. \tag{A.13}$$

In this notation:

$$\left(\boldsymbol{\omega}_b \times \mathbf{A}^b\right)_X = \left(\omega_{3b}A^b_2 - \omega_{2b}A^b_3\right)^1 \tag{A.14}$$

where the minus sign has been introduced following the usage of previous papers.

These results are obtained in the special case:

$$a = \mu \tag{A.15}$$

in Eq. (A.4). This means that the vectors V^a and V^μ are written in the same frame of reference. Thus q^a_μ is diagonal in this special case:

$$V^0 = q^0_0V^0,\ V^1 = q^1_1V^1,\ V^2 = q^2_2V^2,\ V^3 = q^3_3V^3, \tag{A.16}$$

and from Eq. (A.3), A^a_μ must be diagonal also. So in Eq. (A.14)

$$\left(\boldsymbol{\omega}_b \cdot \mathbf{A}^b\right)_X = \left(\omega_{32}A^2_2 - \omega_{23}A^3_3\right)^1 = \omega^1_{32}A^2_2 - \omega^1_{23}A^3_3. \tag{A.17}$$

Similarly:

$$\left(\boldsymbol{\omega}_b \times \mathbf{A}^b\right)_Y = \omega^2_{13}A^3_3 - \omega^2_{31}A^1_1, \tag{A.18}$$

$$\left(\boldsymbol{\omega}_b \times \mathbf{A}^b\right)_Z = \omega^3_{21}A^1_1 - \omega^3_{12}A^2_2. \tag{A.19}$$

Therefore the meaning of the b index is given by Eqs. (A.17) to (A.19). The final result is:

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A} \tag{A.20}$$

as used in previous papers on SCR [1]- [10]. The spin connection has been reduced here to a vector $\boldsymbol{\omega}$. The components of this vector in analogy with Eqs. (A.9) to (A.12) are:

$$\omega_X = \omega^1_1 = \omega^1_{32} = -\omega^1_{23}, \tag{A.21}$$
$$\omega_Y = \omega^2_2 = \omega^2_{31} = -\omega^2_{13}, \tag{A.22}$$
$$\omega_Z = \omega^3_3 = \omega^3_{12} = -\omega^3_{21}. \tag{A.23}$$

So:

$$\boldsymbol{\omega} = \omega_X\mathbf{i} + \omega_Y\mathbf{j} + \omega_Z\mathbf{k}. \tag{A.24}$$

Finally if we adopt the complex circular basis [1]- [10]:

$$\mathbf{B}^{(3)*} = \boldsymbol{\nabla} \times \mathbf{A}^{(3)*} - i\boldsymbol{\omega}^{(1)} \times \mathbf{A}^{(2)} \tag{A.25}$$

and if:

$$\boldsymbol{\omega}^{(1)} = g\mathbf{A}^{(1)} \tag{A.26}$$

we obtain the $\mathbf{B}^{(3)*}$ spin field:

$$\mathbf{B}^{(3)*} = -ig\mathbf{A}^{(1)} \times \mathbf{A}^{(2)}. \tag{A.27}$$

B

Appendix 2: Derivation of the Electric Field in Vector Notation

For the electric field we consider:

$$F^i_{0i} = (\partial_0 A_i - \partial_i A_0)^1 + \omega^i_{0i} A^i_i - \omega^i_{i0} A^0_0,\ \ i = 1, 2, 3 \tag{B.1}$$

which is equivalent in vector notation to:

$$\mathbf{E} = -\boldsymbol{\nabla}\phi - \frac{\partial \mathbf{A}}{\partial t} - c\omega^0 \mathbf{A} + c\phi\boldsymbol{\omega}. \tag{B.2}$$

Therefore

$$-\left(\boldsymbol{\nabla}\phi + \frac{\partial \mathbf{A}}{\partial t}\right)_X = (\partial_0 A_1 - \partial_1 A_0)^1\,, \tag{B.3}$$

$$-\left(\boldsymbol{\nabla}\phi + \frac{\partial \mathbf{A}}{\partial t}\right)_Y = (\partial_0 A_2 - \partial_2 A_0)^2\,, \tag{B.4}$$

$$-\left(\boldsymbol{\nabla}\phi + \frac{\partial \mathbf{A}}{\partial t}\right)_Z = (\partial_0 A_3 - \partial_3 A_0)^3\,, \tag{B.5}$$

and

$$-\left(c\omega^0 \mathbf{A} - c\phi\boldsymbol{\omega}\right)_X = \omega^1_{01} A^1_1 - \omega^1_{10} A^0_0, \tag{B.6}$$

$$-\left(c\omega^0 \mathbf{A} - c\phi\boldsymbol{\omega}\right)_Y = \omega^2_{02} A^2_2 - \omega^2_{20} A^0_0, \tag{B.7}$$

$$-\left(c\omega^0 \mathbf{A} - c\phi\boldsymbol{\omega}\right)_Z = \omega^3_{03} A^3_3 - \omega^3_{30} A^0_0. \tag{B.8}$$

Thus:

$$\mathbf{A} = A_X \mathbf{i} + A_Y \mathbf{j} + A_Z \mathbf{k}. \tag{B.9}$$

where

$$A_X = A^1_1,\ A_Y = A^2_2,\ A_Z = A^3_3. \tag{B.10}$$

and

$$\boldsymbol{\omega} = \omega_X \mathbf{i} + \omega_Y \mathbf{j} + \omega_Z \mathbf{k}. \tag{B.11}$$

where

$$\omega_X = \omega^1_{10},\ \omega_Y = \omega^2_{20},\ \omega_Z = \omega^3_{30}. \tag{B.12}$$

The scalar part of the spin connection is defined by:

$$c\omega^0 = -\omega^1_{01} = -\omega^2_{02} = -\omega^3_{03} \tag{B.13}$$

and the scalar potential is defined by:

$$c\phi = -A^0_0. \tag{B.14}$$

So the electric and magnetic field in general relativity (ECE theory) are:

$$\mathbf{E} = -\boldsymbol{\nabla}\phi - \frac{\partial \mathbf{A}}{\partial t} - c\omega^0 \mathbf{A} + c\boldsymbol{\omega}\phi. \tag{B.15}$$

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A}. \tag{B.16}$$

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